Planar k-NodeTrix Graphs
A New Family of Beyond Planar Graphs

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- *Planar NodeTrix representations* are hybrid planar drawings where clusters of vertices are represented by adjacency matrices [1, 2, 3]
- *Planar k-NodeTrix graphs* admit a planar NodeTrix representation where the matrices have size at most k
- The *planar NodeTrix number* of a graph G, denoted as $nt(G)$, is the minimum k for which G is planar k-NodeTrix

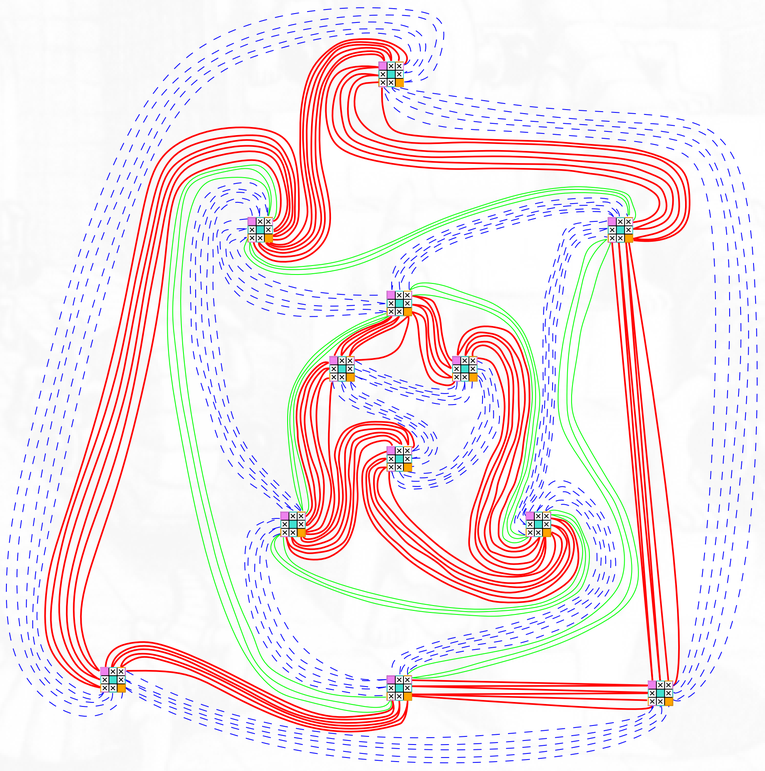
Maximum density of some families of beyond planar graphs

1-planar graphs	$m \leq 4n - 8$ [4]
NIC-planar graphs	$m \leq 3.6n - 7.2$ [4]
IC-planar graphs	$m \leq 3.25n - 6$ [4]

What is the relationship between Planar 2-NodeTrix graphs and 1-planar graphs?

Maximum density of planar k-NodeTrix graphs

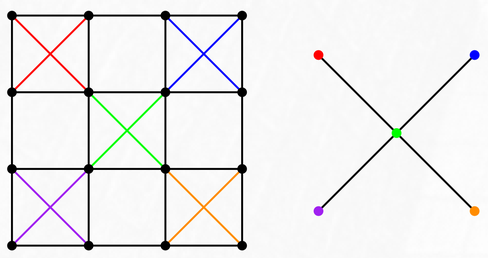
- A planar k-NodeTrix graph $(k \geq 2)$ has at most
$$m = n\left(\frac{k}{2} + \frac{7}{2} + \frac{1}{k}\right) - 6 \quad \text{edges}$$
- This bound is tight for infinitely many values of n



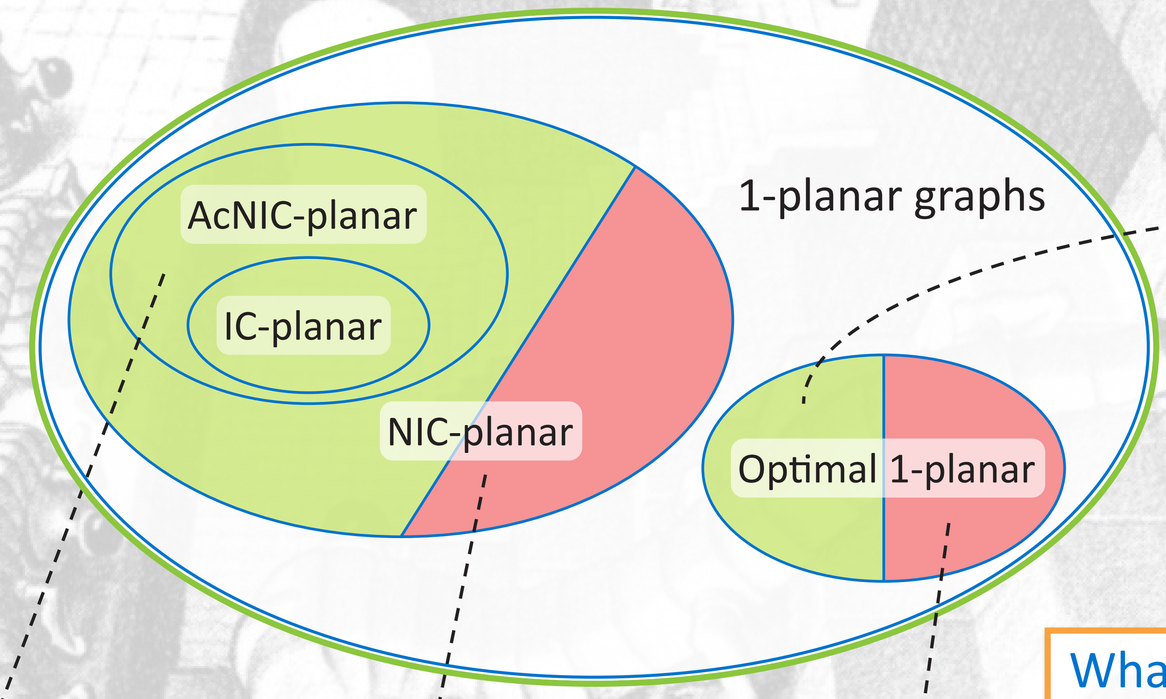
- Planar 2-NodeTrix graphs $(k = 2) \Longrightarrow m \leq 5n - 6$

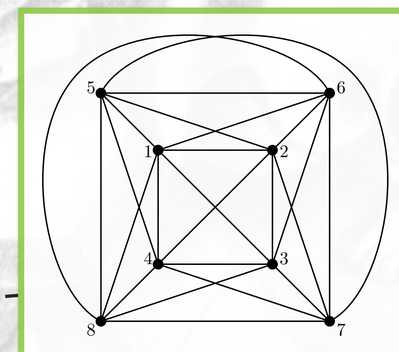
AcNIC-planar graphs

- The *crossing pairs graph* (*cp-graph*) of G is a graph with a vertex w_i for each pair $< e_i, f_i >$ of crossing edges e_i and f_i, and an edge between w_i and w_j if the pairs $< e_i, f_i >$ and $< e_j, f_j >$ share a vertex.
- An *AcNIC-planar* graph is a NIC-planar graph whose *cp*-graph is acyclic.

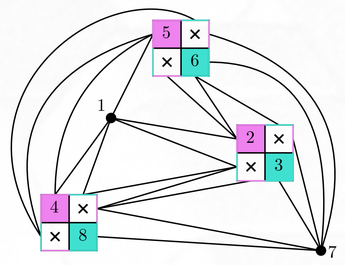


AcNIC-planar graph ⟶ *cp*-graph



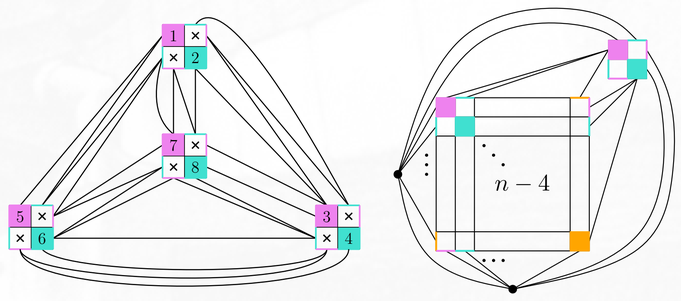


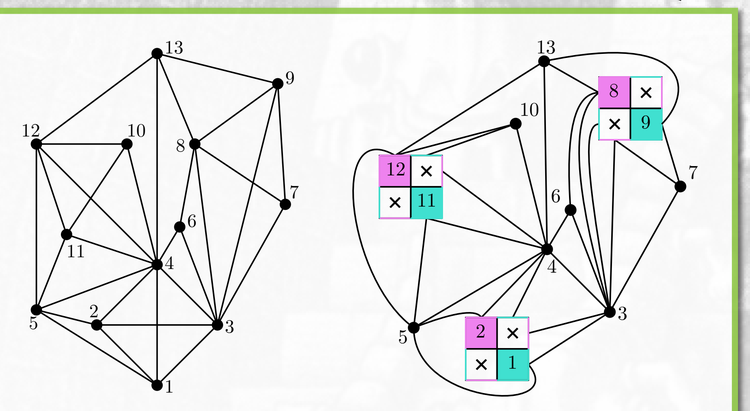
Planar NodeTrix number = 2



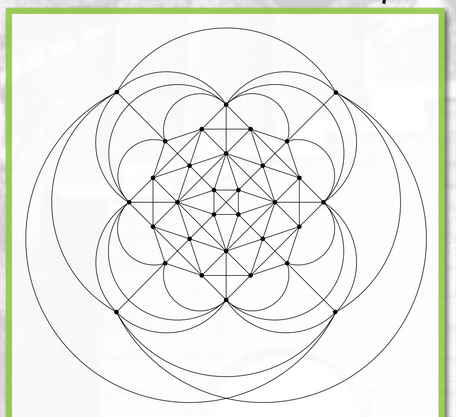
What about **complete graphs**?

- $5 \leq n \leq 8 \Longrightarrow nt(K_n) = 2$
- $8 < n < 32 \Longrightarrow nt(K_n) \leq n - 4$
- $n \geq 32 \Longrightarrow nt(K_n) = n - 4$

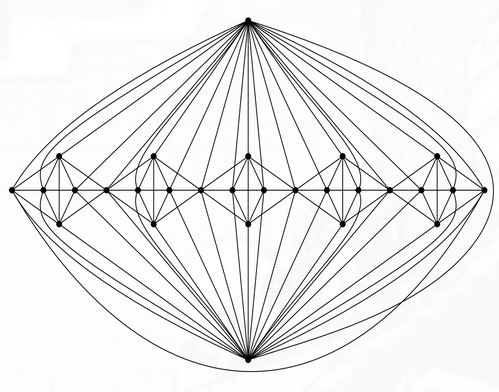




Planar NodeTrix number = 2



Planar NodeTrix number > 2



Planar NodeTrix number > 2

Open problems

- What is the planar NodeTrix number of RAC graphs?
- What is the planar NodeTrix number of K_n for $8 < n < 32$?

References

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[4] Kobourov, S.G., Liotta, G., Montecchiani, F.: An annotated bibliography on 1-planarity. CoRR abs/1703.02261 (2017)