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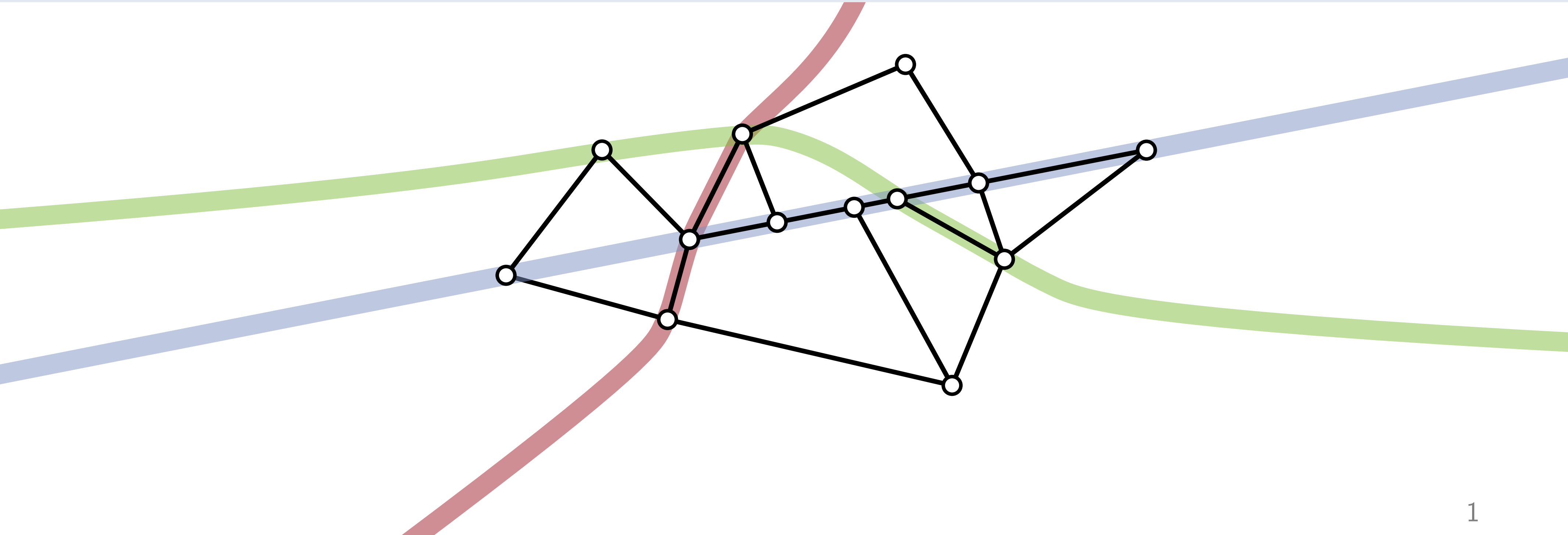
Marcel Radermacher
Karlsruhe Institute of Technology

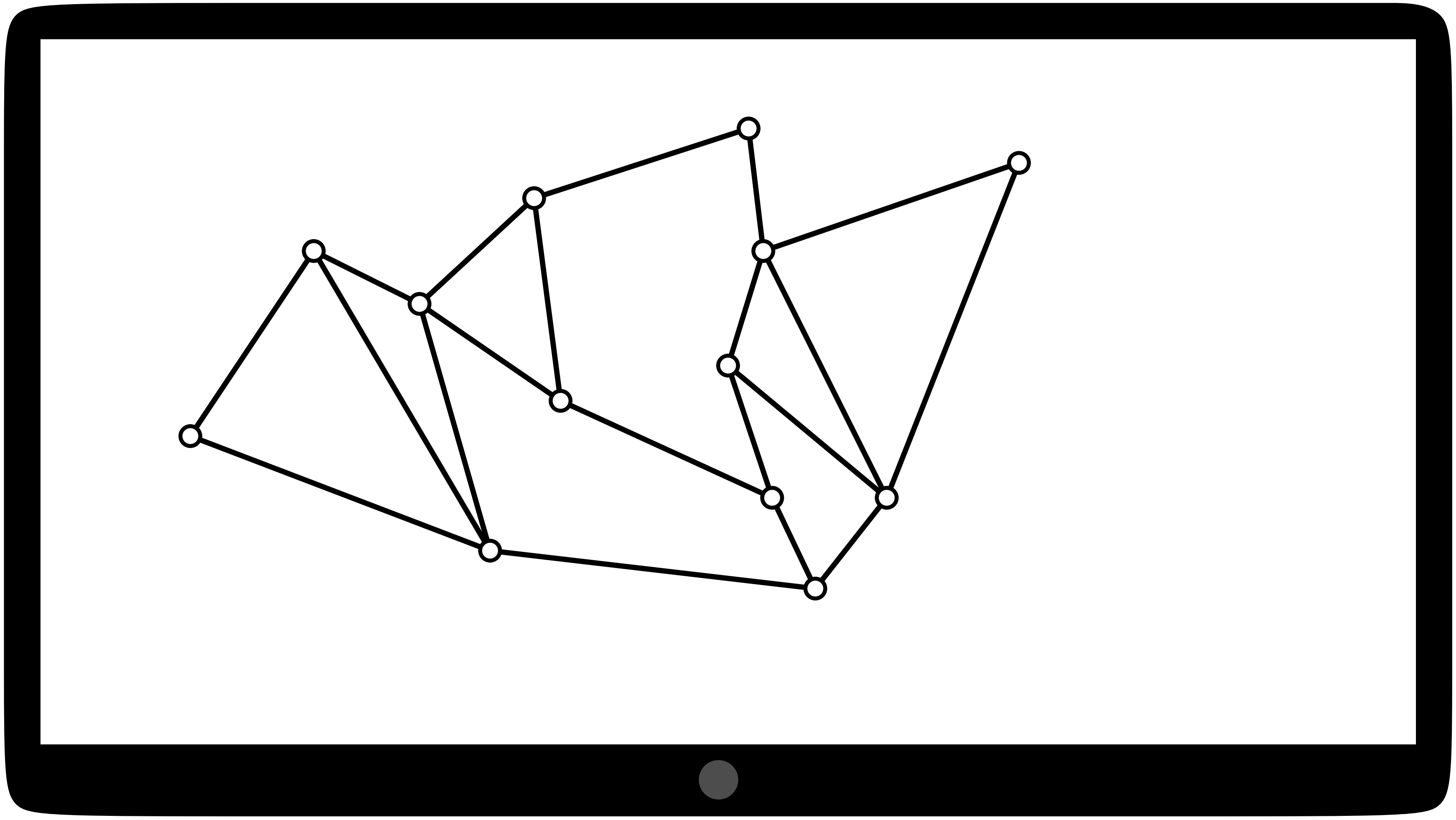
Ignaz Rutter
TU Eindhoven

Aligned Drawings of Planar Graphs

GD'17 · Sept. 25, 2017



Aligned Drawings of Graphs



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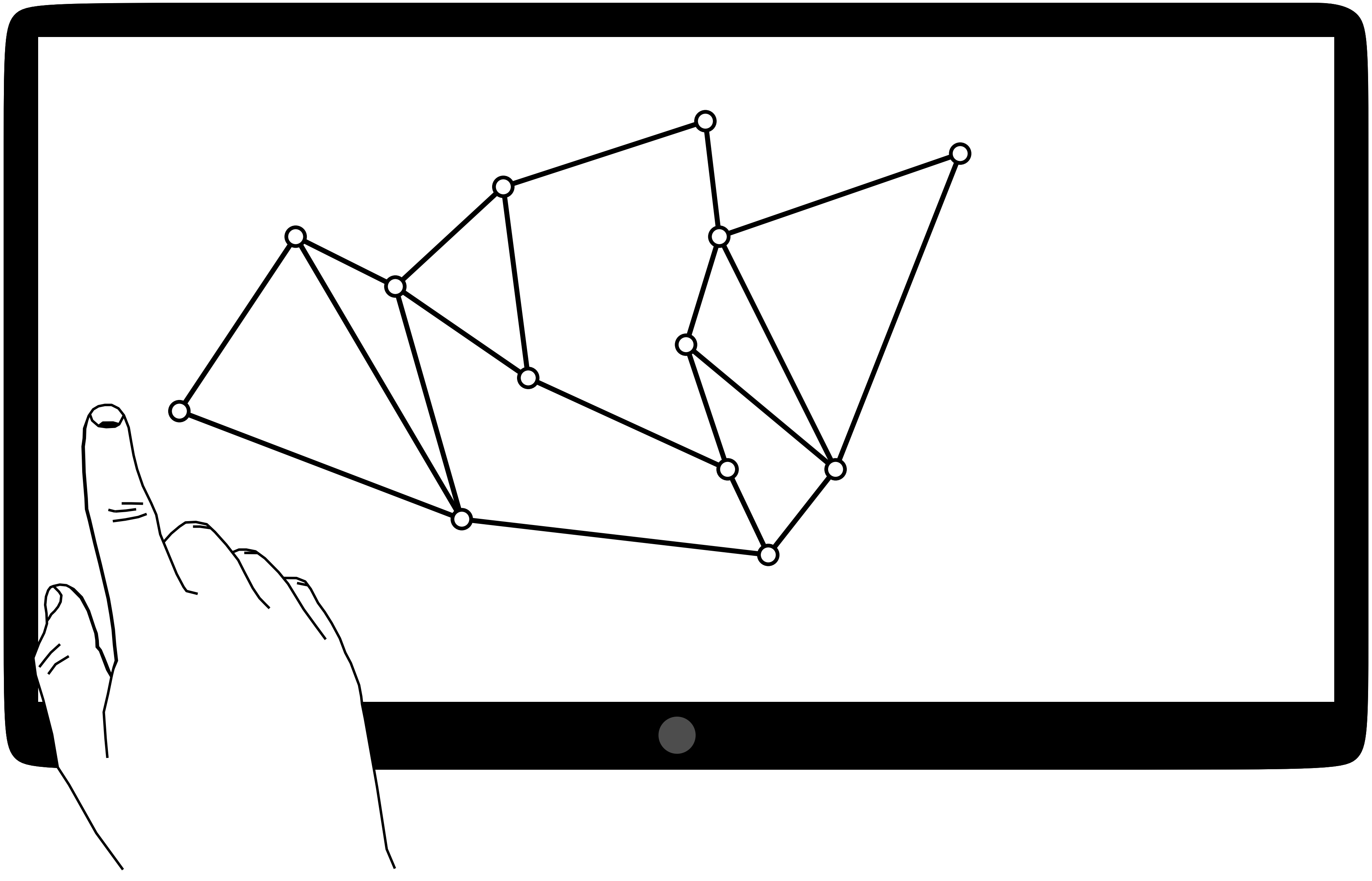

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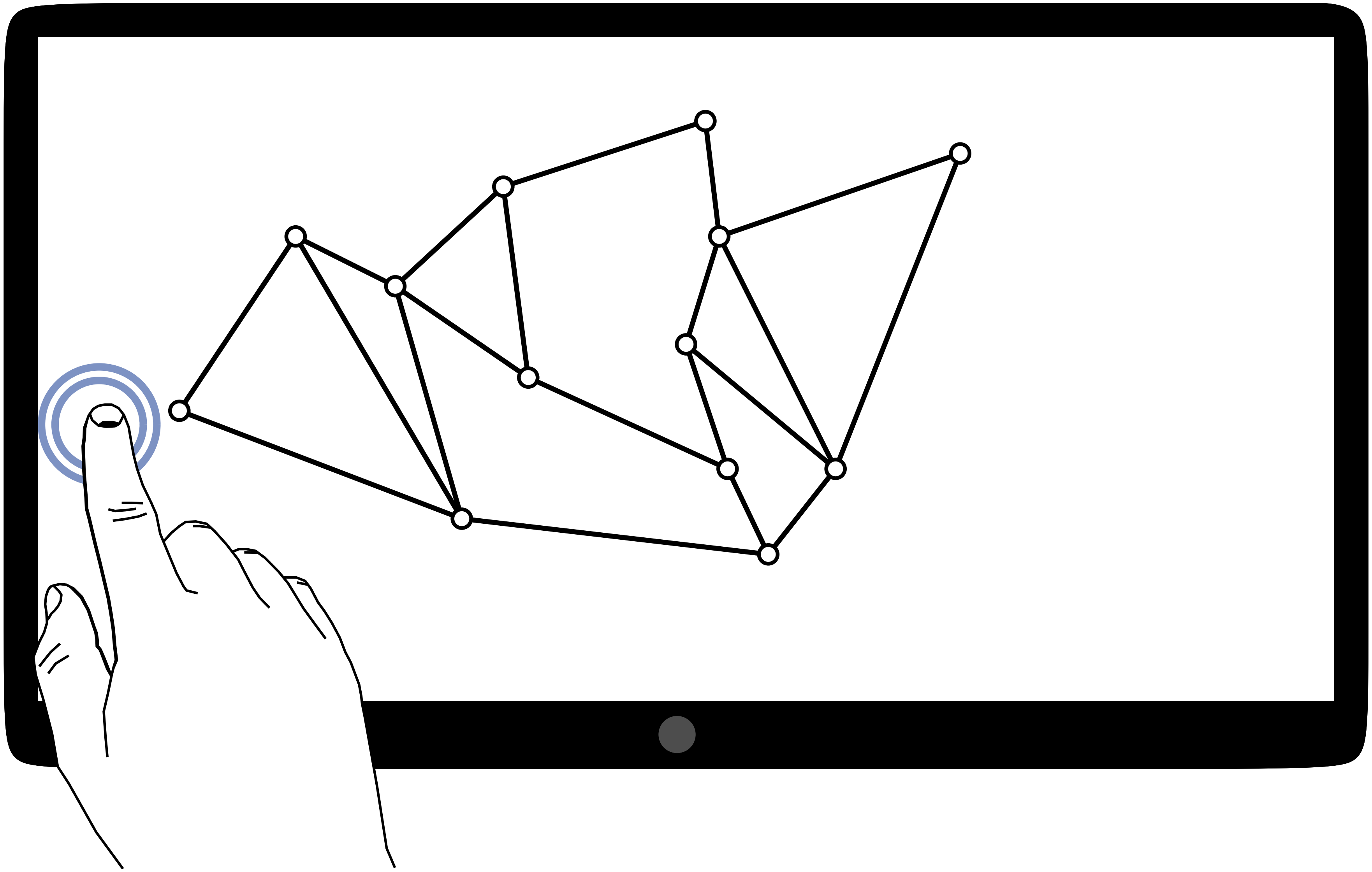

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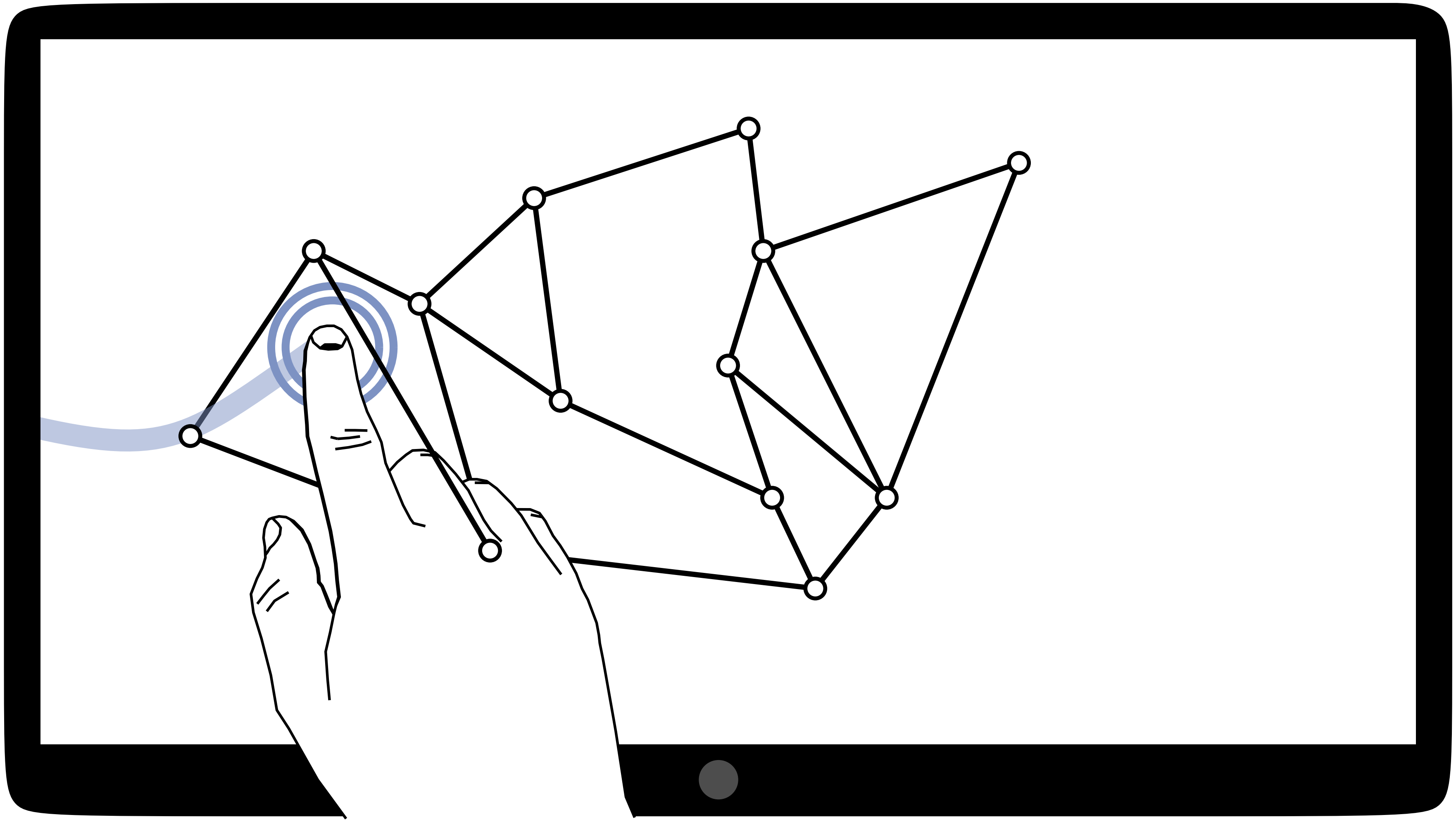

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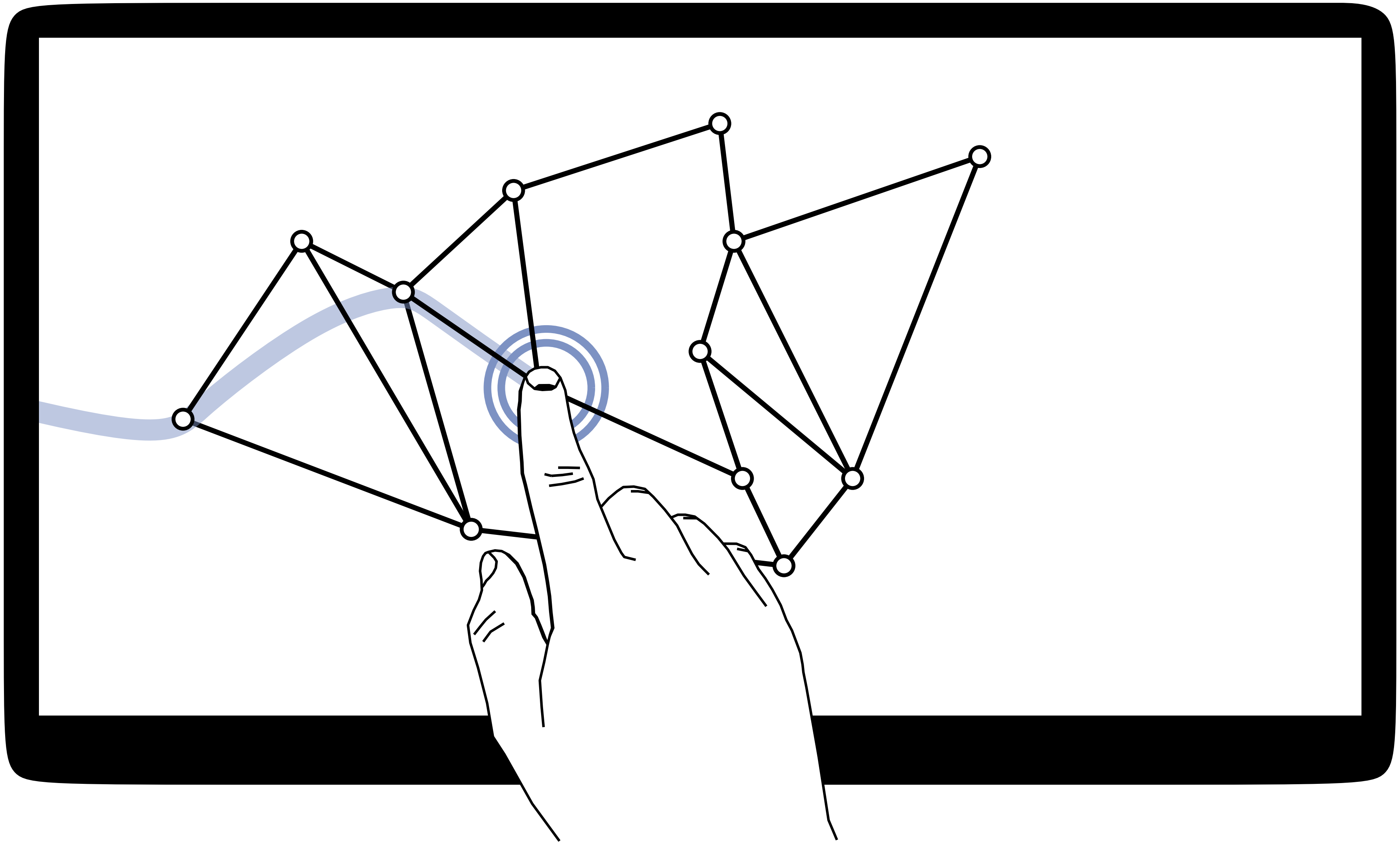

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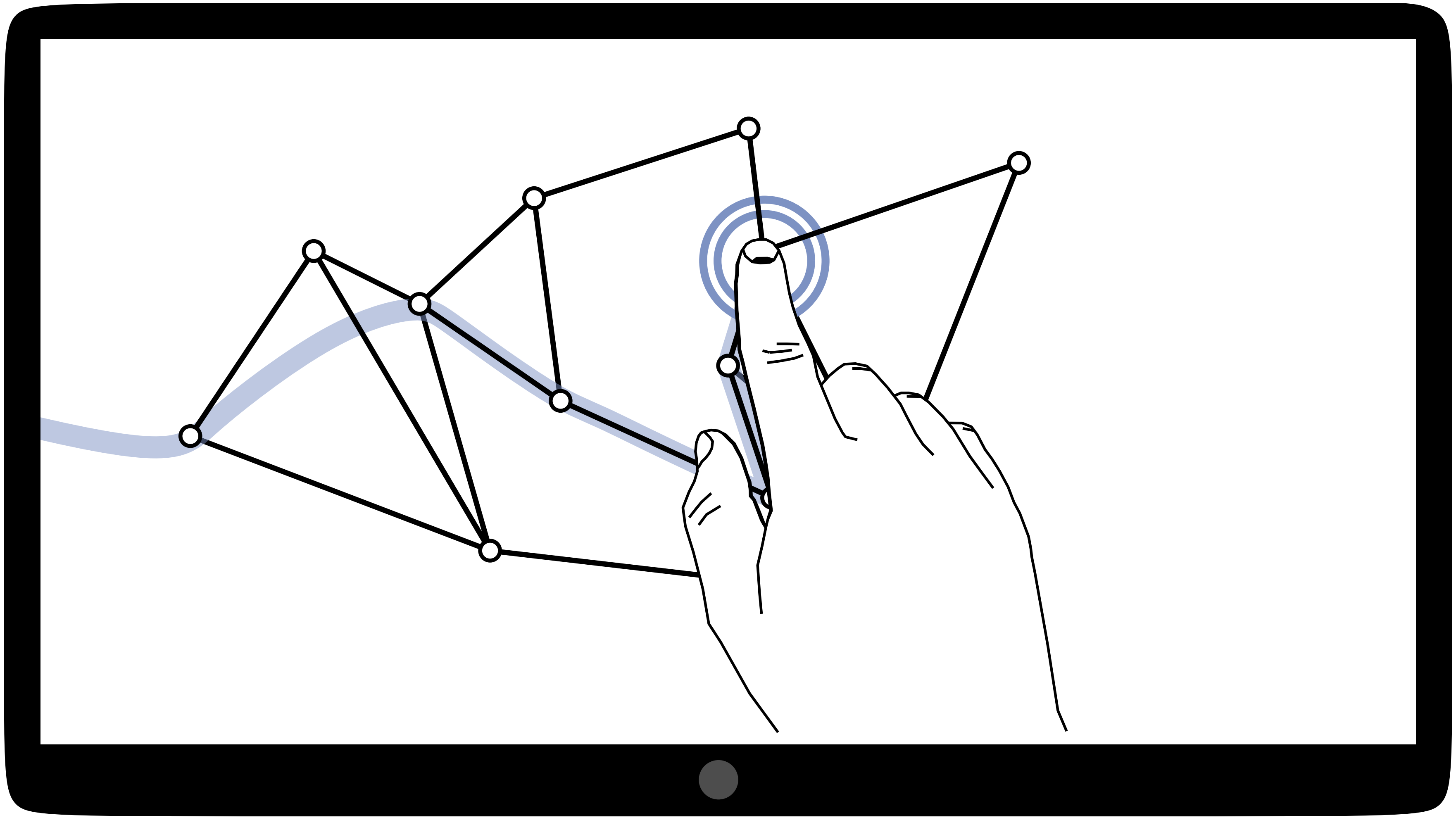

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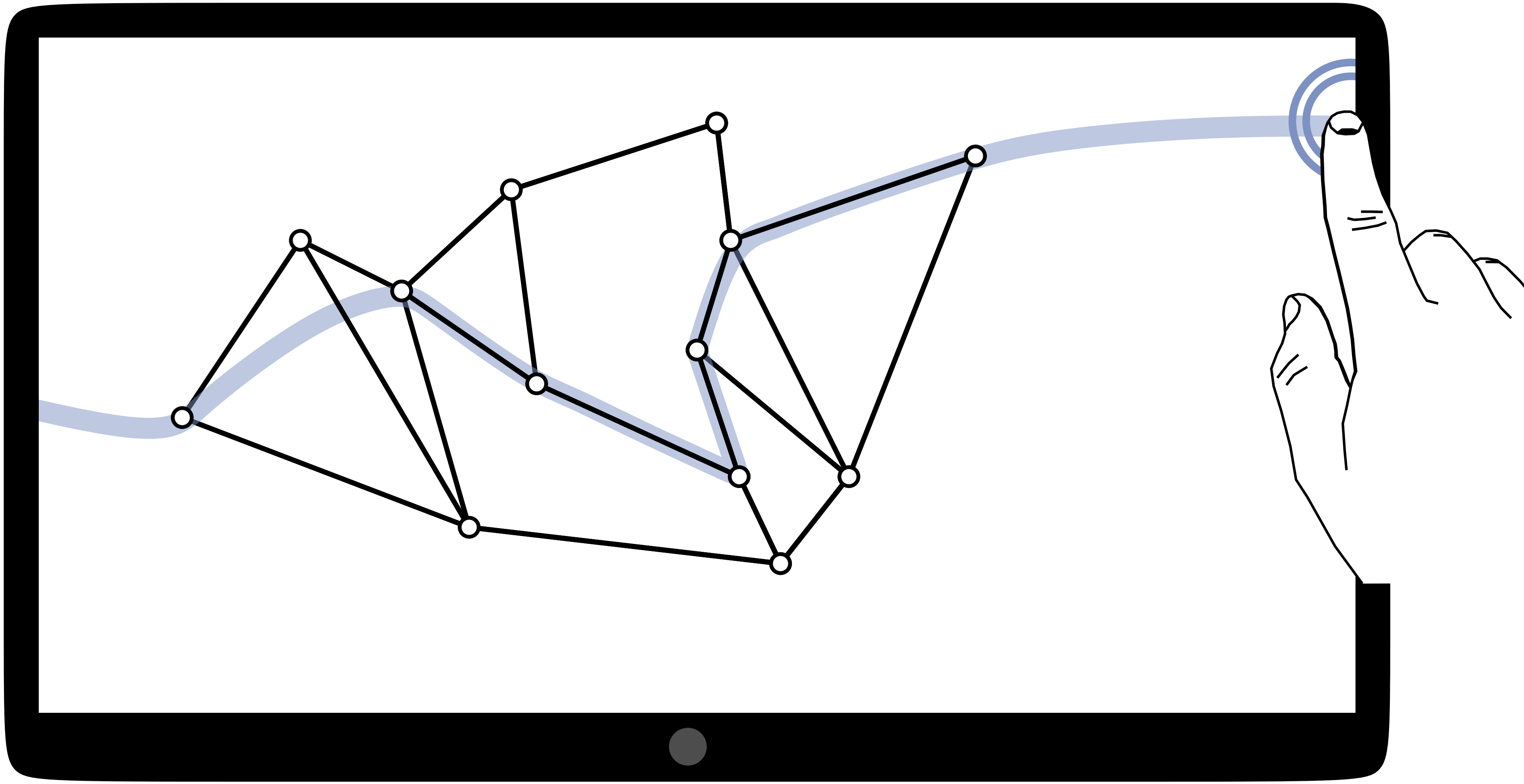

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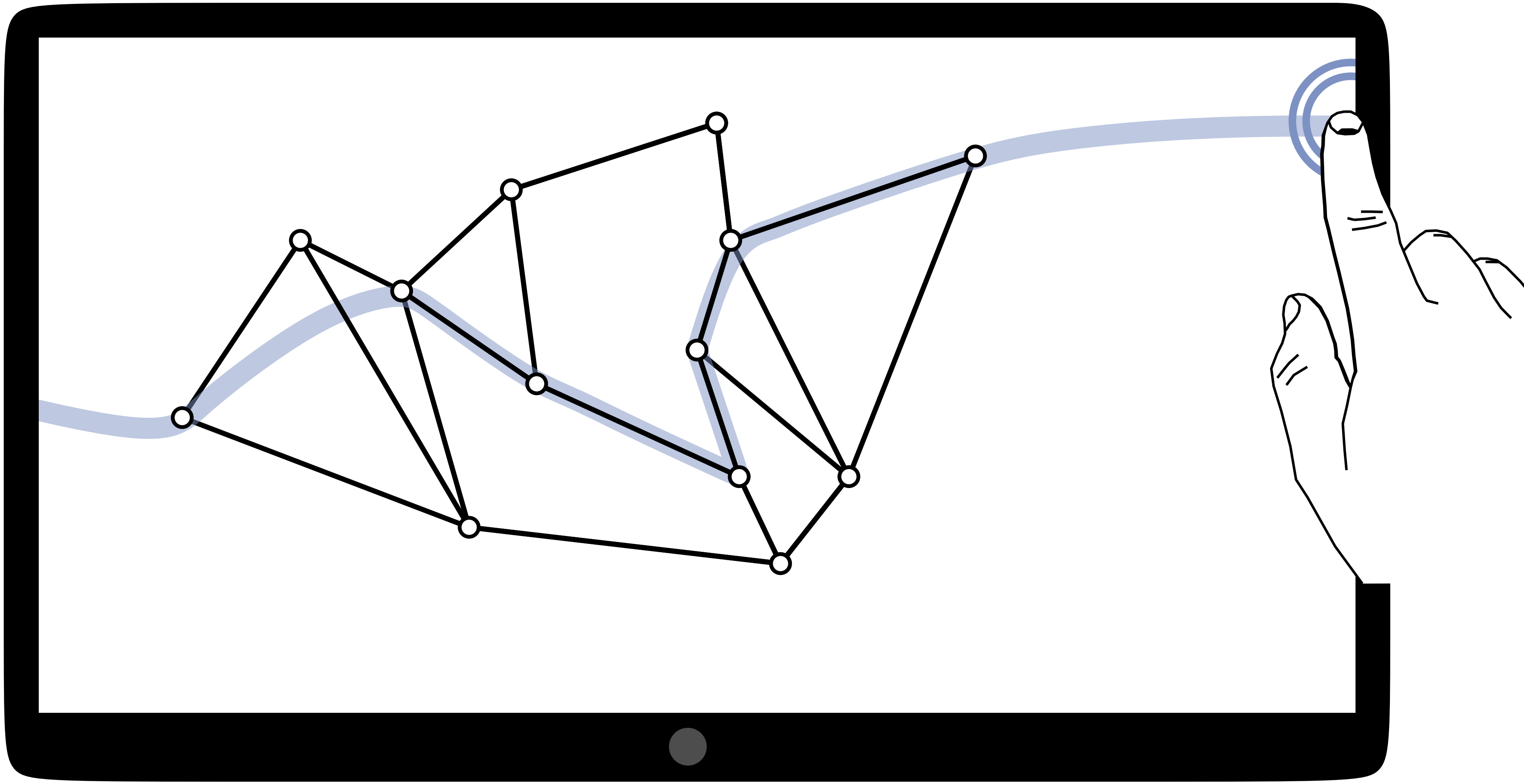

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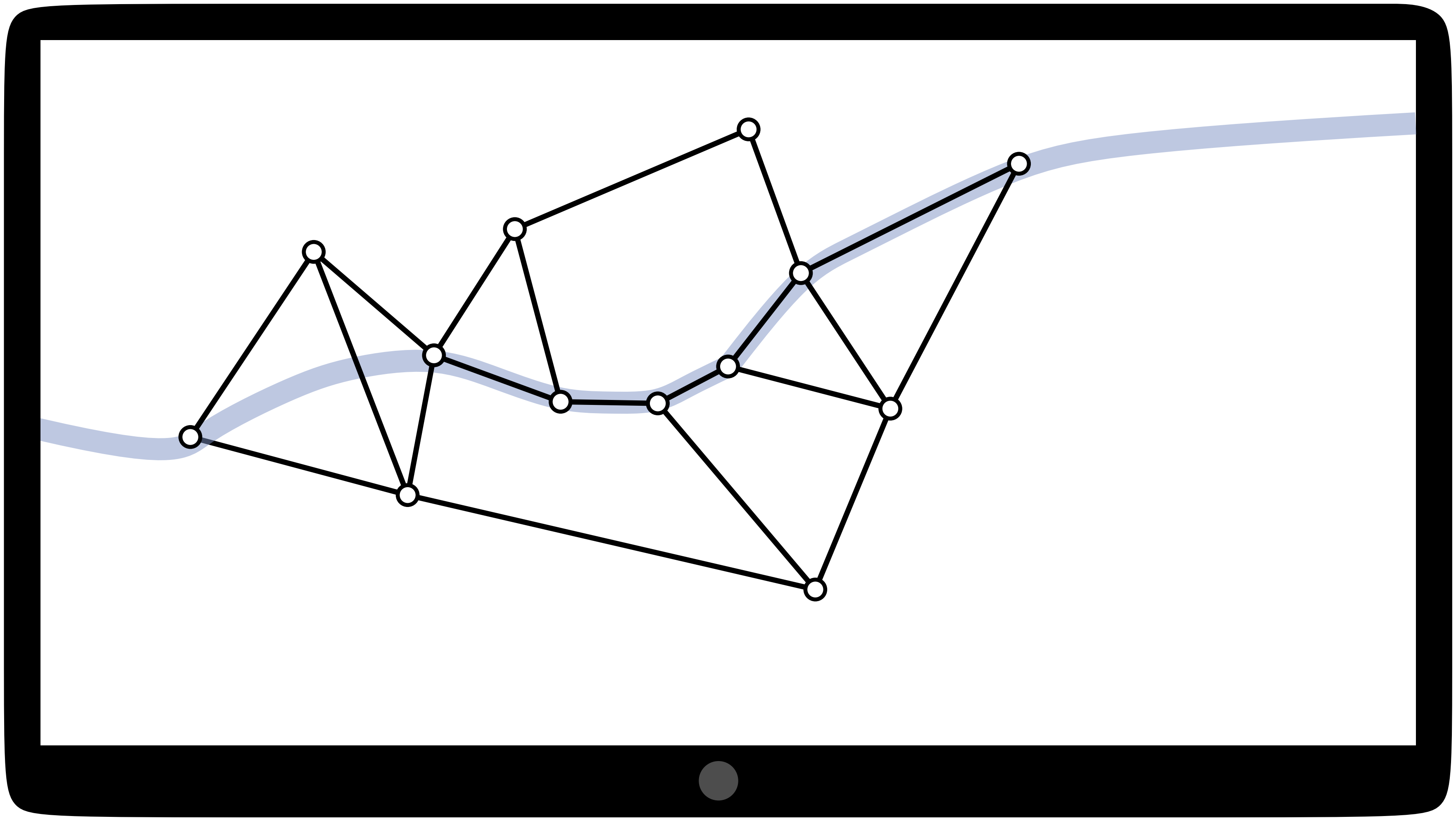

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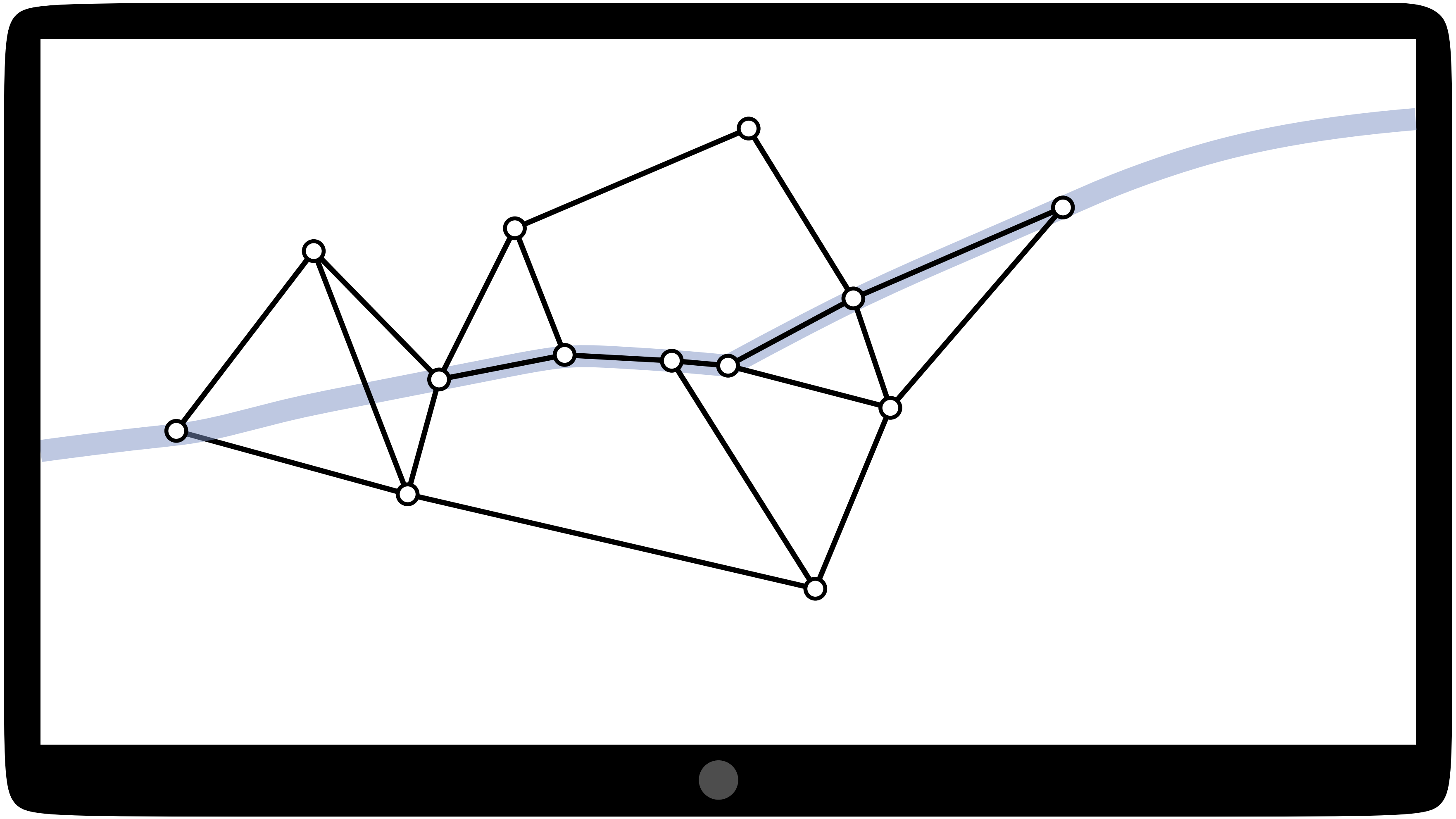

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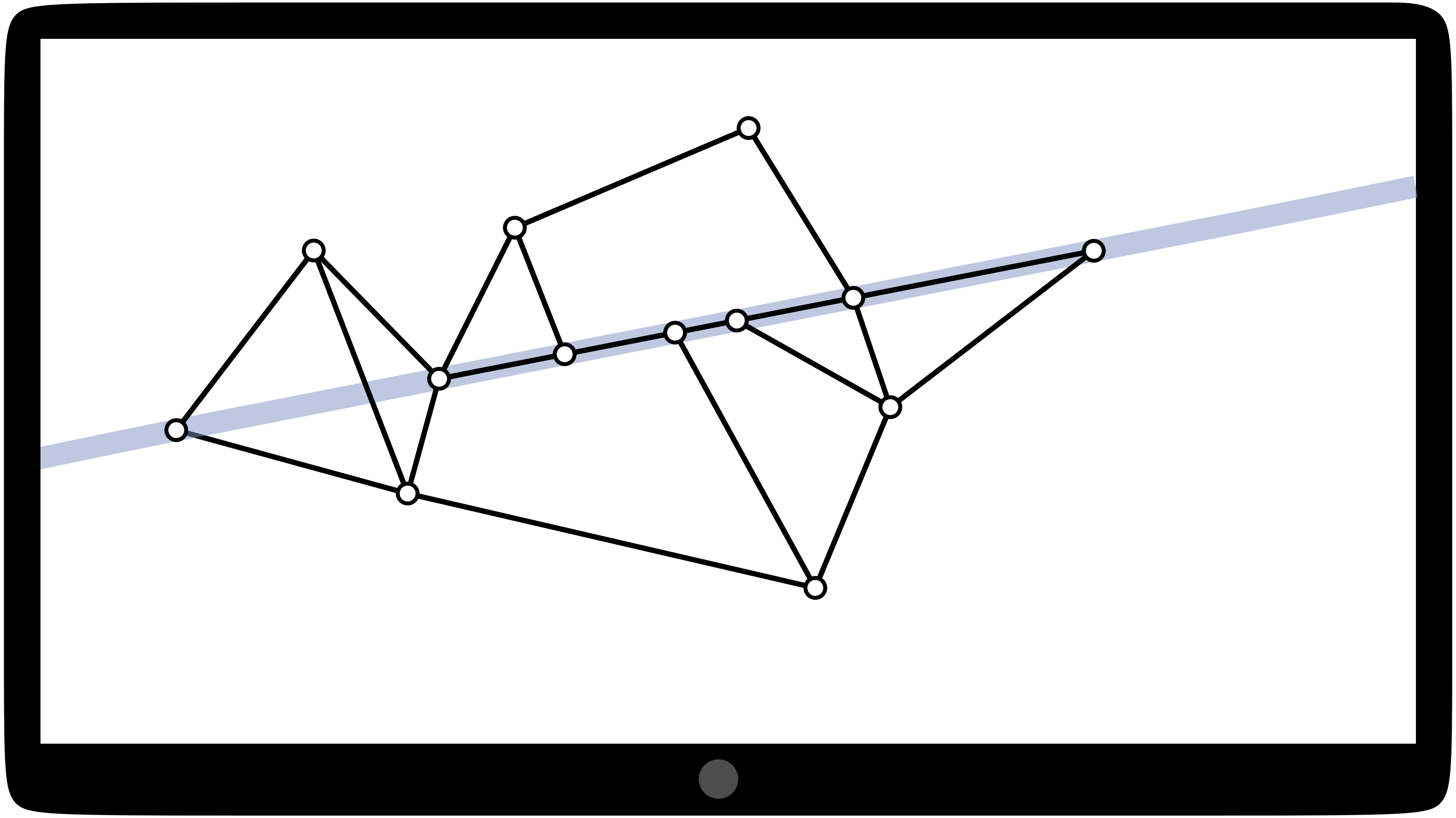

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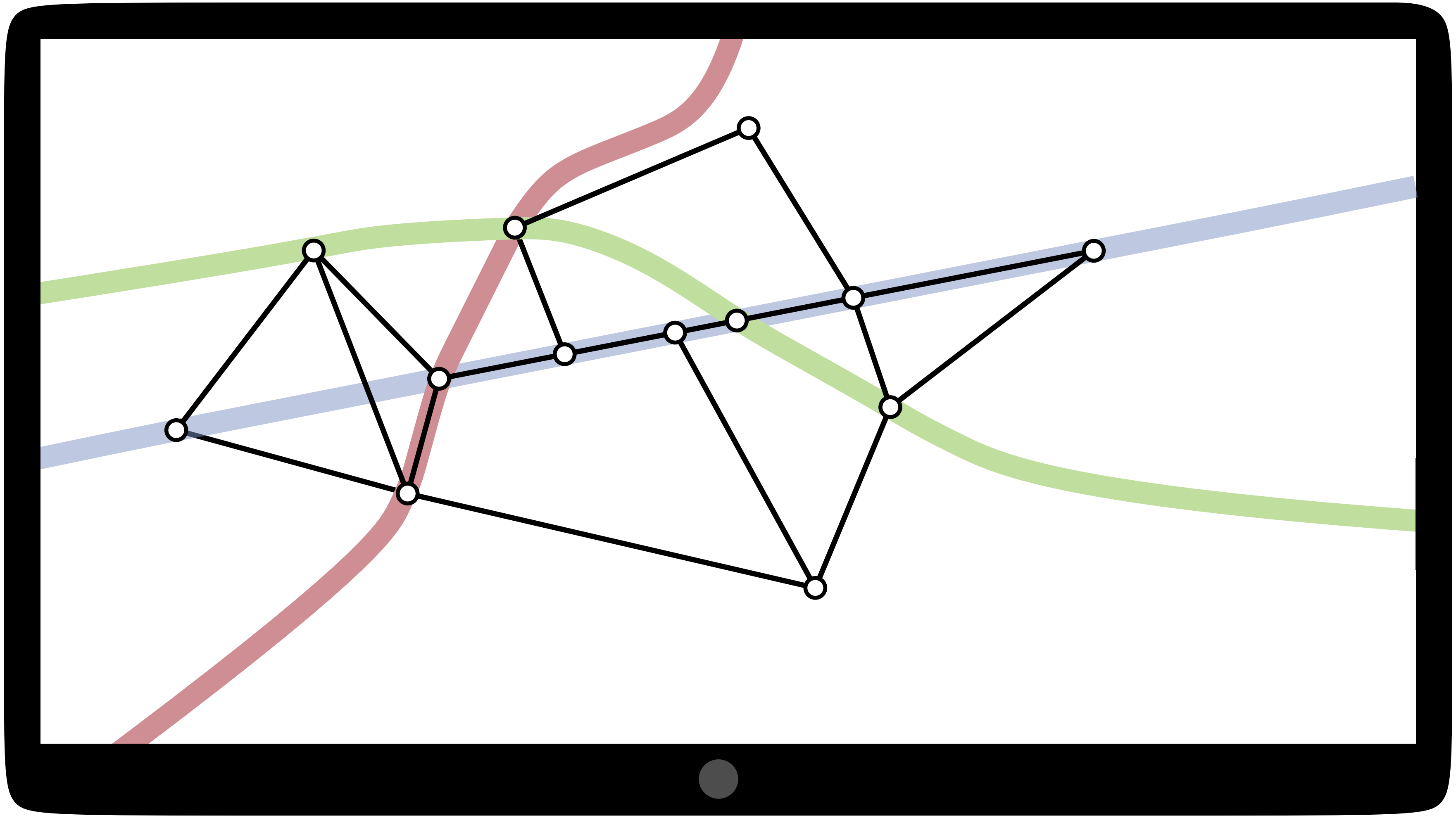
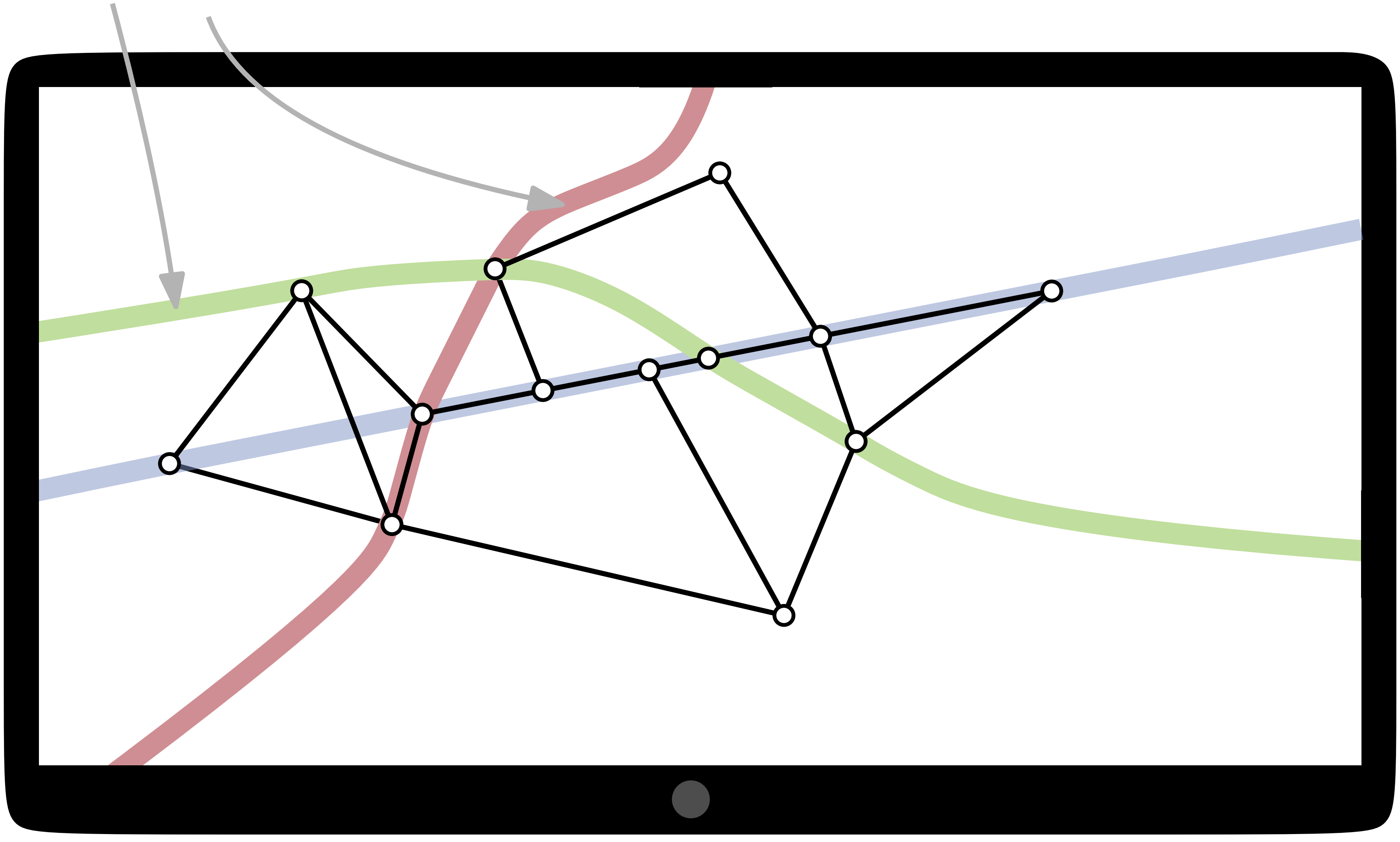


Aligned Drawings of Graphs

pseudolines



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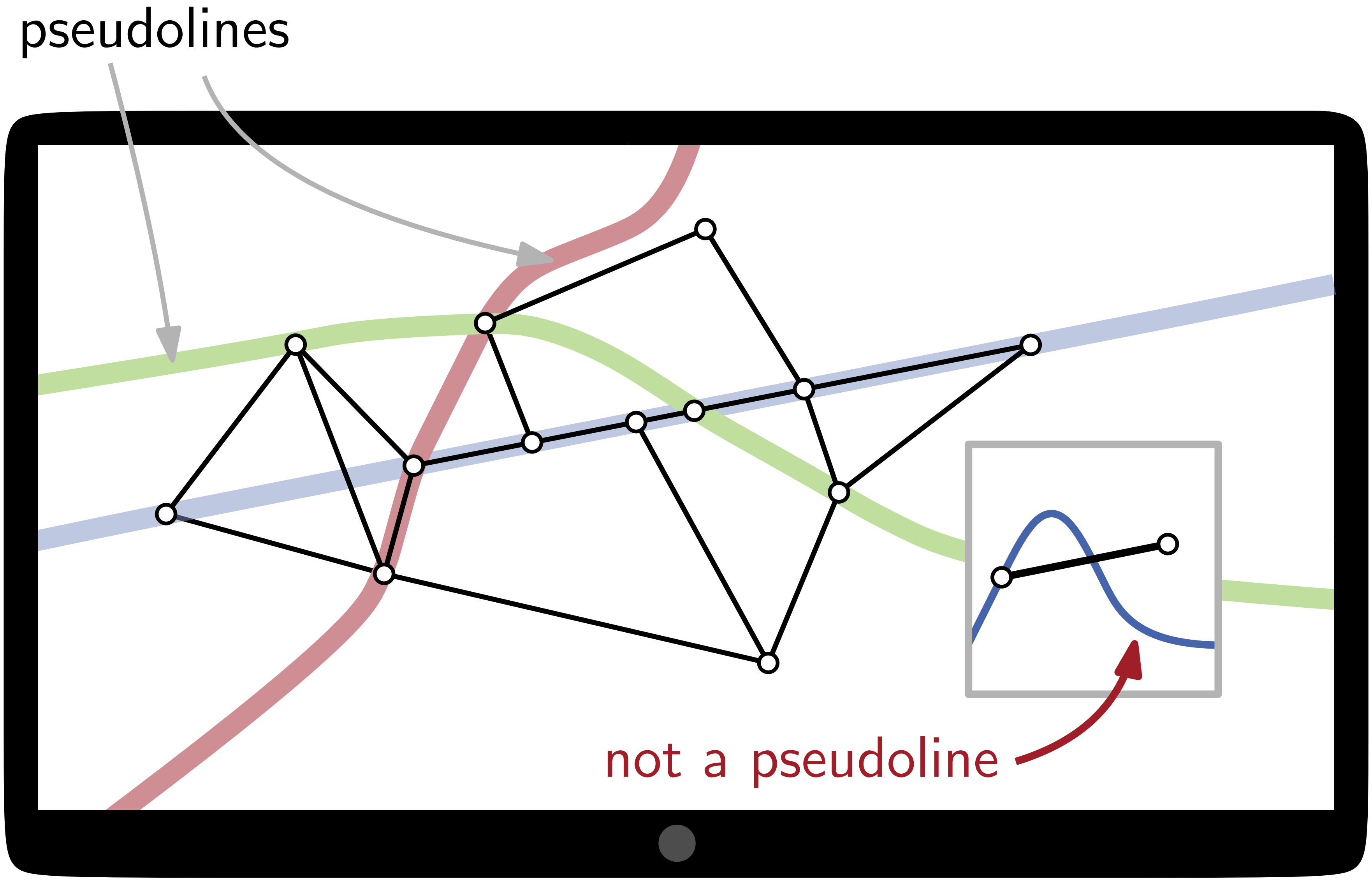

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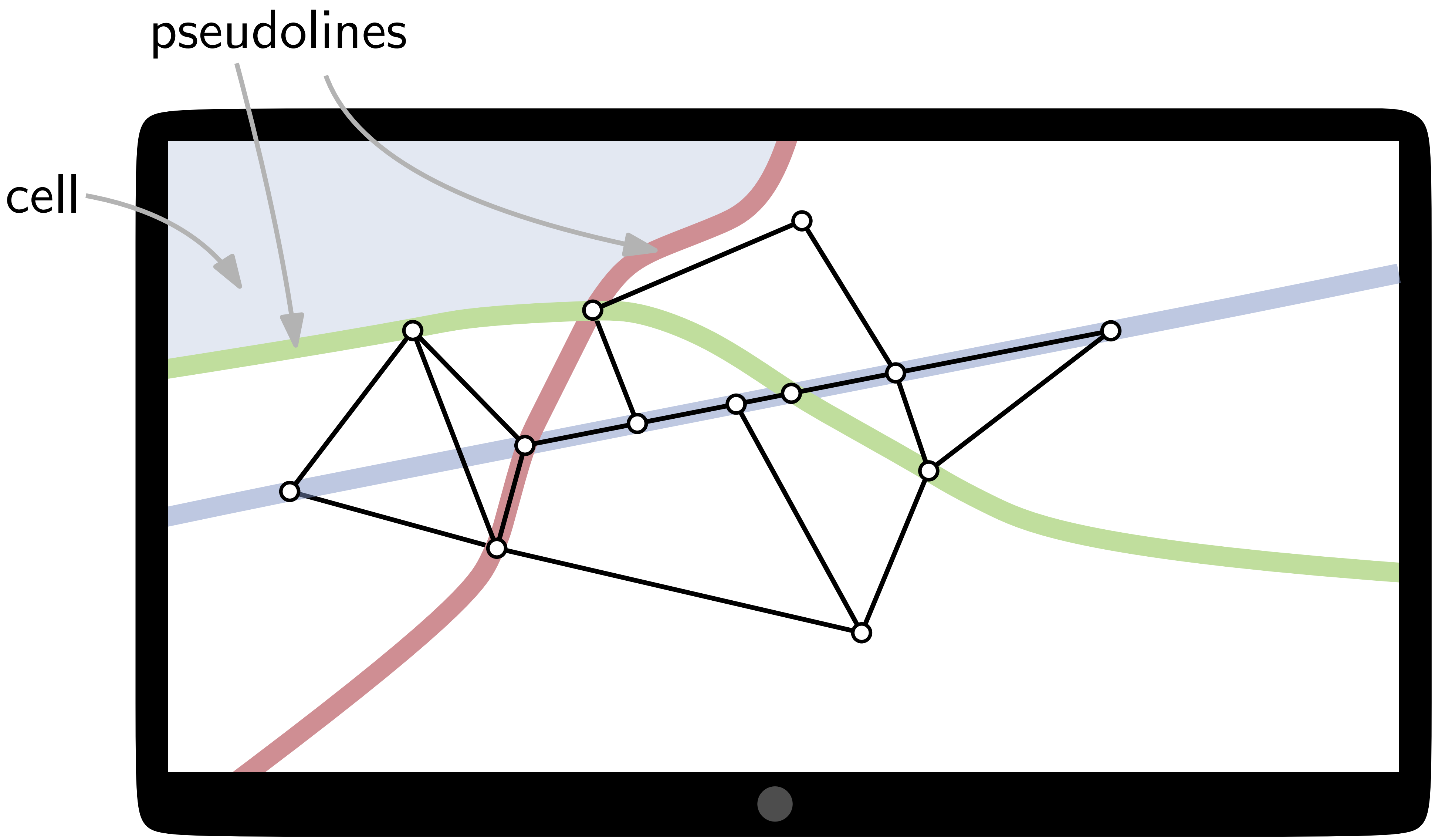

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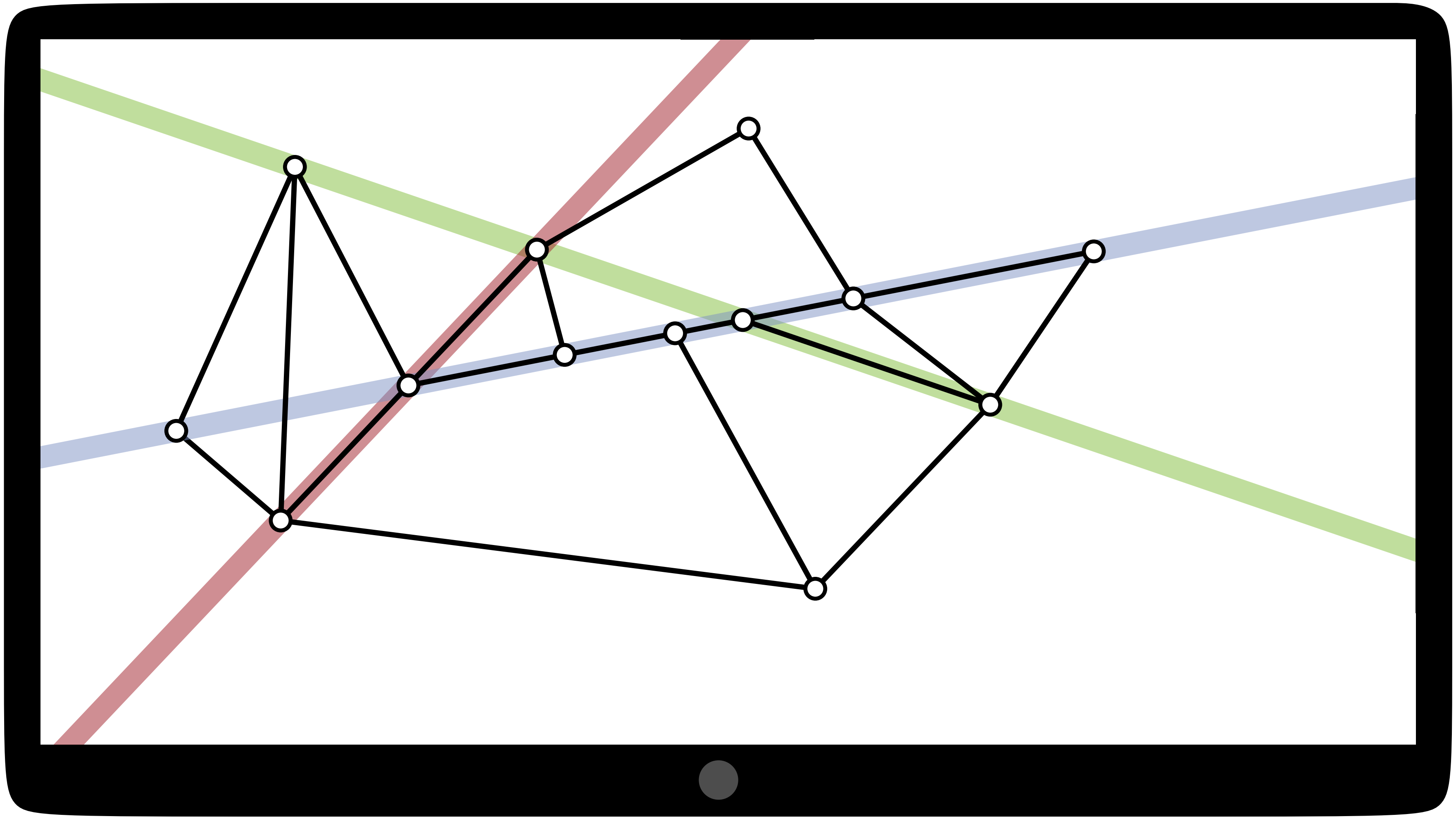


Aligned Graphs with Aligned Drawings

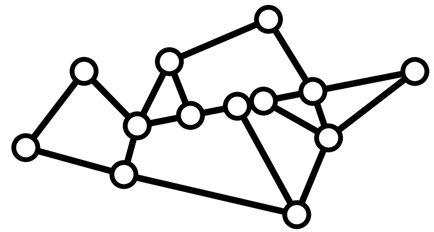
Pseudolines

$\mathcal{A}$

$+$

Planar embedded graph

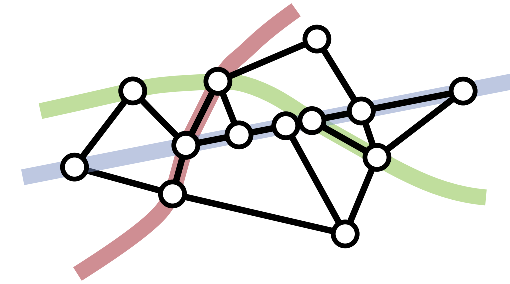
$G = (V, E)$



$=$

Aligned Graph

$(G, \mathcal{A})$



Aligned Graphs with Aligned Drawings

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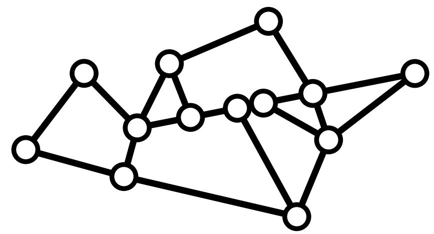
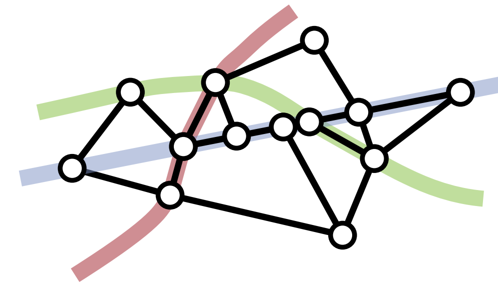
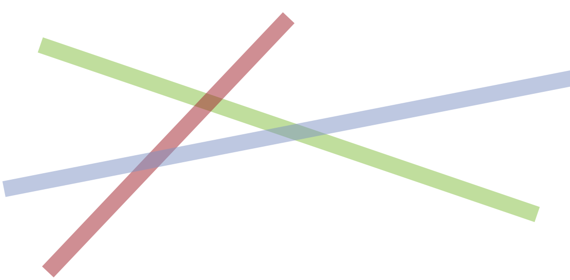


$=$

Aligned Graph $(G, \mathcal{A})$

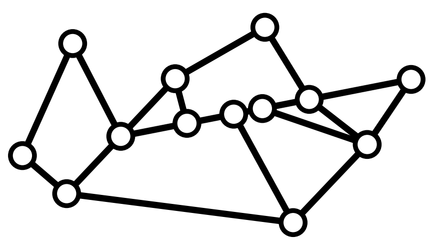


Lines A



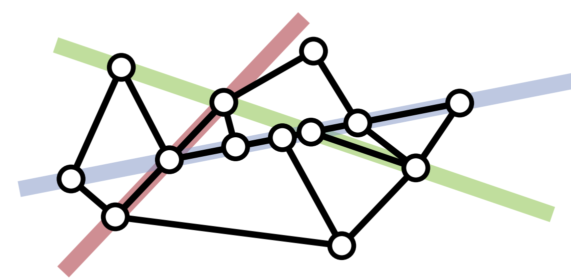
$+$

Straight-line drawing Γ

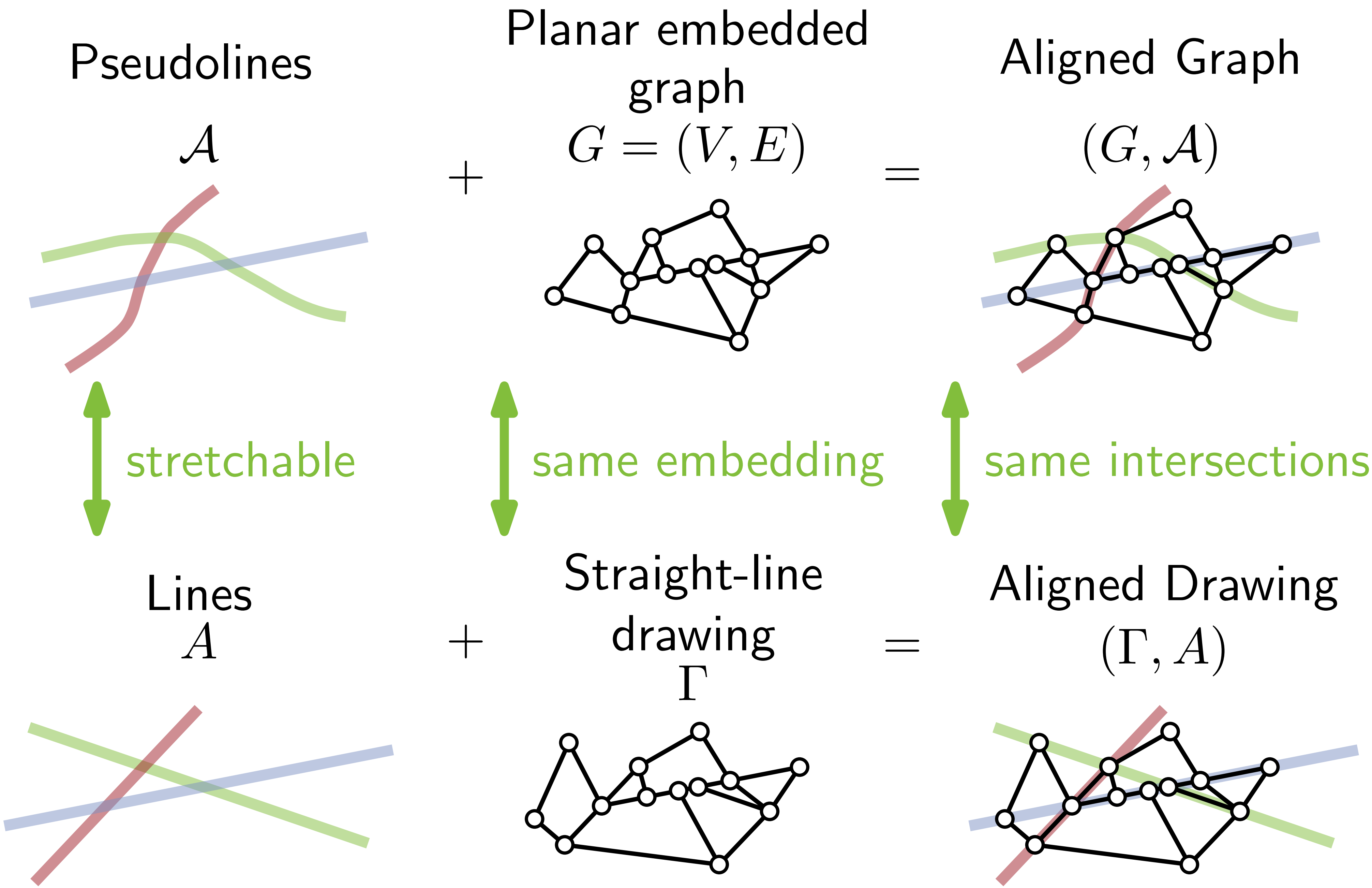


$=$

Aligned Drawing (Γ, A)



Aligned Graphs with Aligned Drawings



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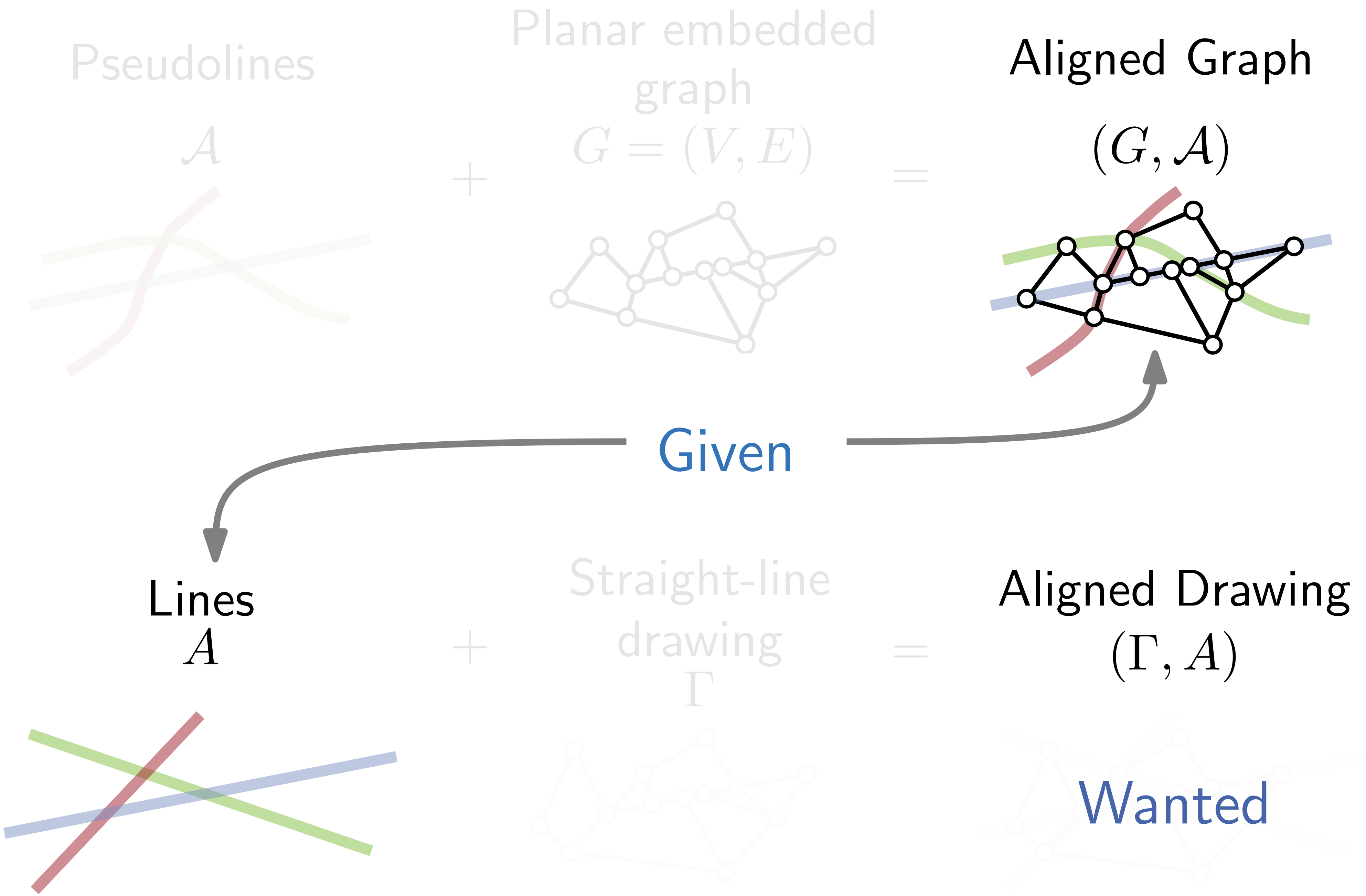
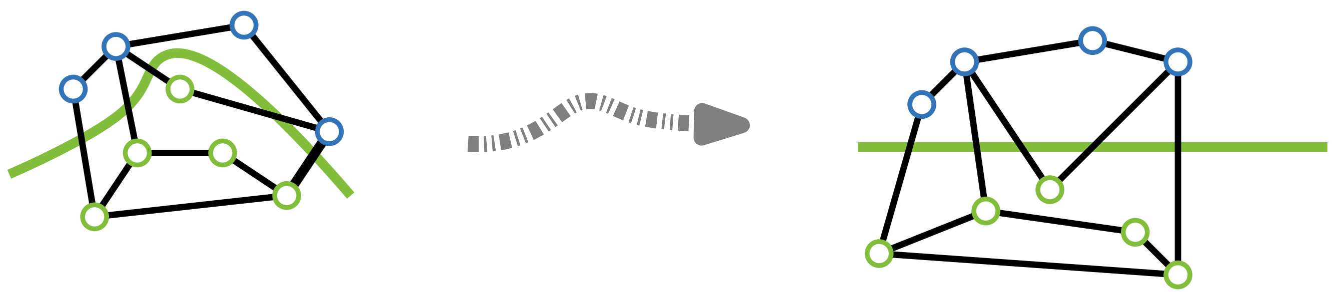


Drawing Aligned Graphs on One Line

Planar embedded graph G, bicoloring $V = A \dot{\cup} B$
A and B separable by a pseudoline $\Leftrightarrow$ A, B linear separable.

[Biedl et al. '98]



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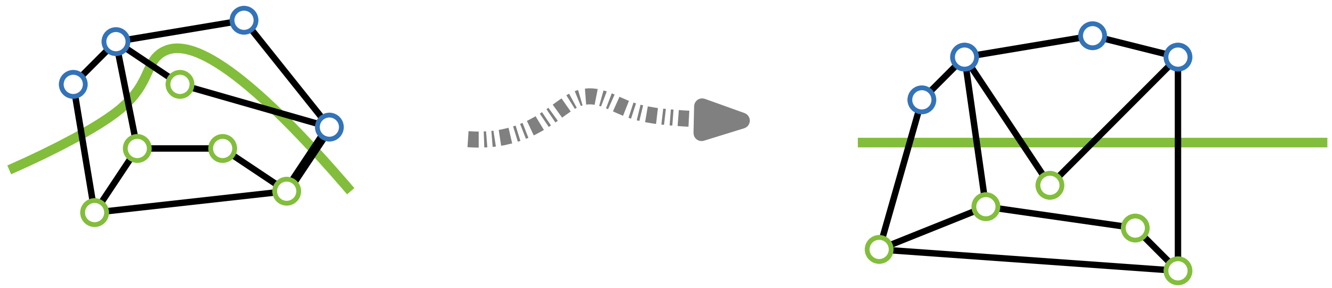
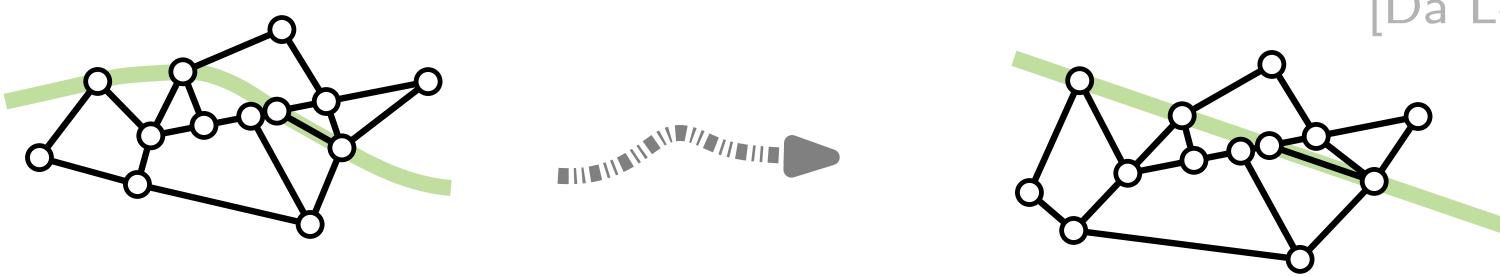


Every aligned graph $(G, \{\mathcal{C}\})$ has an aligned drawing.

[Da Lozzo et al. '16]



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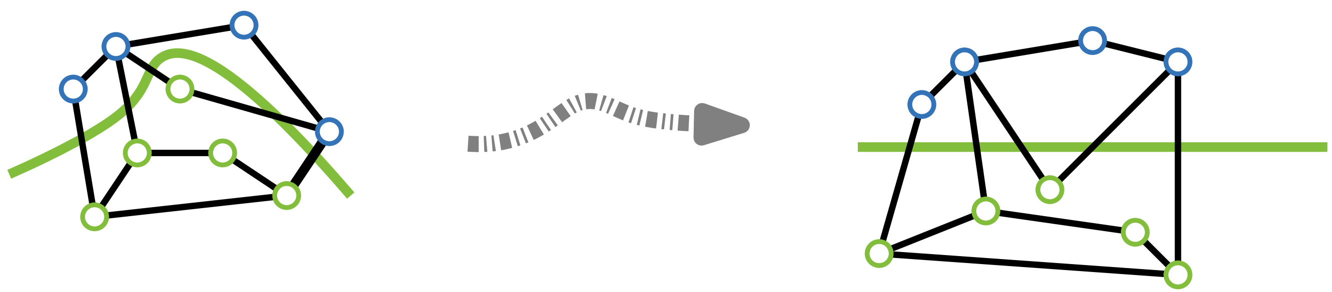

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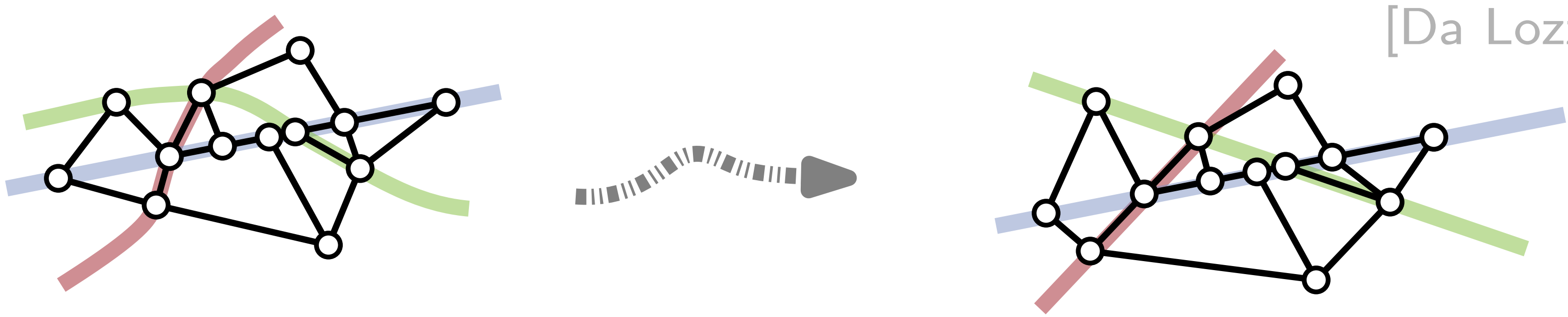
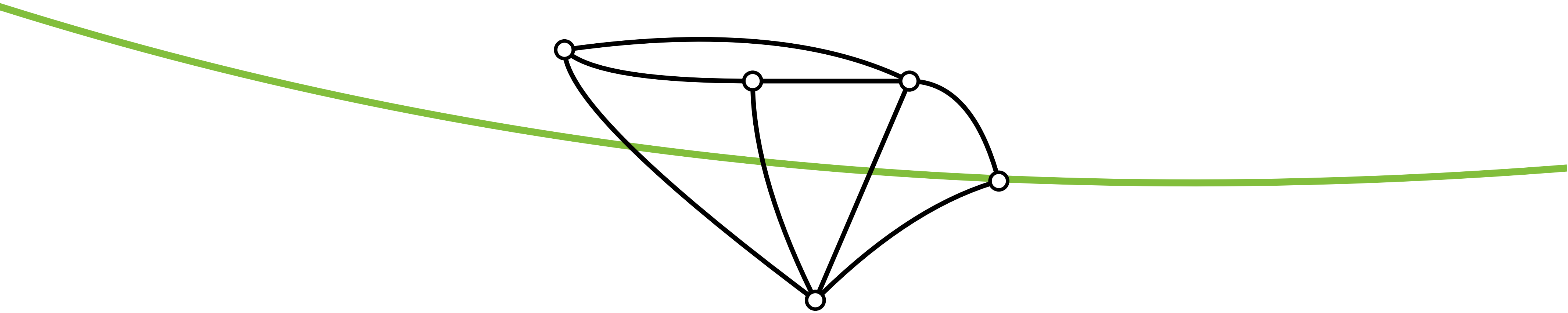


Complexity of Aligned Graphs



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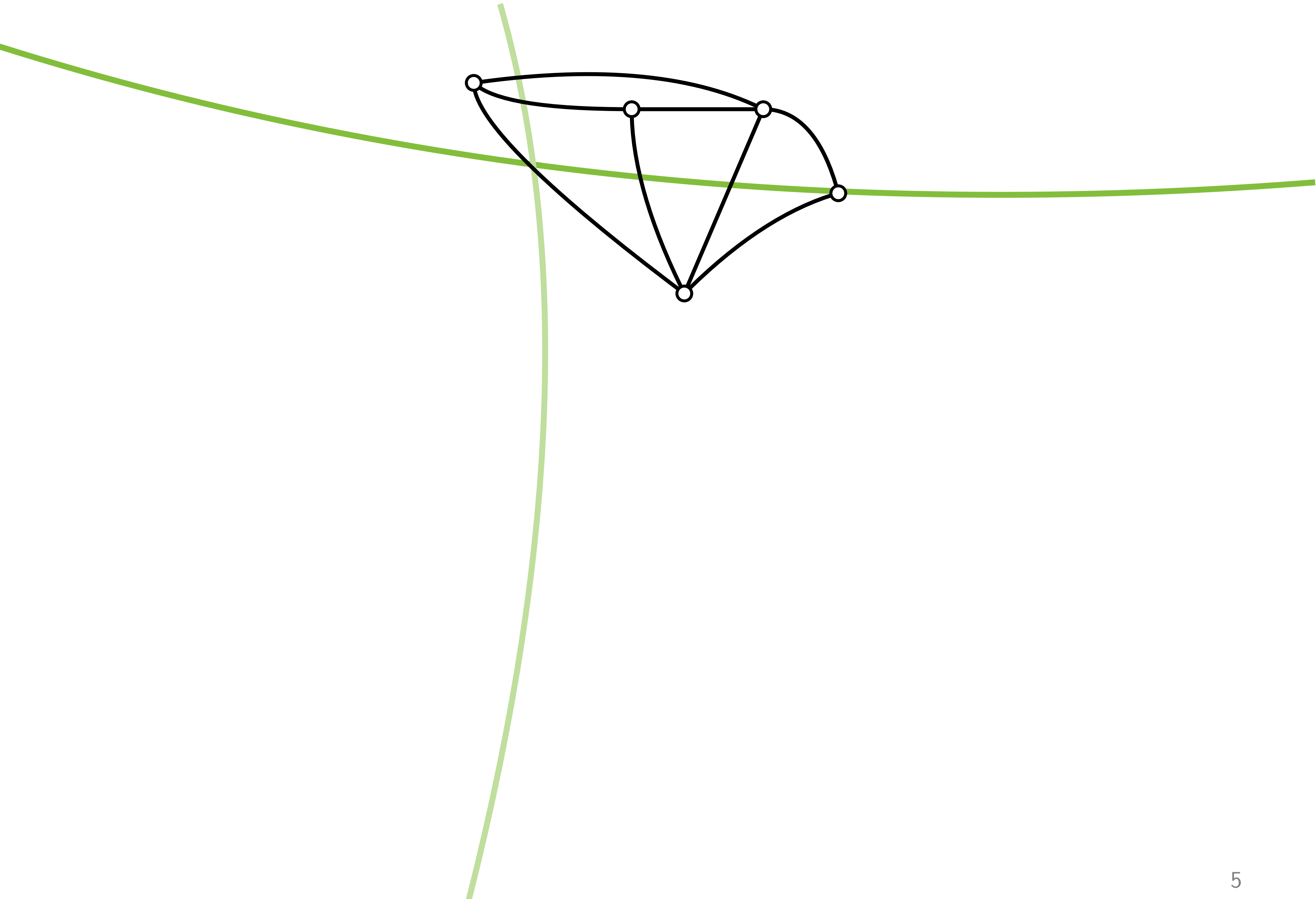

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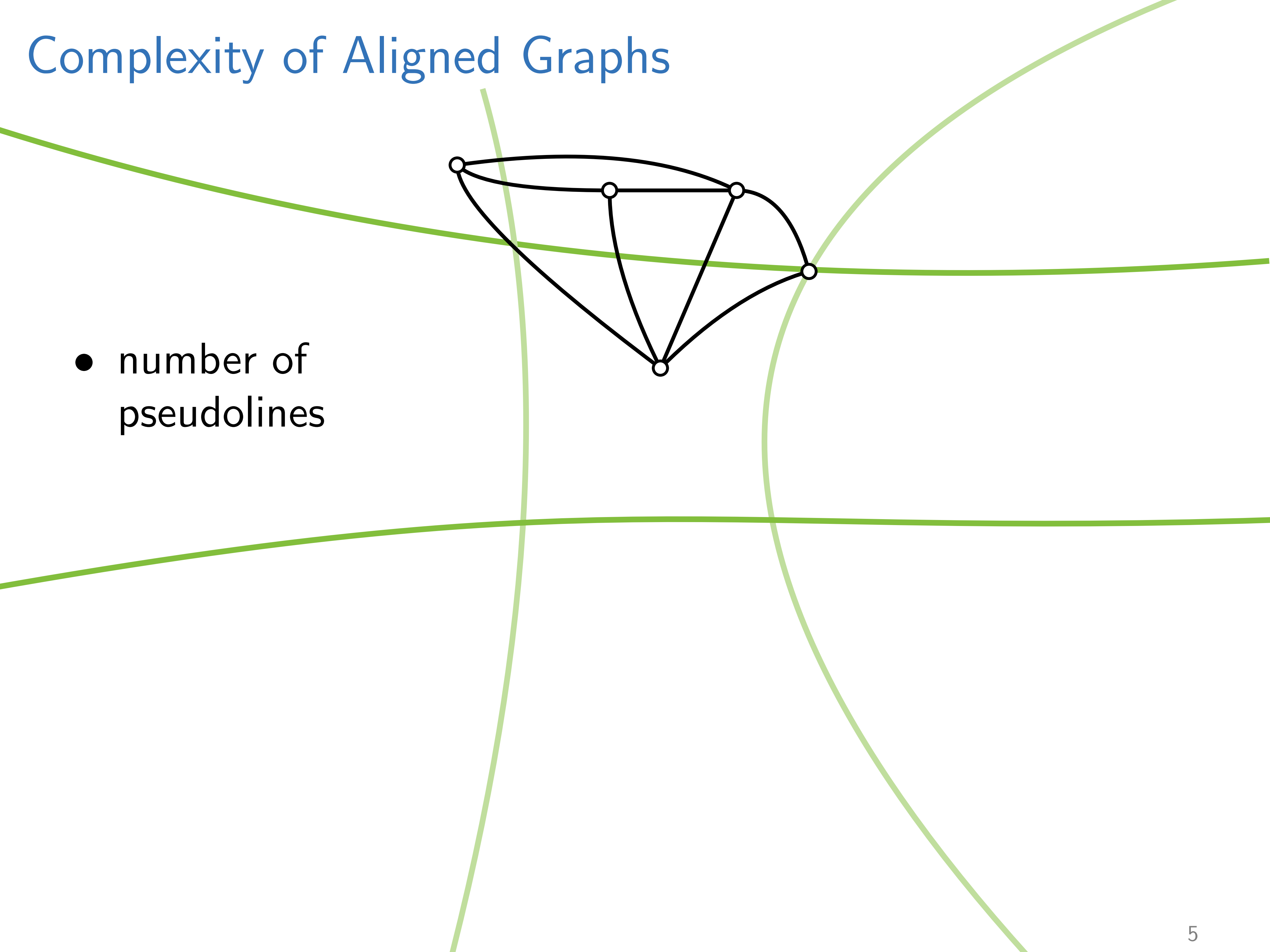
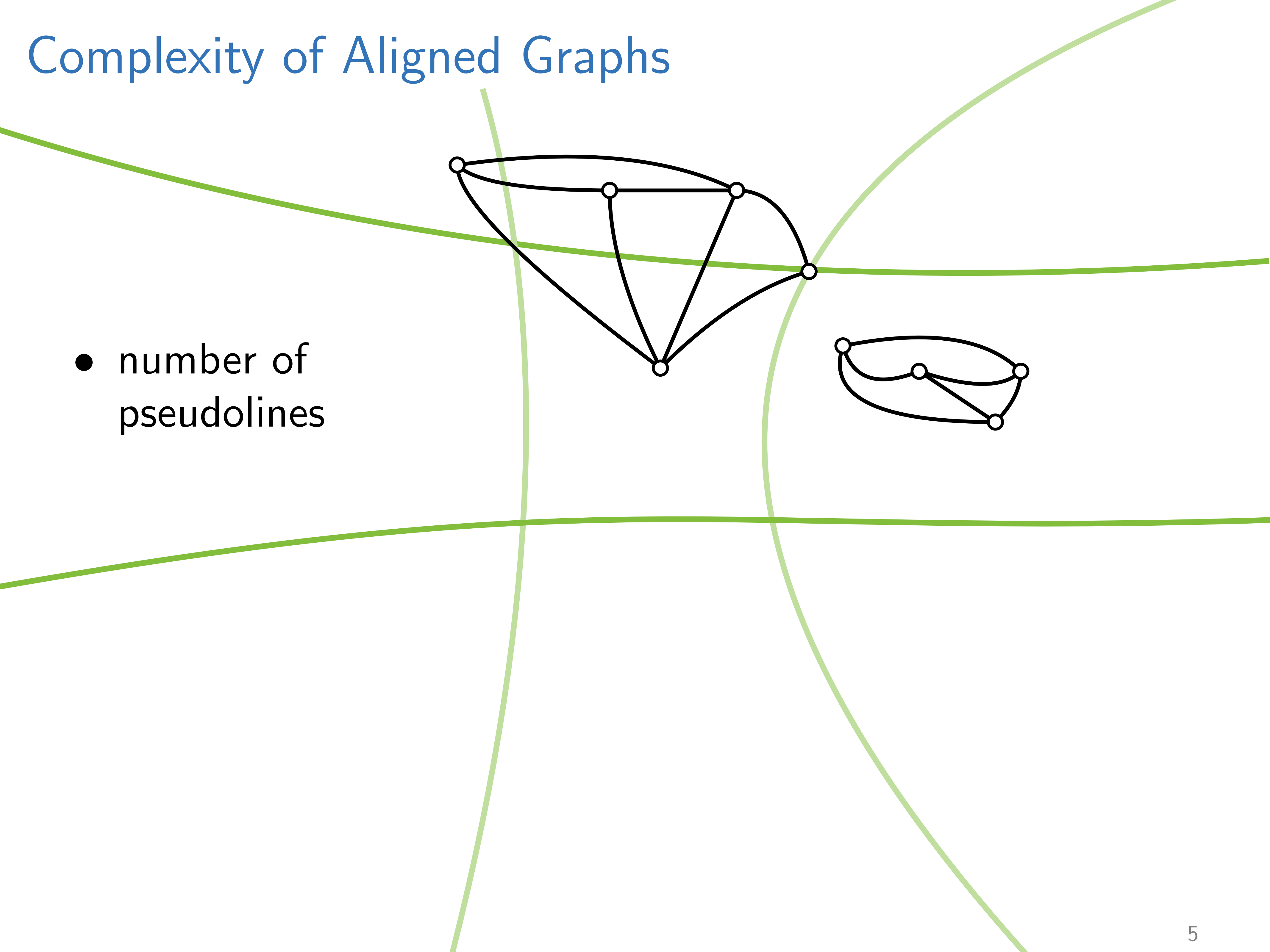


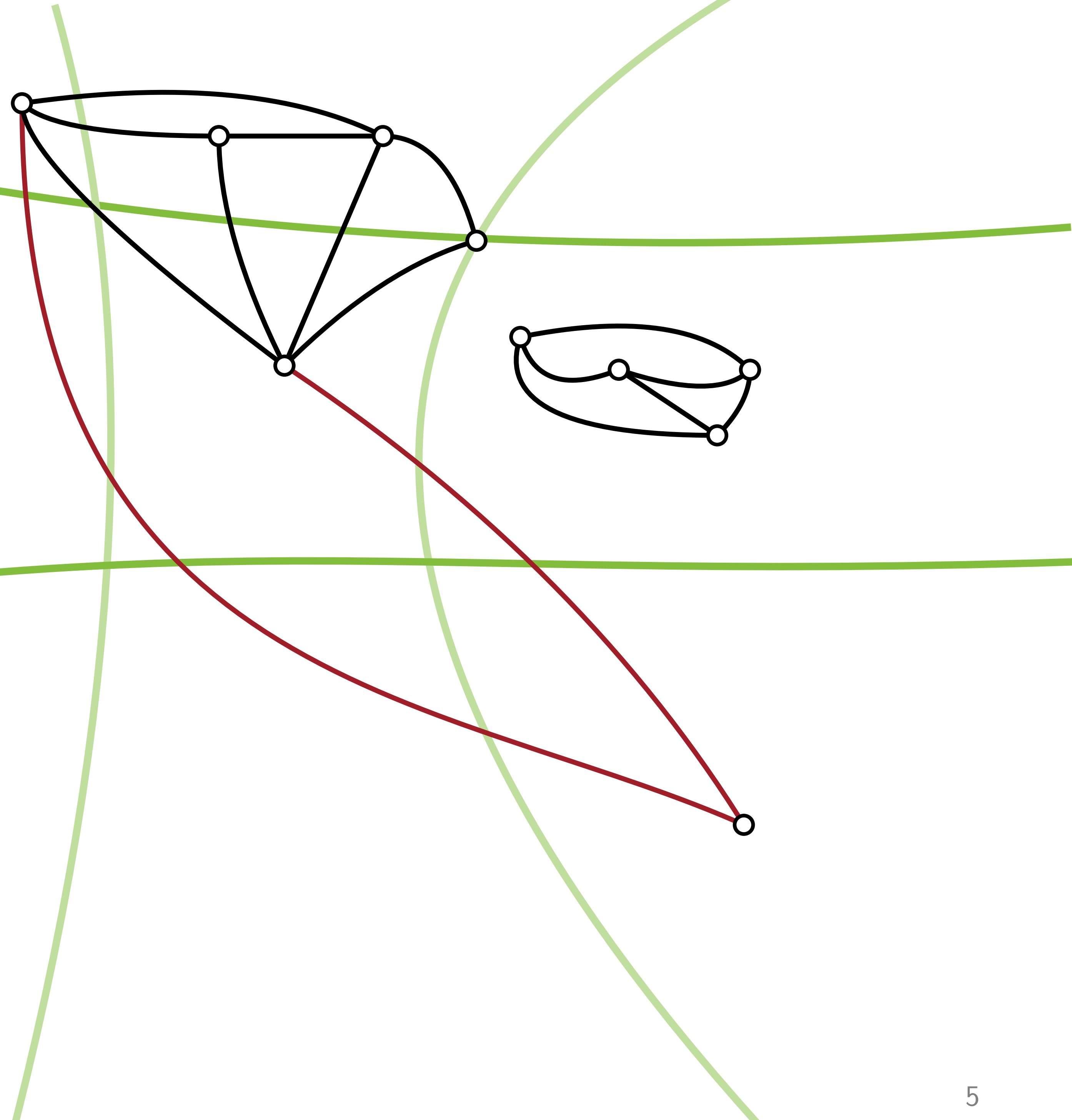
- number of pseudolines

Complexity of Aligned Graphs



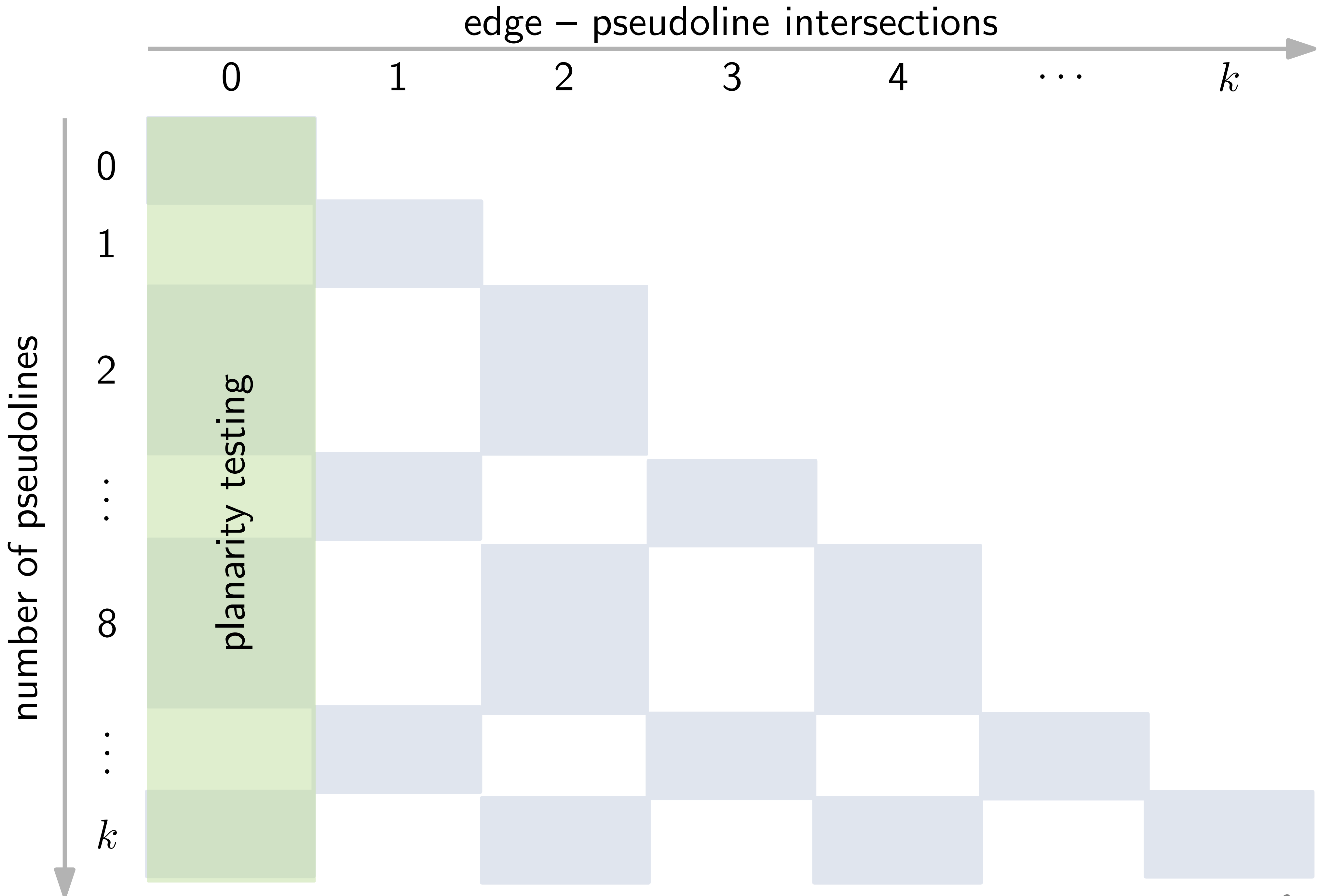
- number of pseudolines

Complexity of Aligned Graphs



- number of pseudolines
- intersections between edges and pseudolines

Complexity of Aligned Graphs



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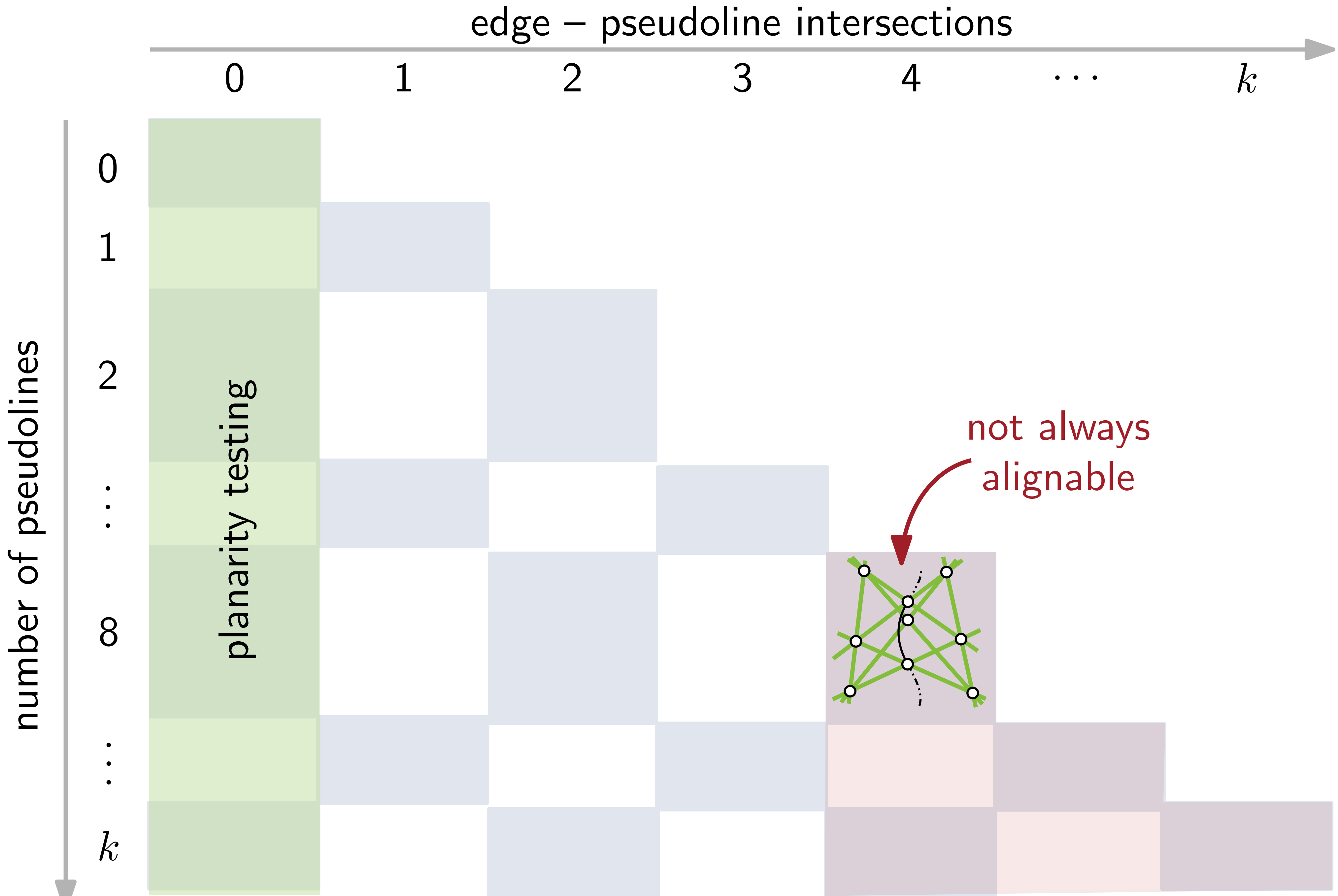

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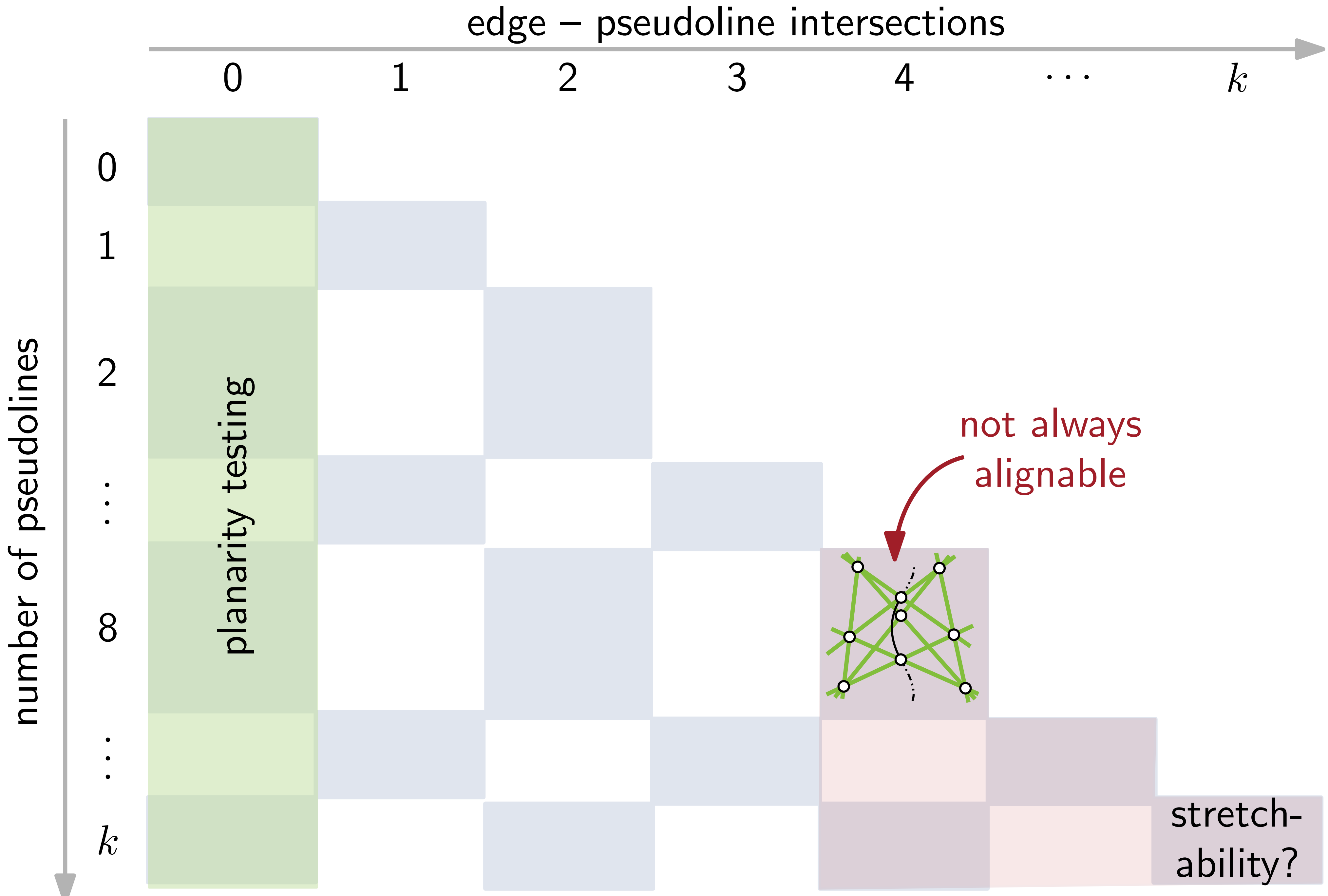

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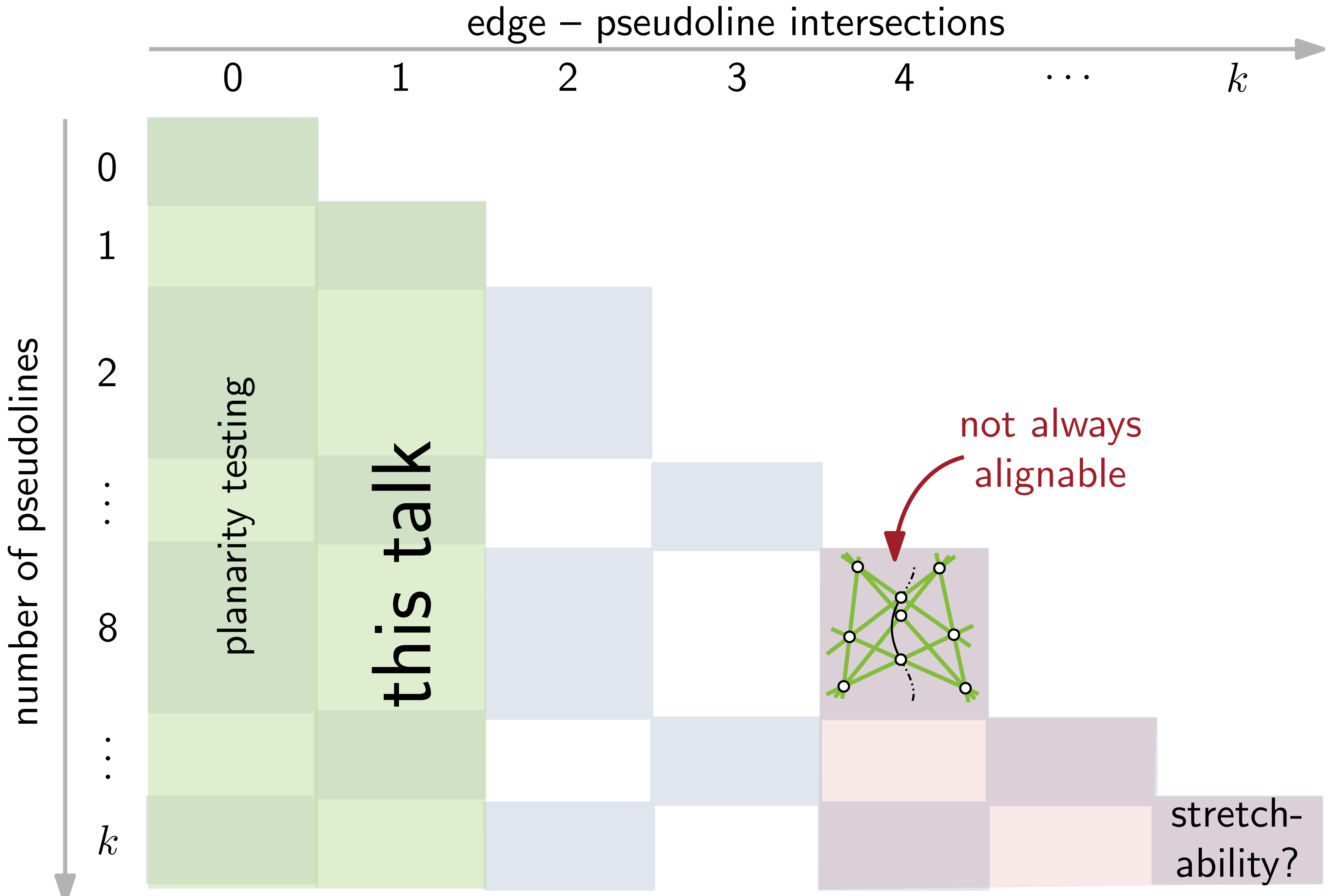
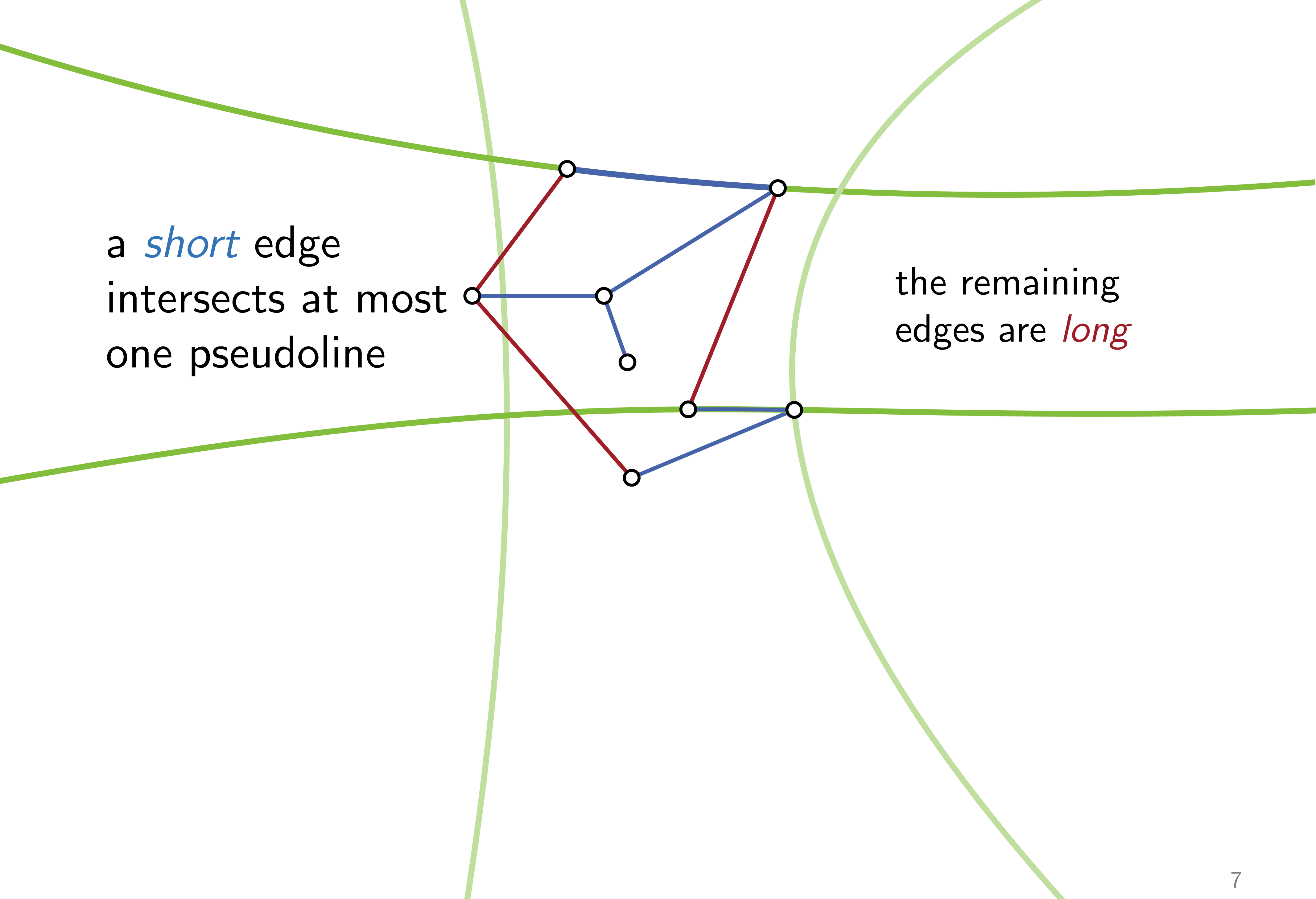
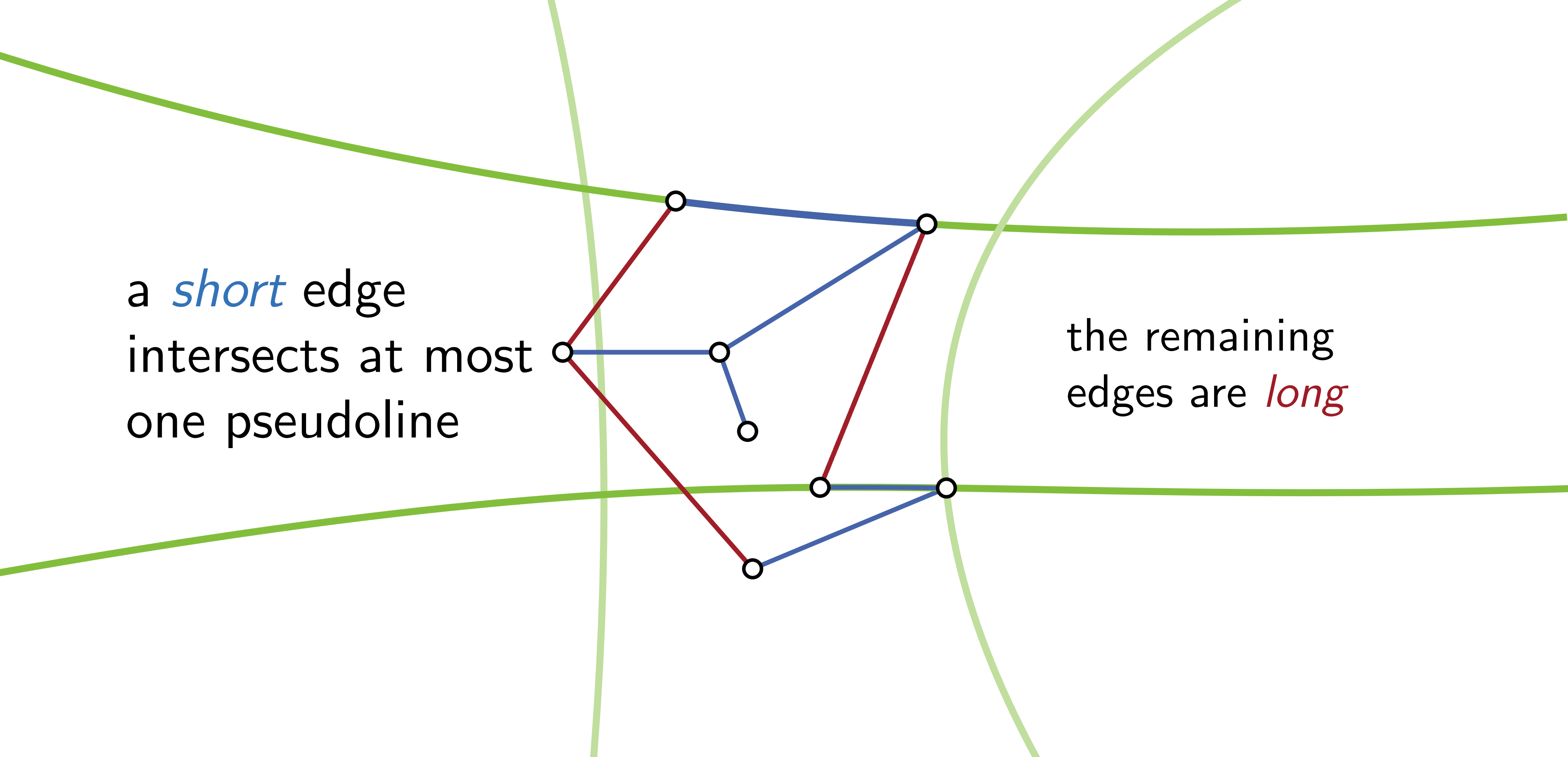


Drawing k-Aligned Graphs with Short Edges

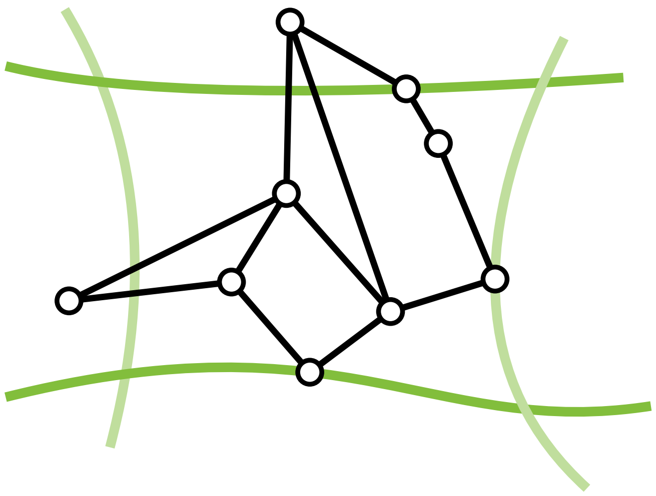


Drawing k-Aligned Graphs with Short Edges



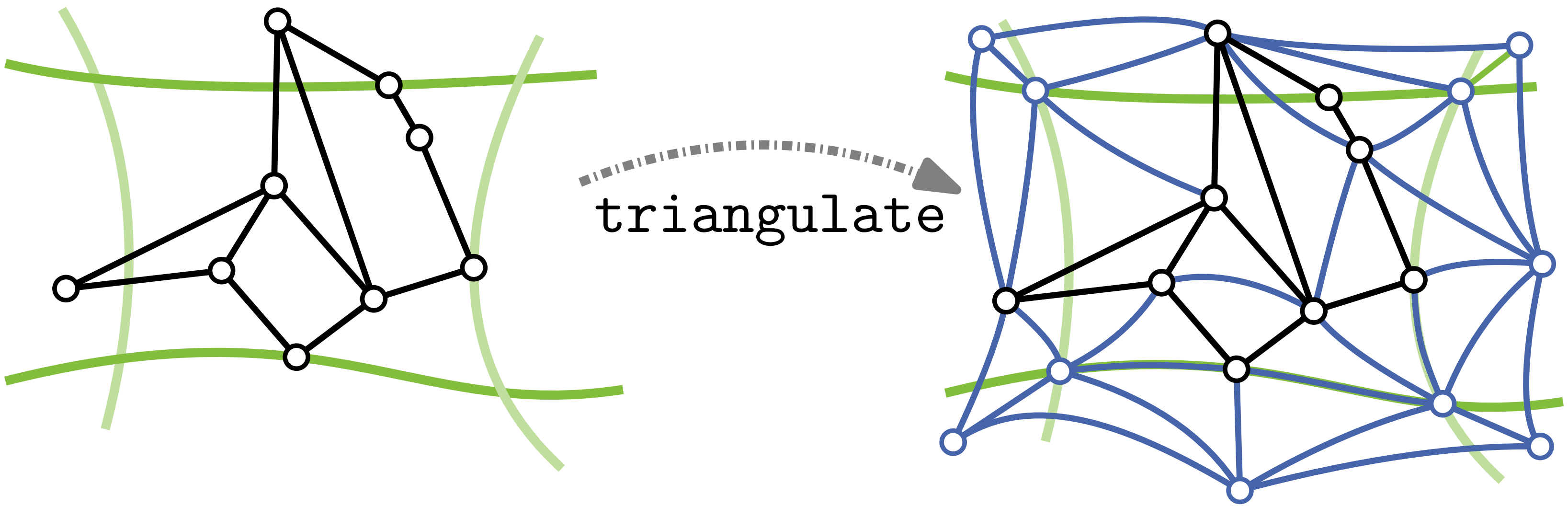
Theorem Every k-aligned graph without long edges has an aligned drawing.

Proof Sketch



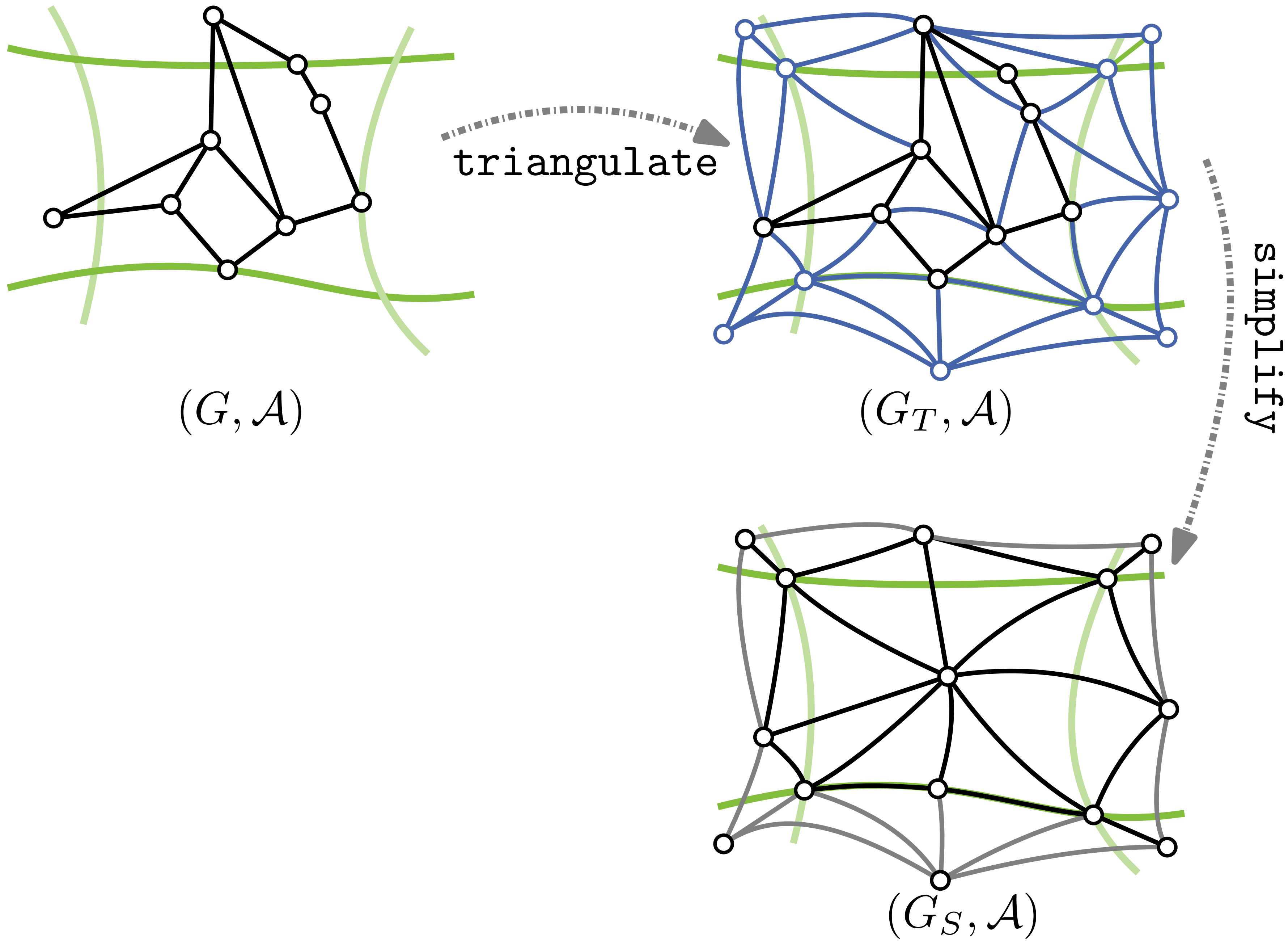
$(G, \mathcal{A})$

Proof Sketch



$(G, \mathcal{A})$ $(G_T, \mathcal{A})$

Proof Sketch



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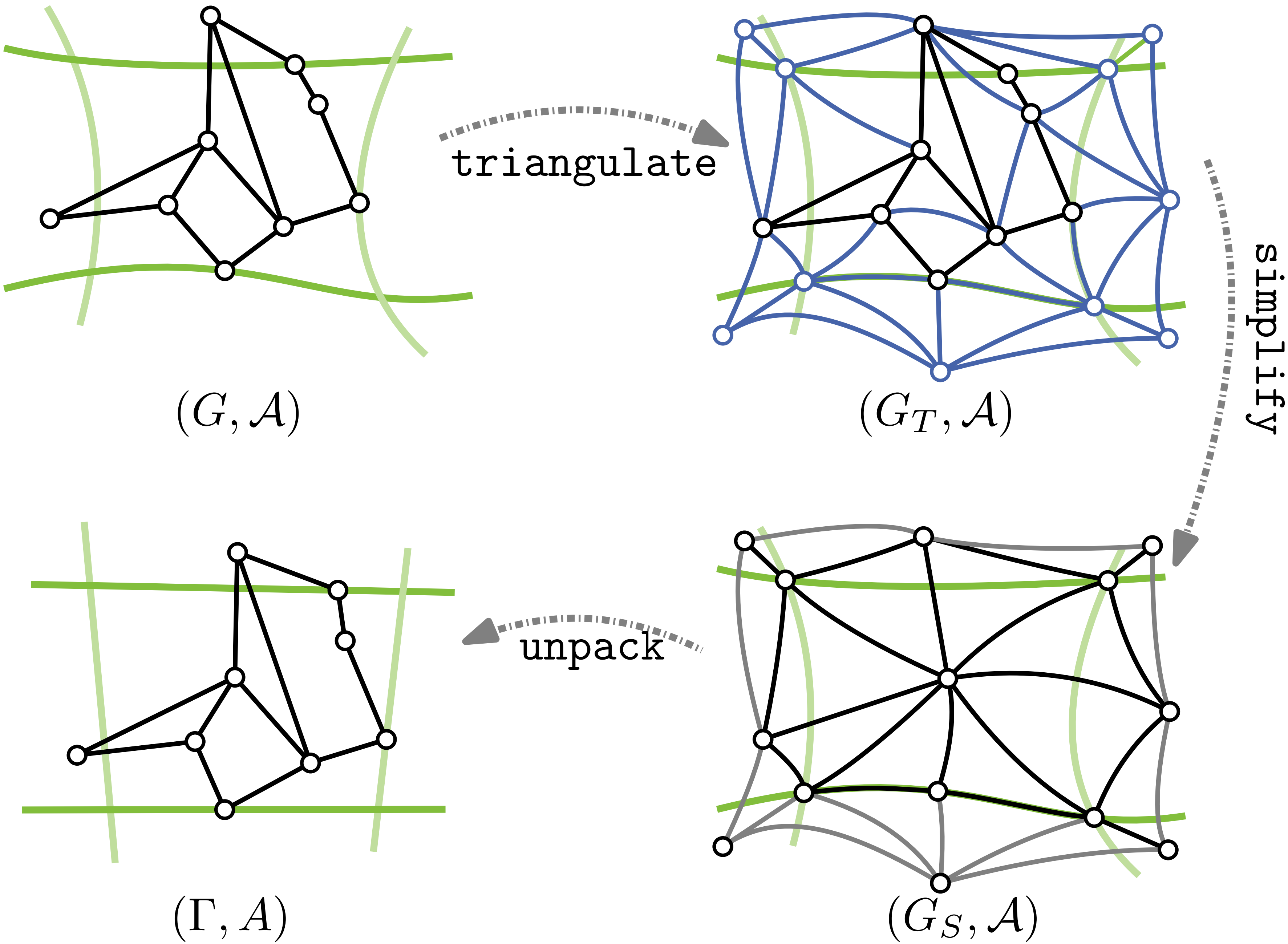

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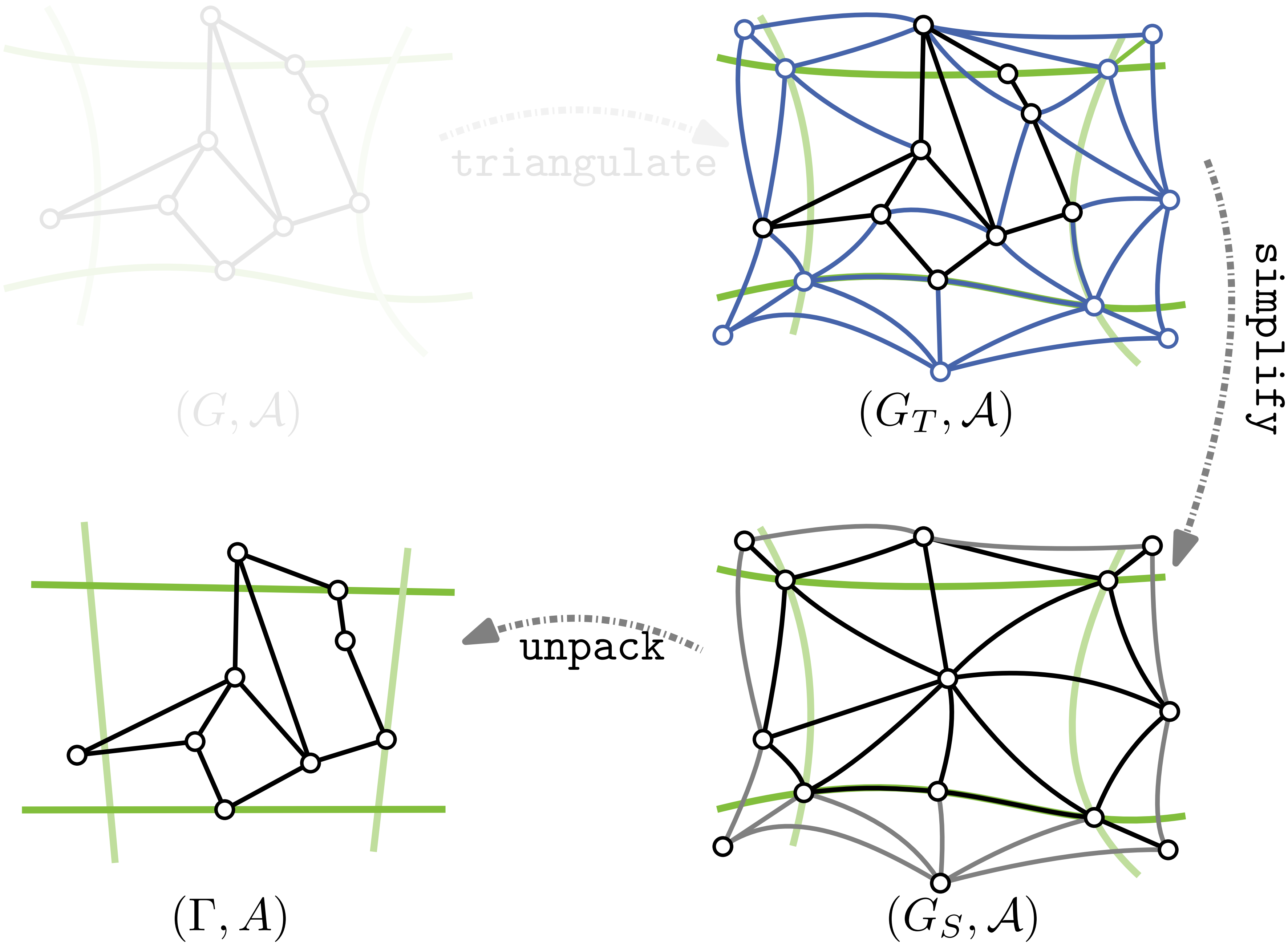
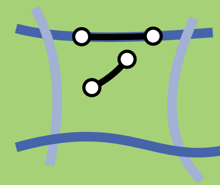
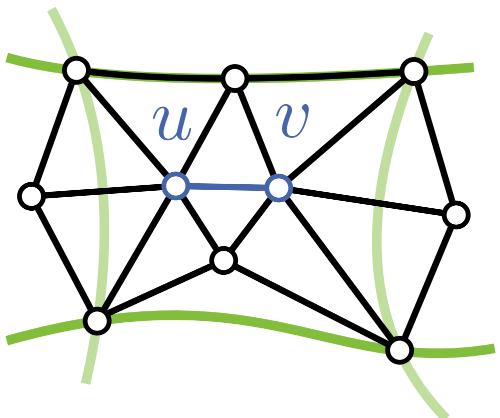


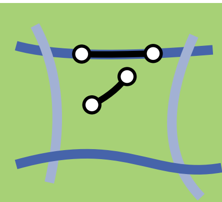
Simplify

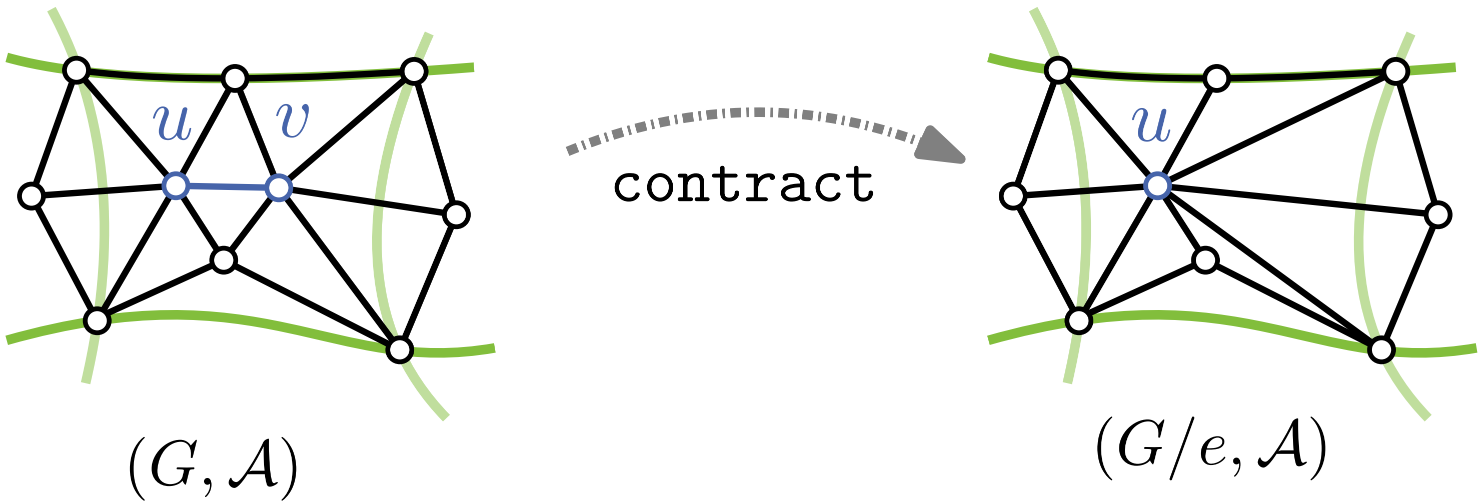
Contract `aligned` / `free`  edges.



$(G, \mathcal{A})$

Simplify

Contract `aligned` / `free`  edges.



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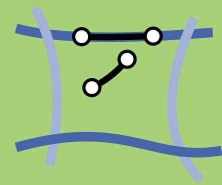
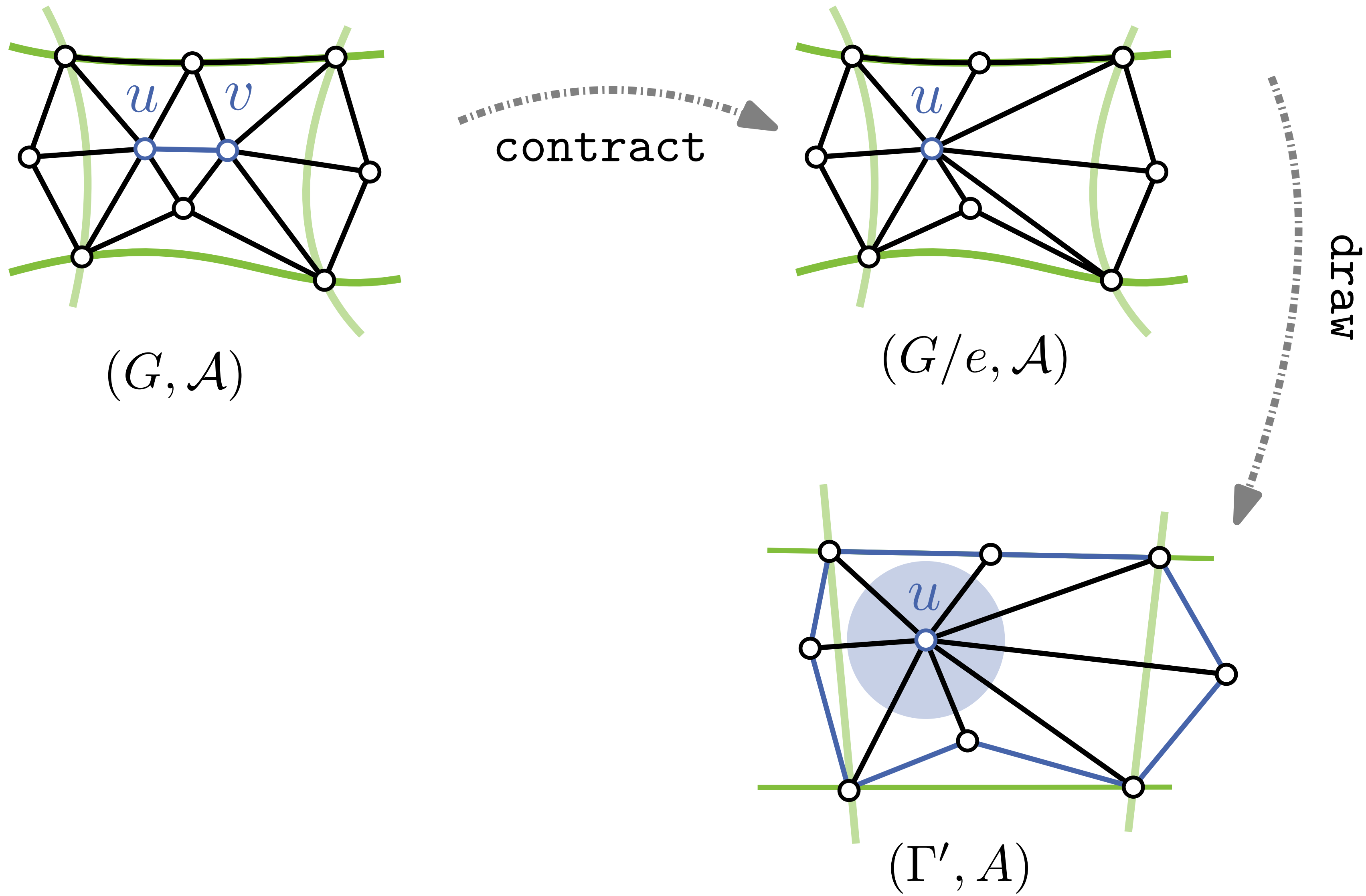
Contract `aligned` / `free`  edges.



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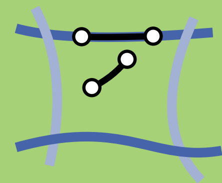
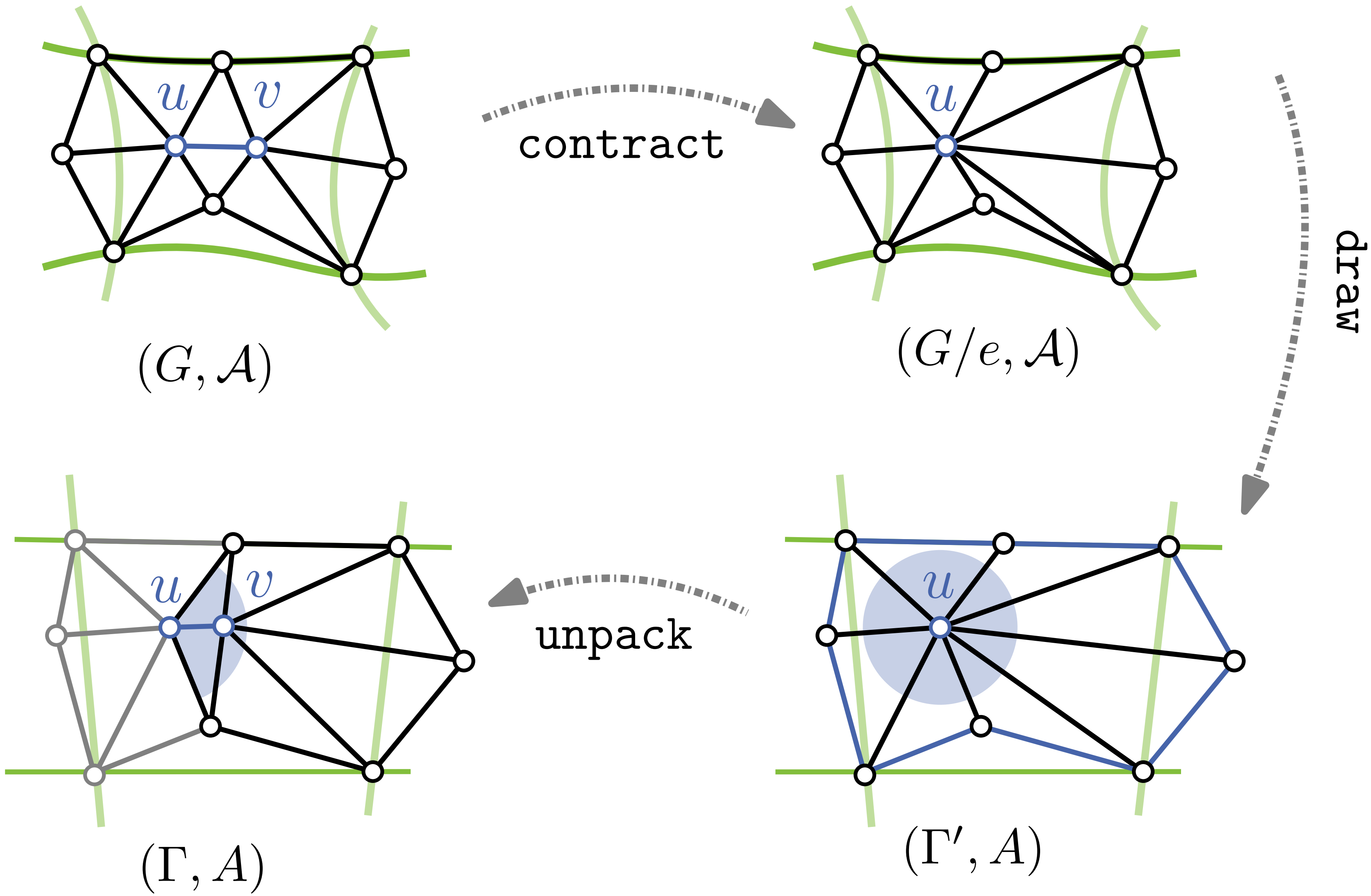
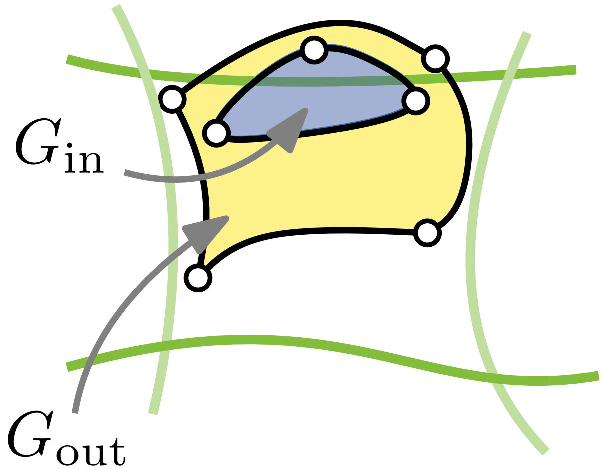
Contract aligned / free  edges.



Simplify

Split at separating triangles.



$(G, \mathcal{A})$

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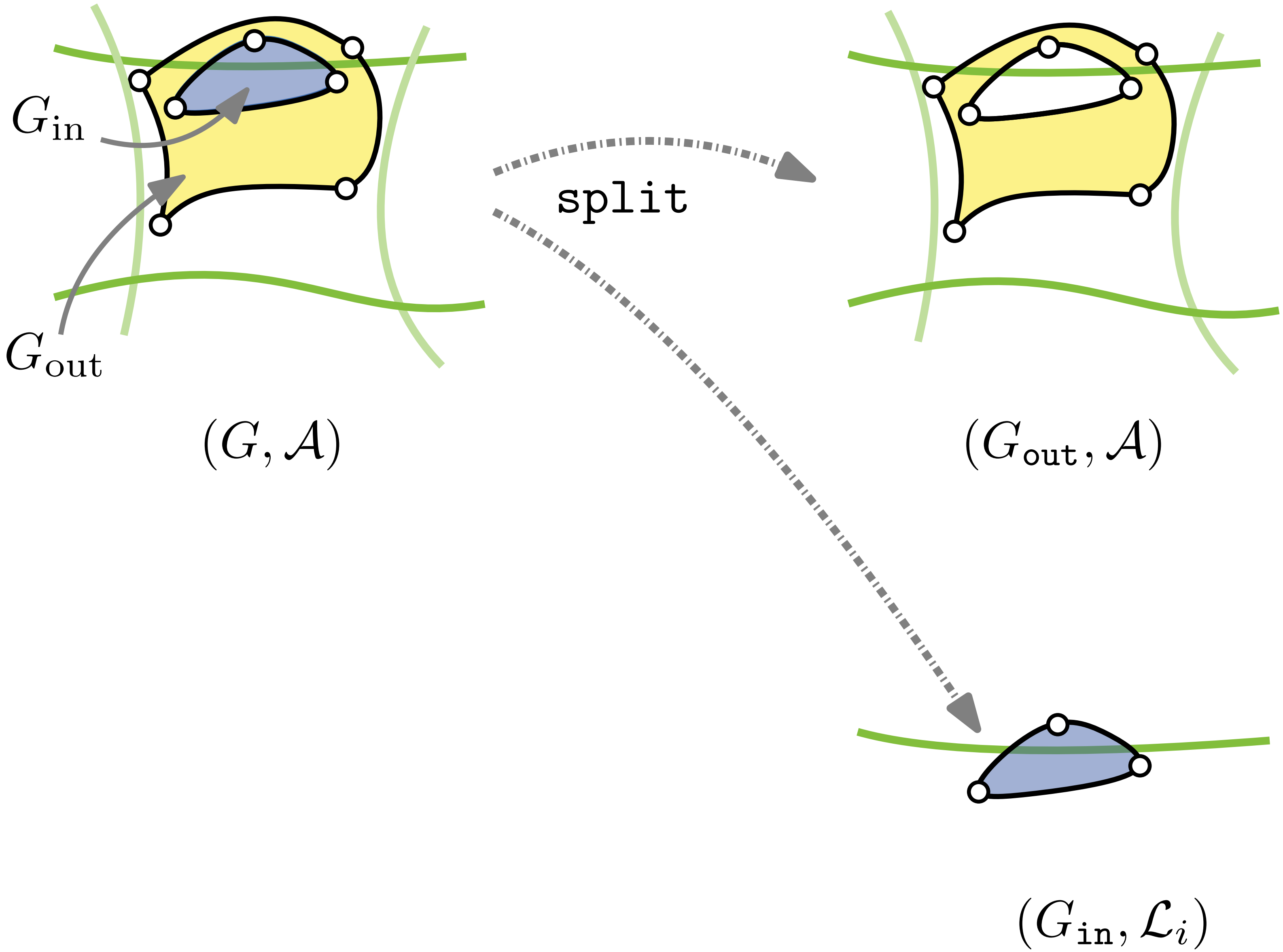

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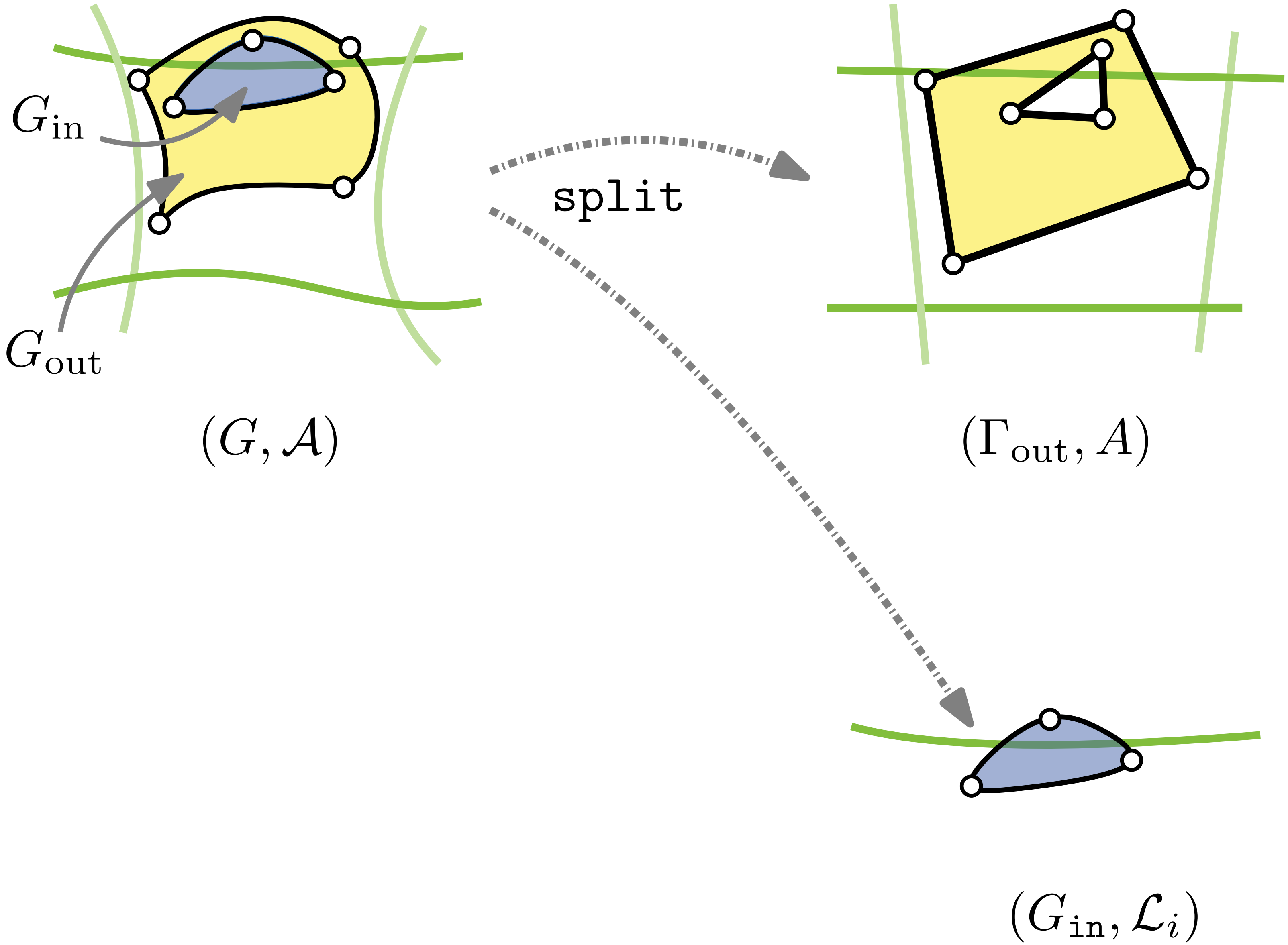

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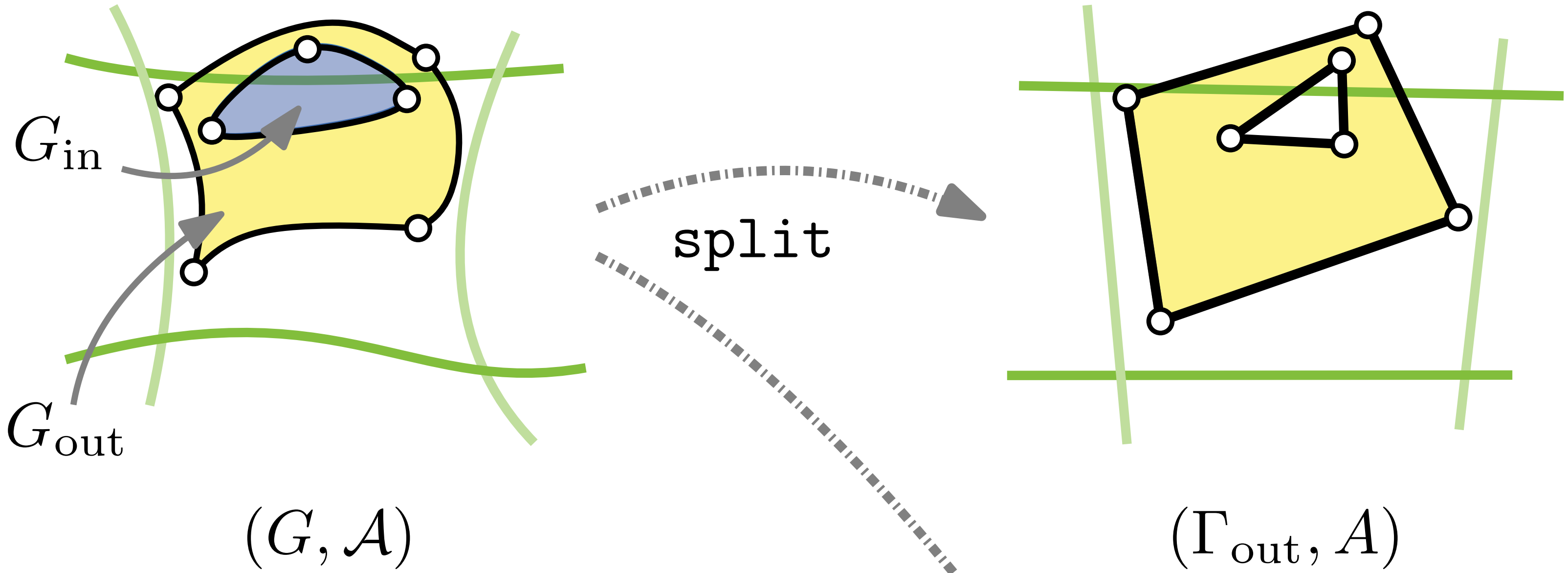
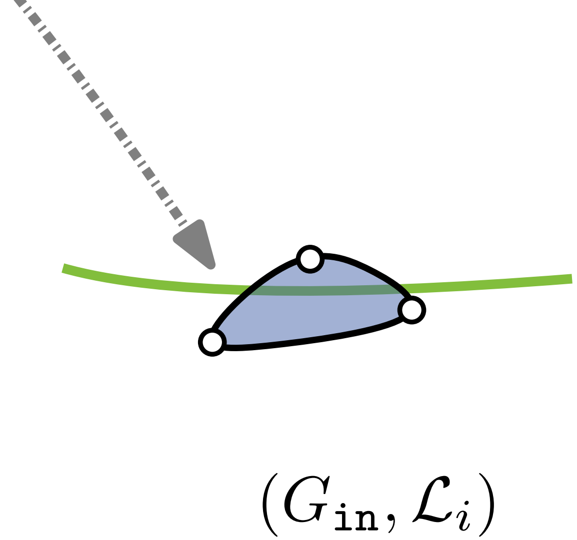


Theorem
Every⋆ aligned graph $(G, \{\mathcal{C}\})$ has an aligned drawing with a fixed line and convex outer face.



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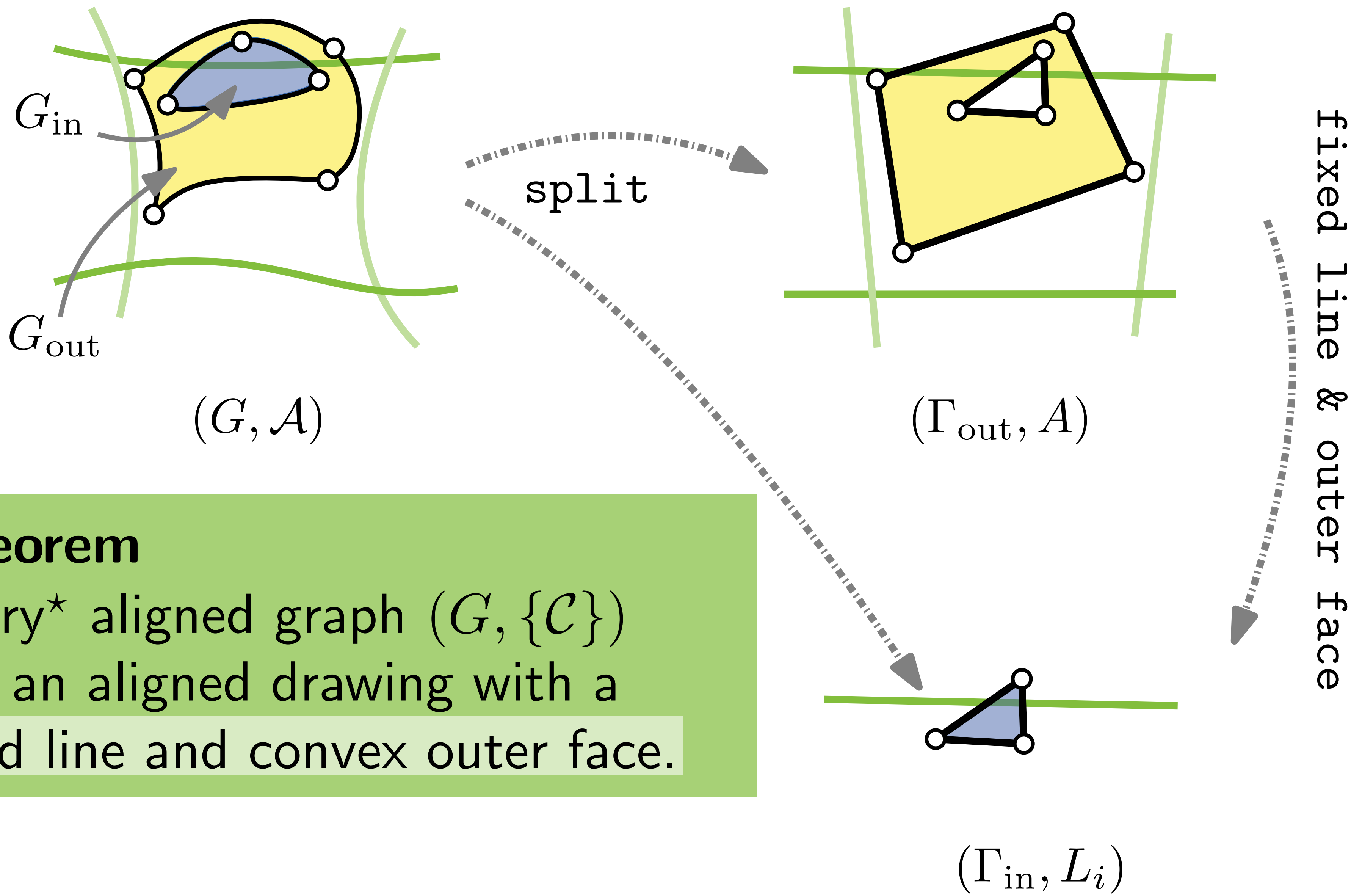

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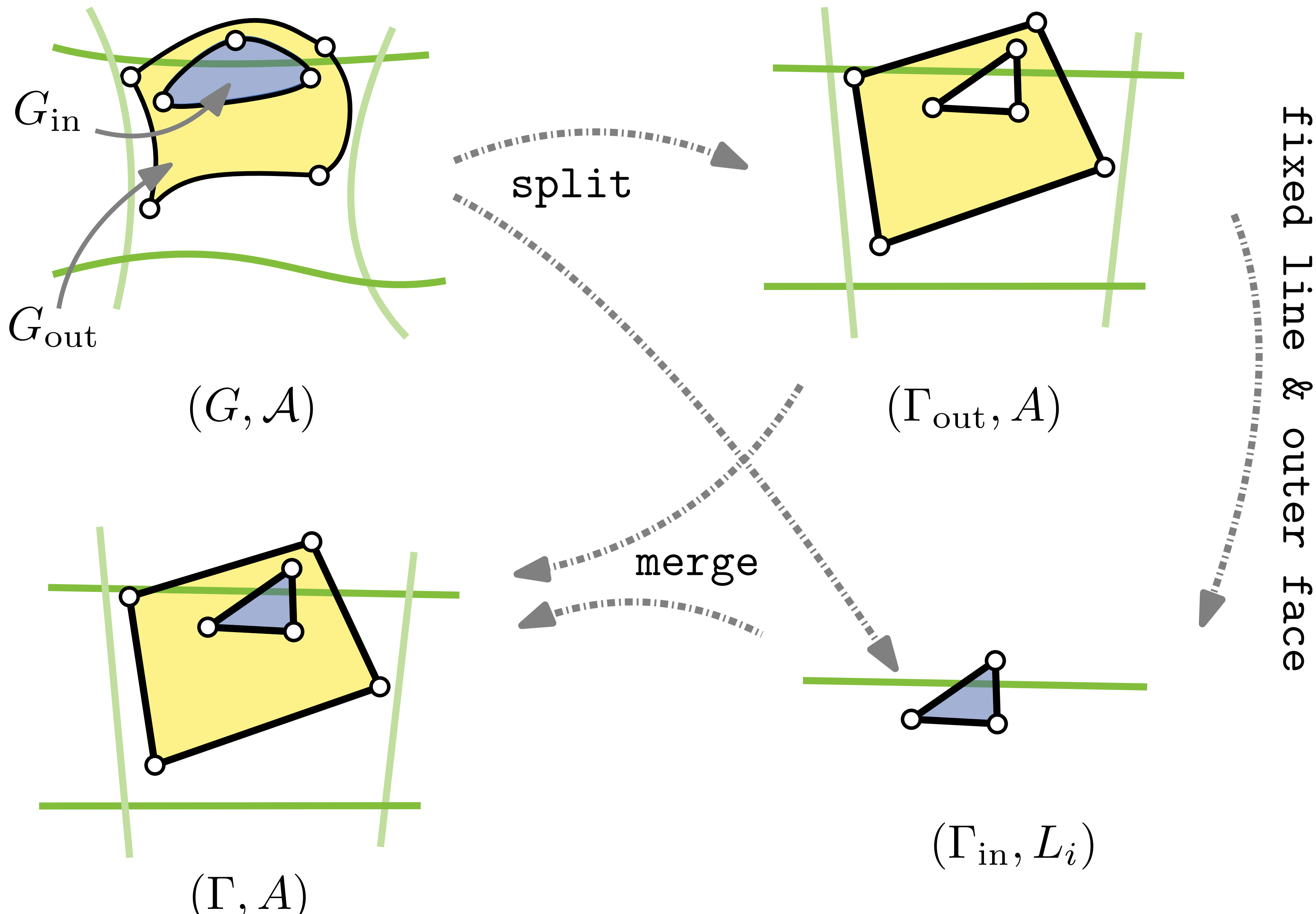
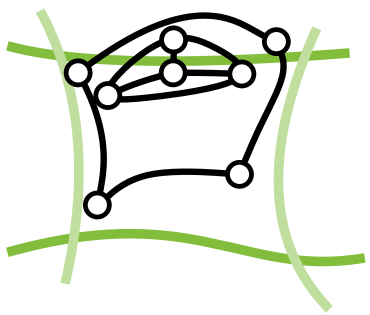


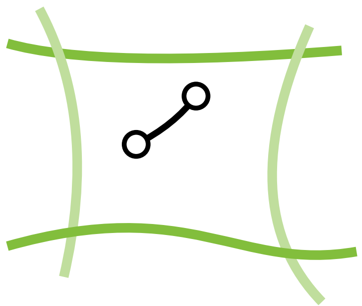
Simplified Aligned Graphs

$(G, \mathcal{A})$ is a triangulation.

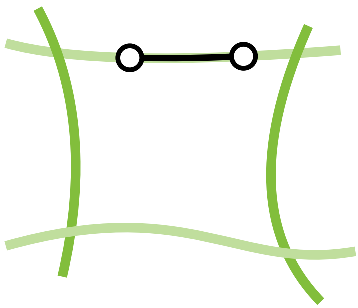
separating triangle



free edge



floating aligned edge

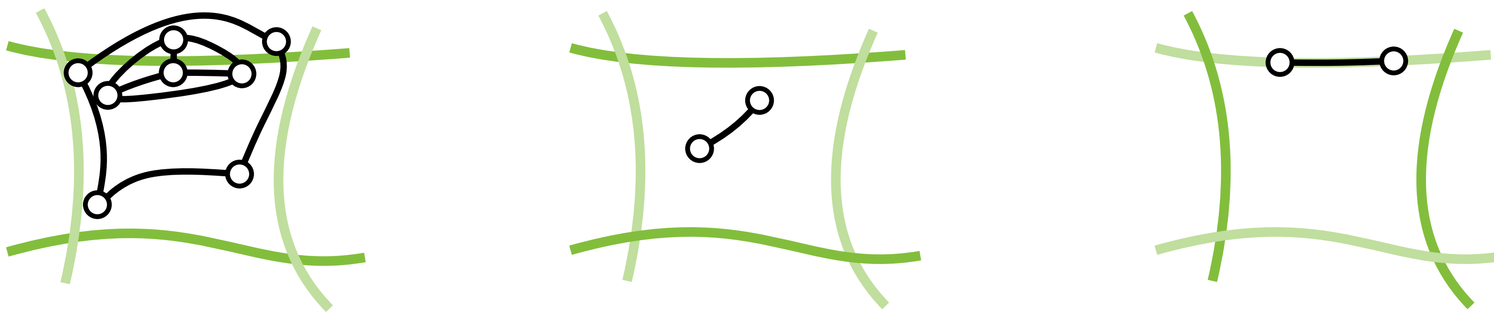


$\not\subset (G, \mathcal{A})$

Simplified Aligned Graphs

$(G, \mathcal{A})$ is a triangulation.

separating triangle | free edge | floating aligned edge



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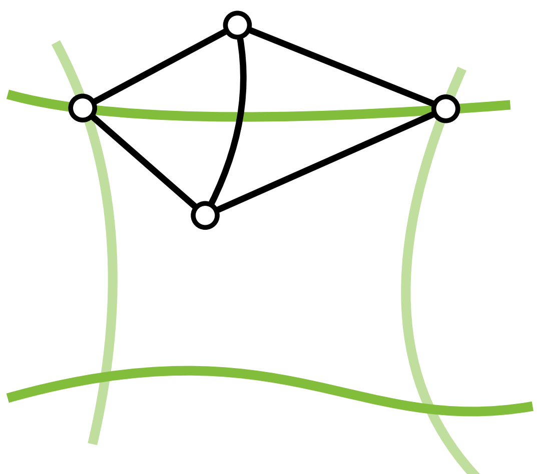
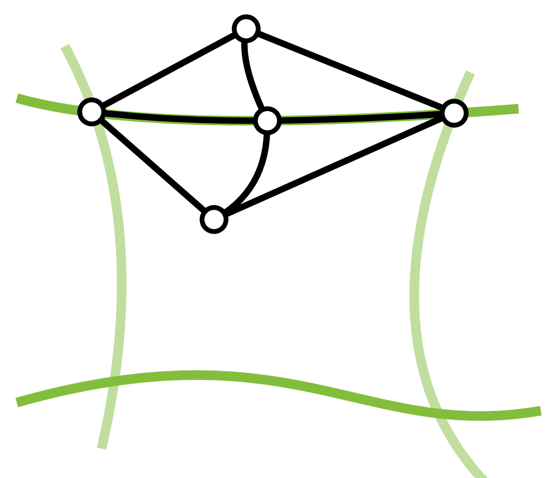
$\Rightarrow$



$\cup$



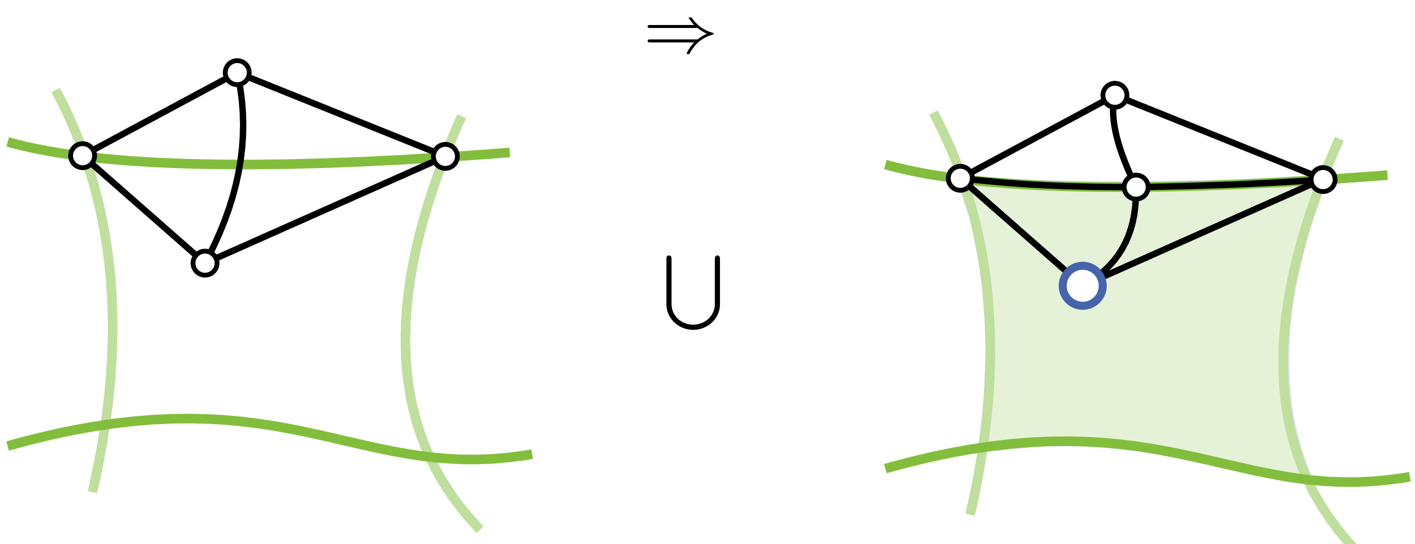
$= (G, \mathcal{A})$

Simplified Aligned Graphs

$(G, \mathcal{A})$ is a triangulation.

Claim 1 Every cell C contains exactly one vertex.

Claim 2 An aligned vertex is incident to two aligned edges.



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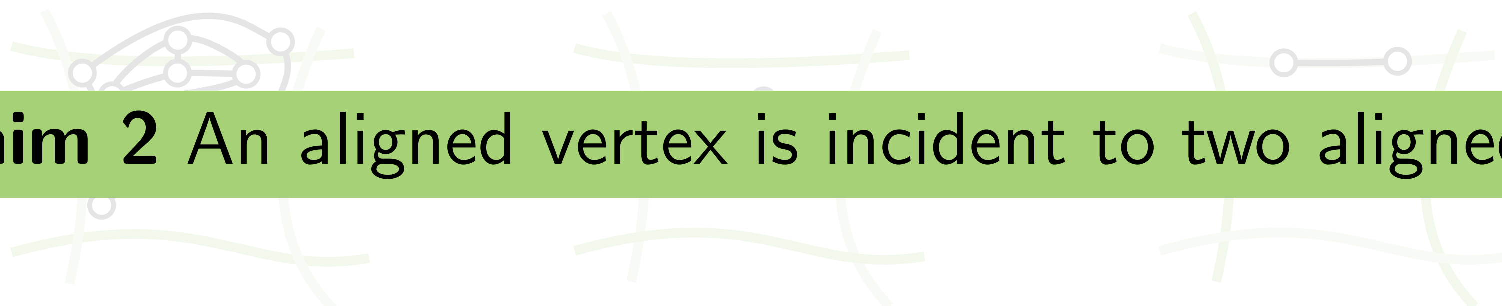

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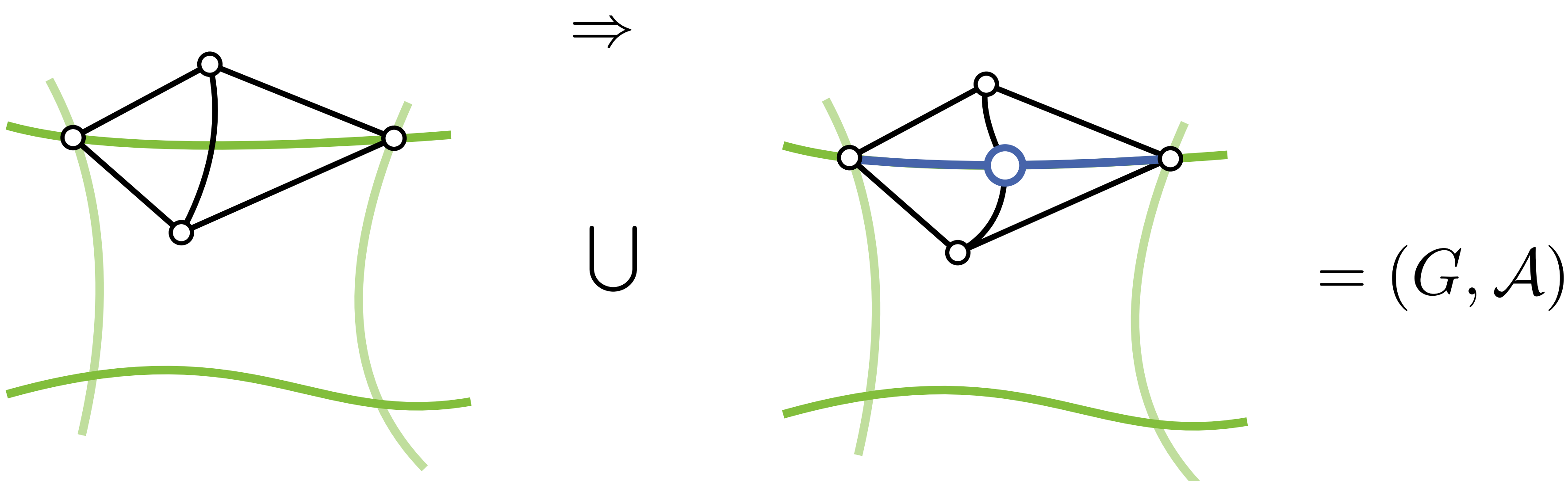

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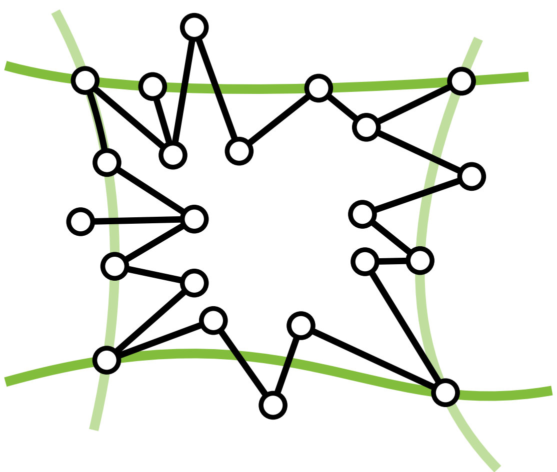
Proof by contradiction



- Assume $|V(C)| > 1$

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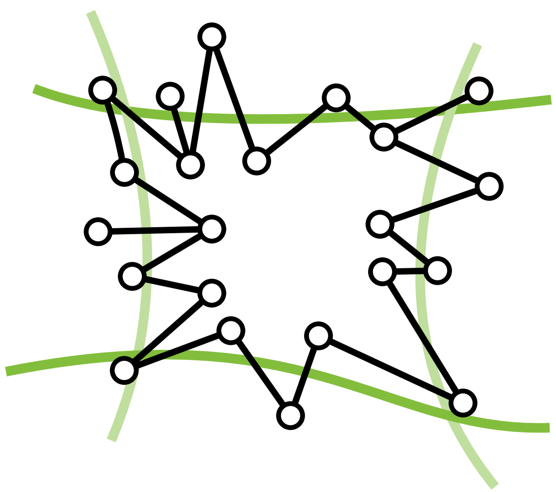

- Assume $|V(C)| > 1$
- move vertices from pseudoline

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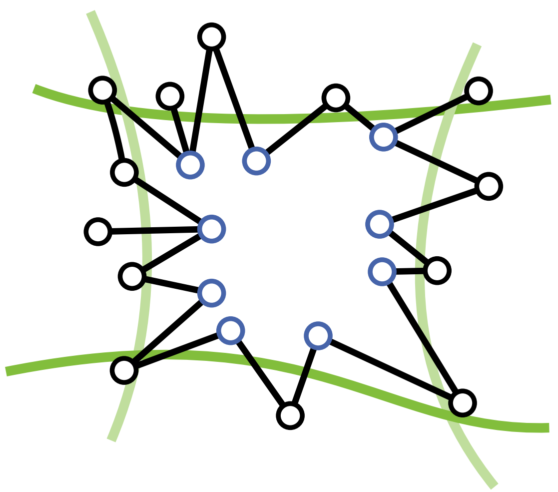

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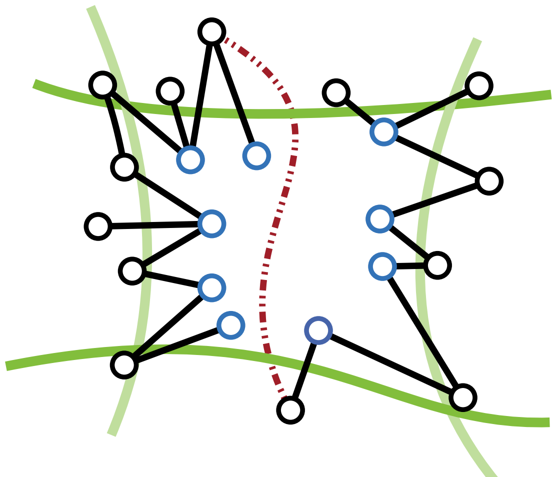

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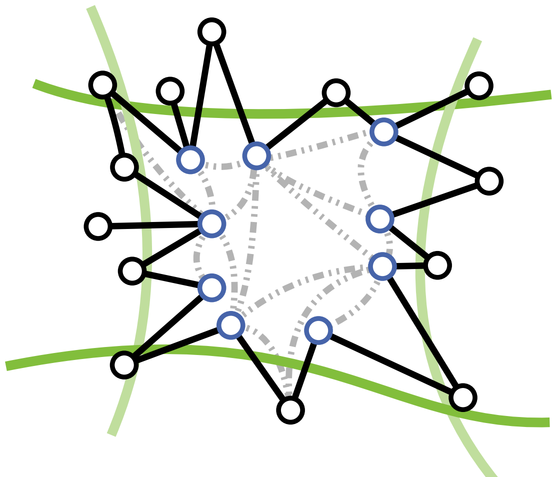

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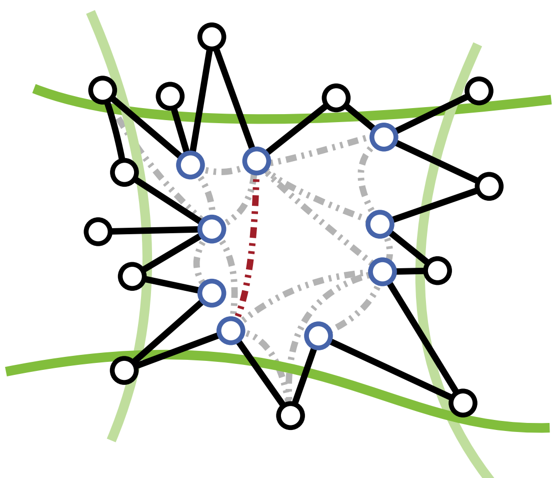


- Assume $|V(C)| > 1$
- move vertices from pseudoline

$\Rightarrow$ pseudolines form a simple cut

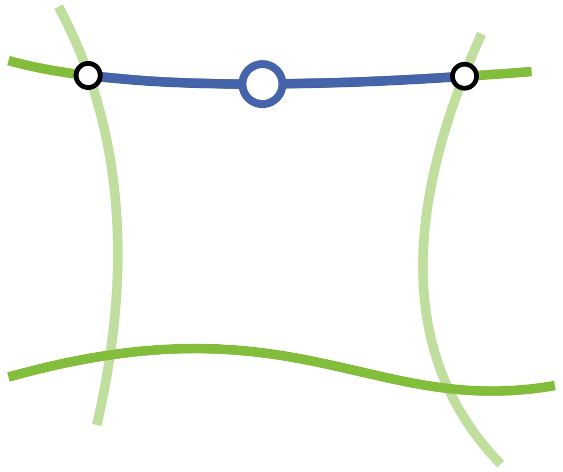
- no long edges

$\Rightarrow$ C is connected

$\Rightarrow$ G contains a free edge ⚡

Claim 2

Claim 2 An aligned vertex is incident to two aligned edges.



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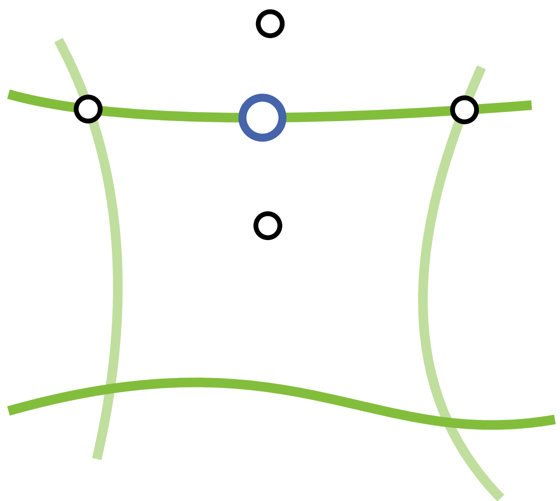

- a vertex on each intersection and in each cell

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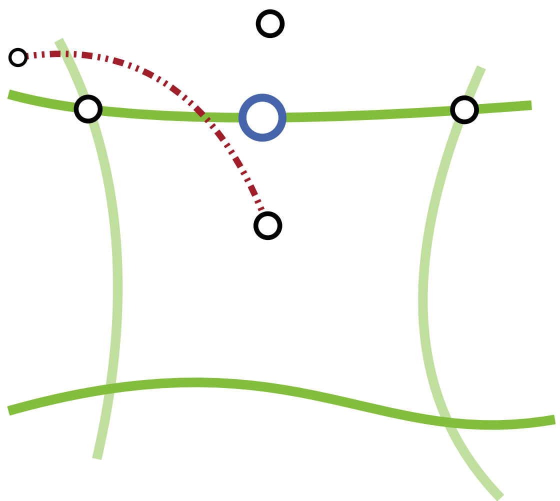

- a vertex on each intersection and in each cell
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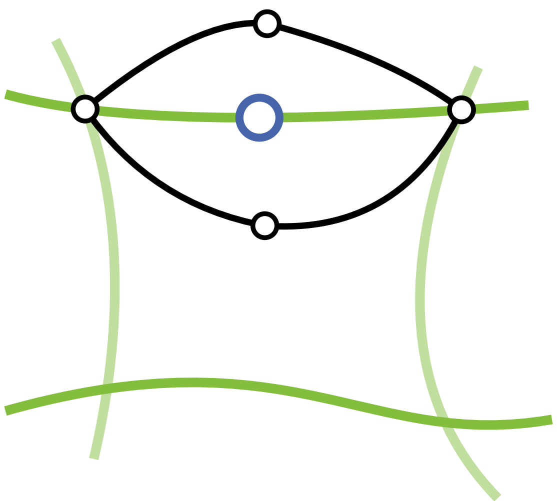

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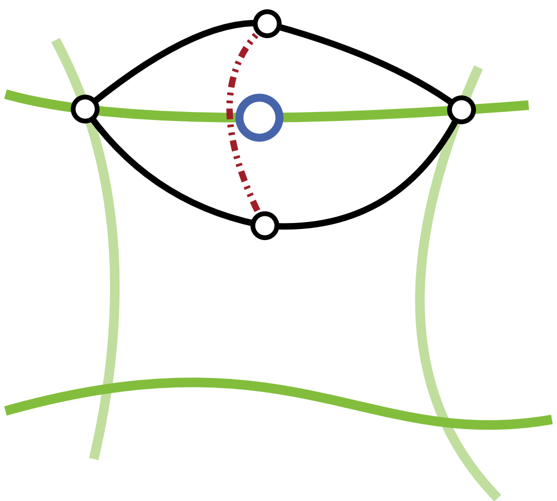

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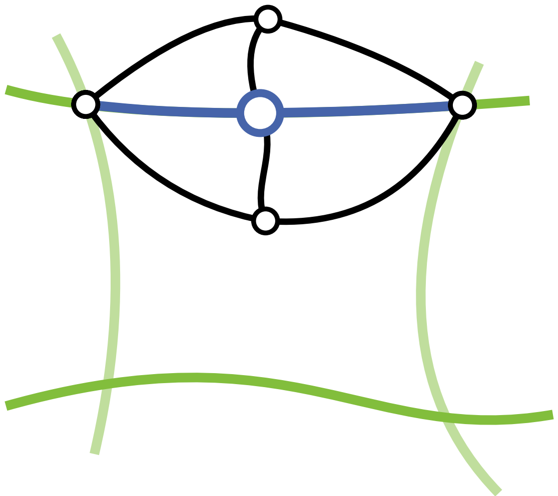

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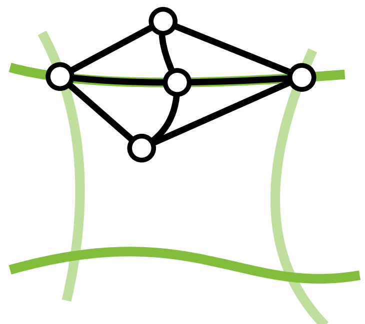
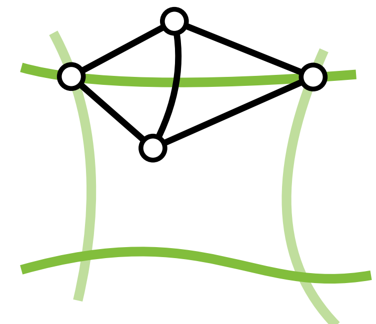
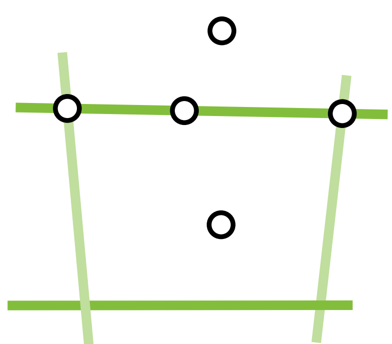
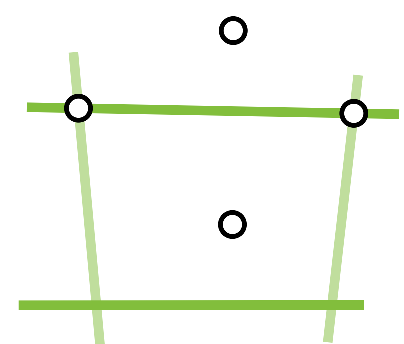
Aligned Drawings of Aligned Graphs

Theorem Every simplified aligned graph has an aligned drawing.

$(G, \mathcal{A}) =$  $\cup$ 

$(\Gamma, A) =$  $\cup$ 

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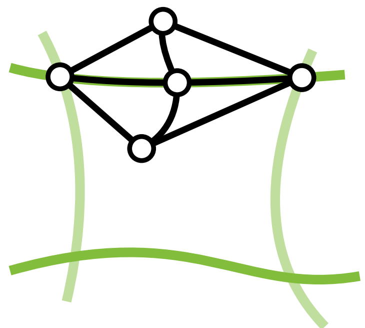
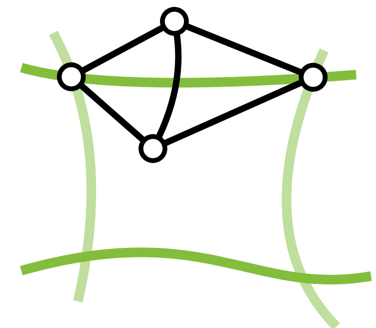
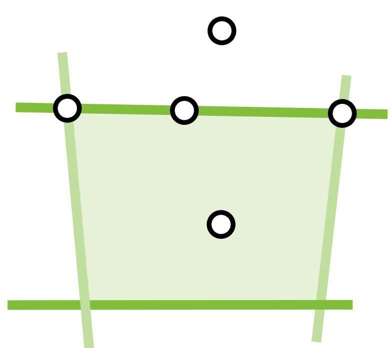
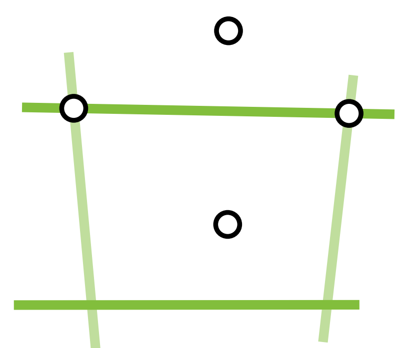
$(G, \mathcal{A}) =$  $\cup$ 

Each cell
is convex

$(\Gamma, A) =$  $\cup$ 

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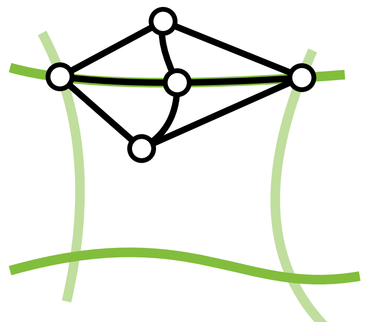
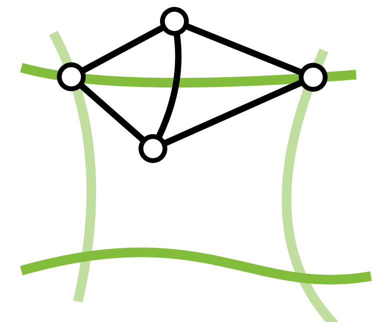
$(G, \mathcal{A}) =$  $\cup$ 

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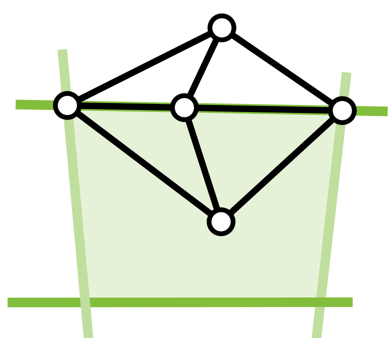
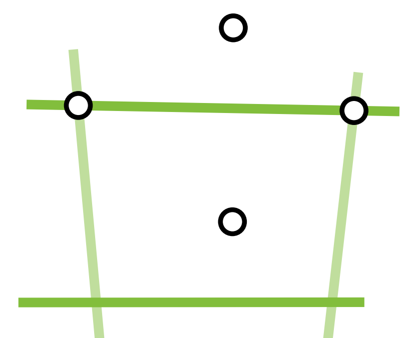
$(\Gamma, A) =$  $\cup$ 

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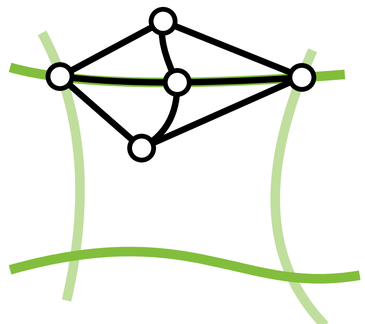
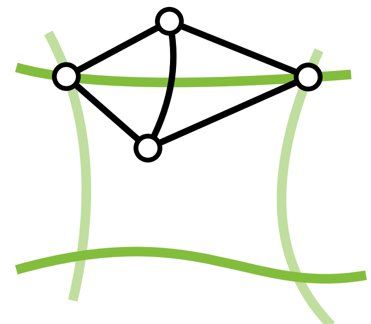
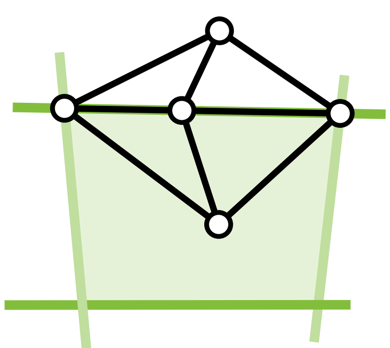
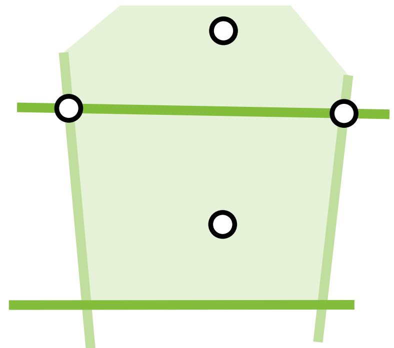
$(G, \mathcal{A}) =$  $\cup$ 

Each cell is convex

Union of two cells is convex

$(\Gamma, A) =$  $\cup$ 

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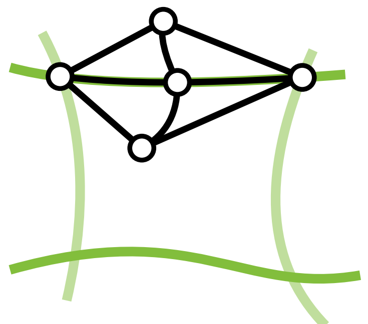

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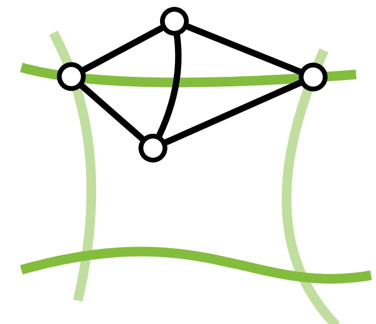

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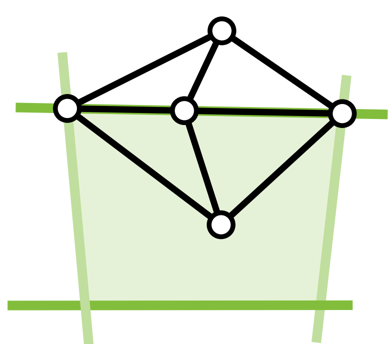

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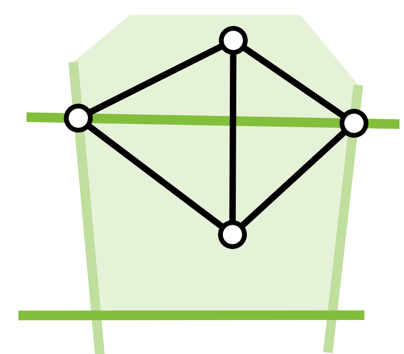
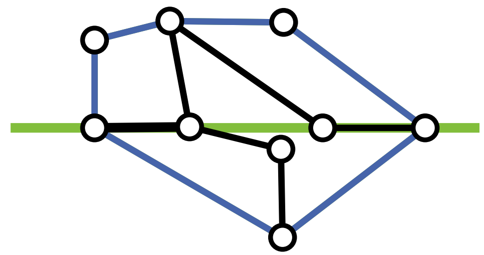


Conclusion

Theorem Every* aligned graph $(G, \{\mathcal{C}\})$ has an aligned drawing with a fixed line and a convex outer face.



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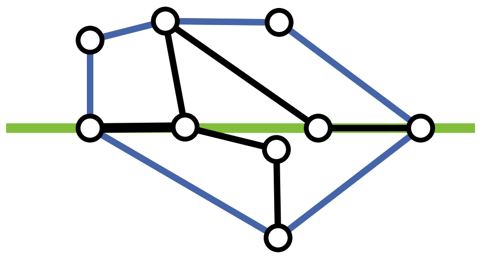
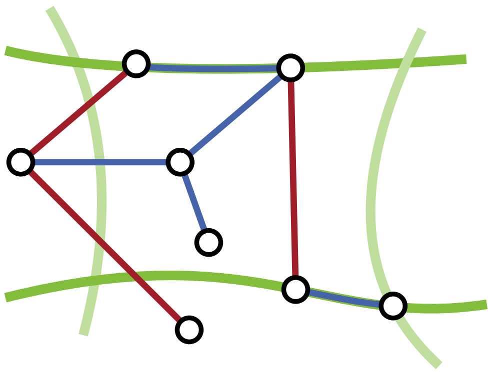


Theorem Every k-aligned graph without long edges has an aligned drawing.



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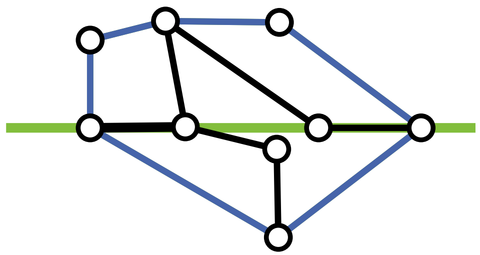

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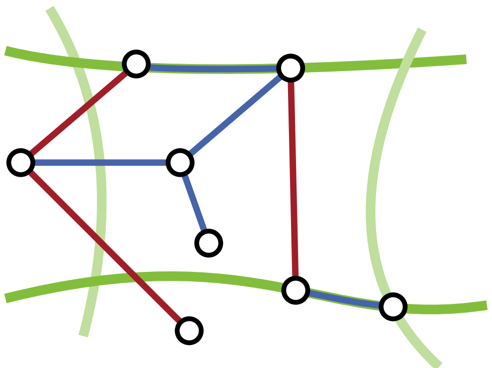
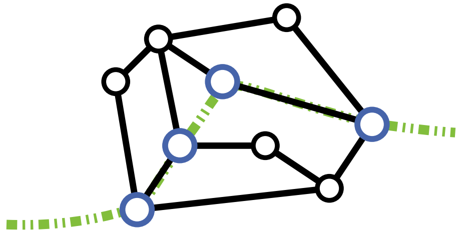
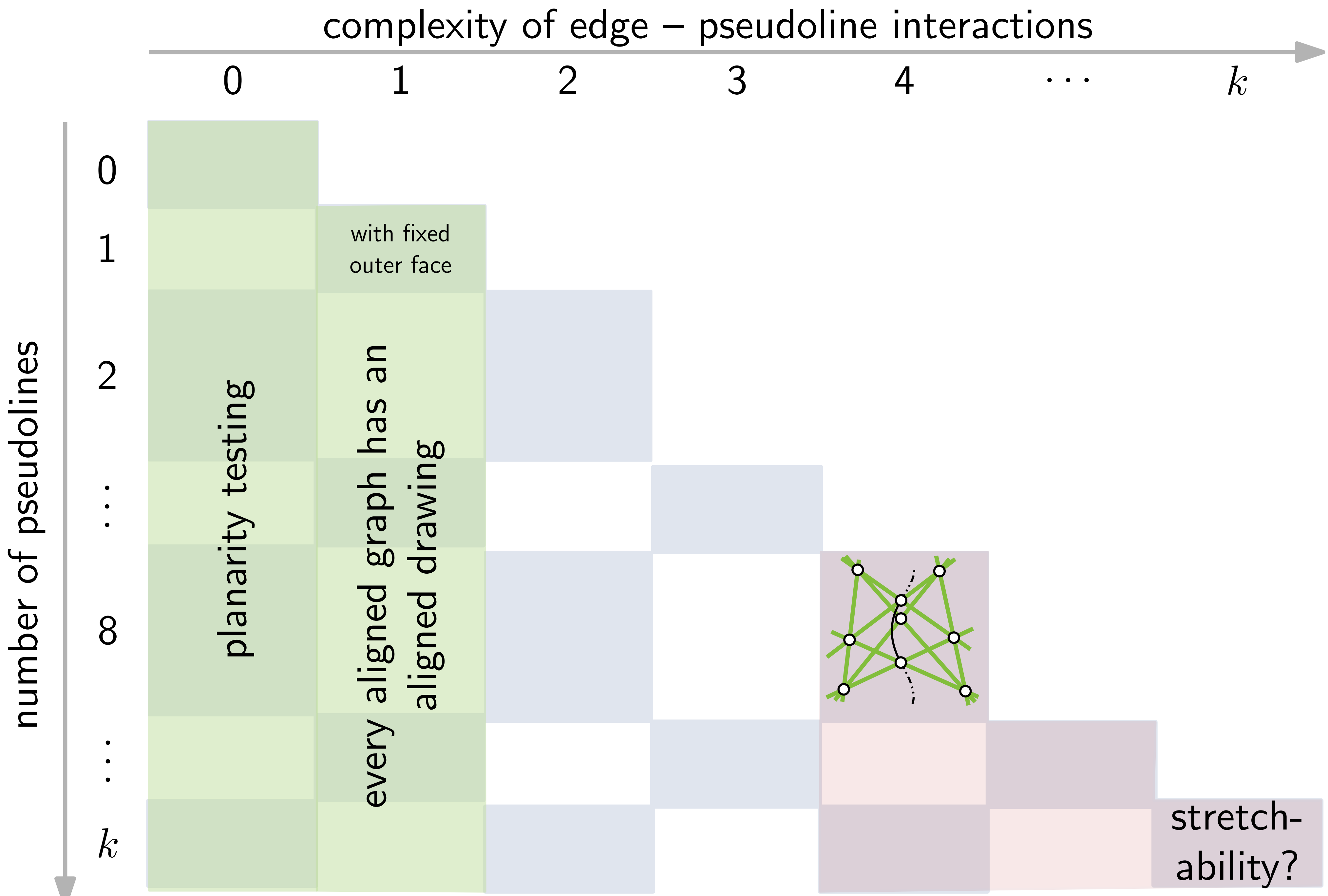


Theorem Passing a pseudoline through a given set S of vertices is $\mathcal{NP}$-hard but FPT in the size of S.



Future Research



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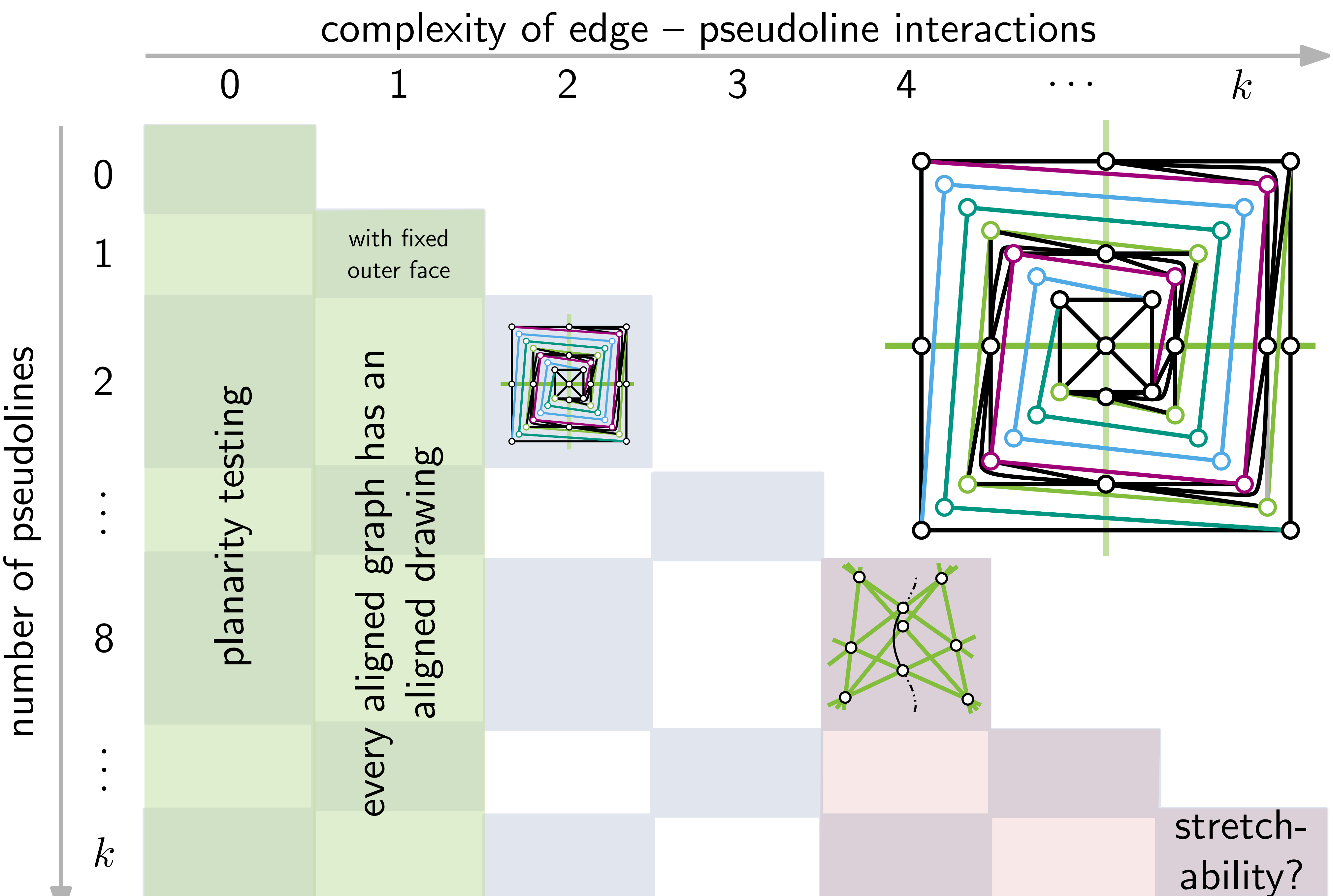

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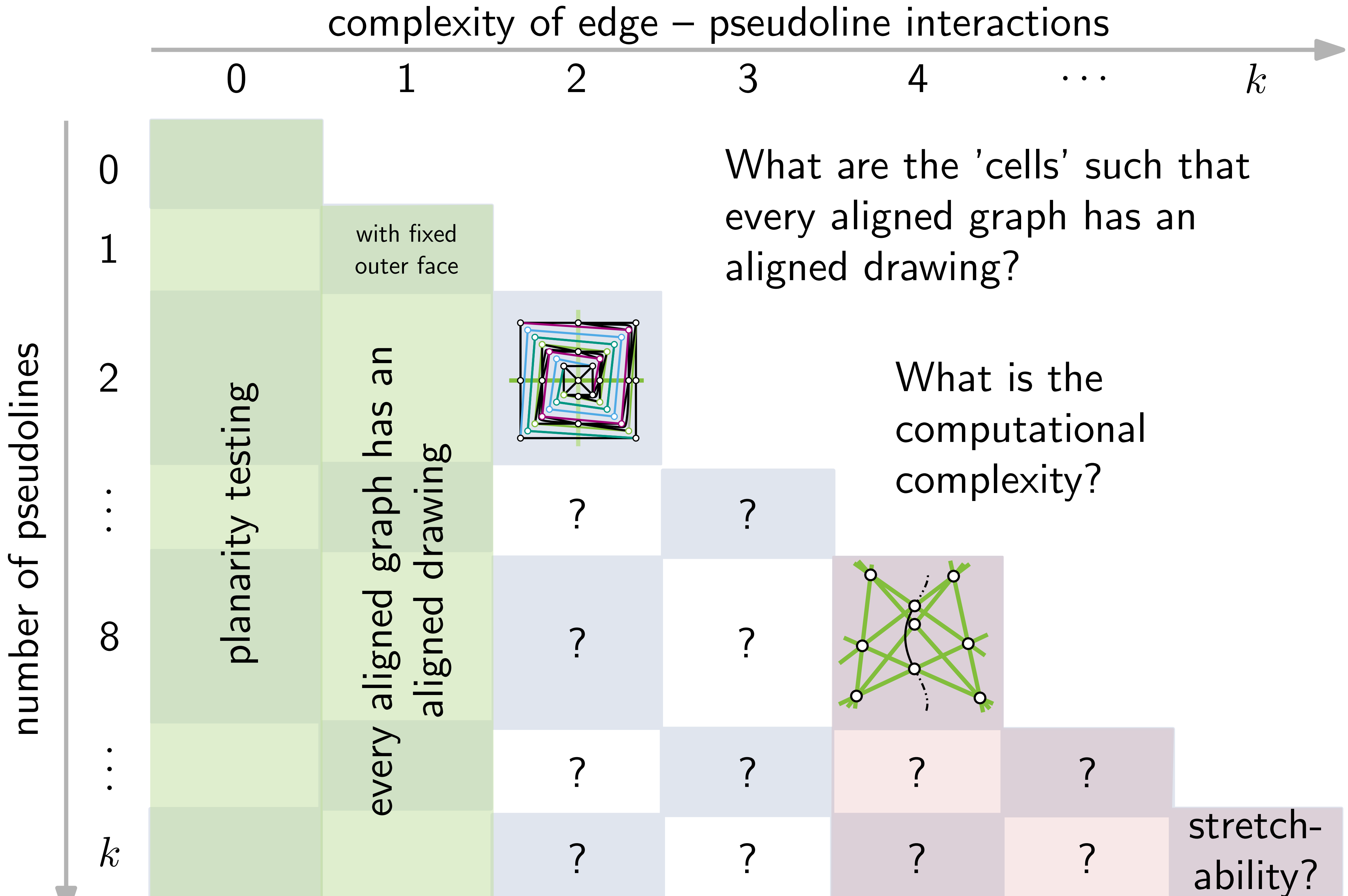

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