Reconstructing Generalized Staircase Polygons with Uniform Step Length

Nodari Sitchinava and **Darren Strash**

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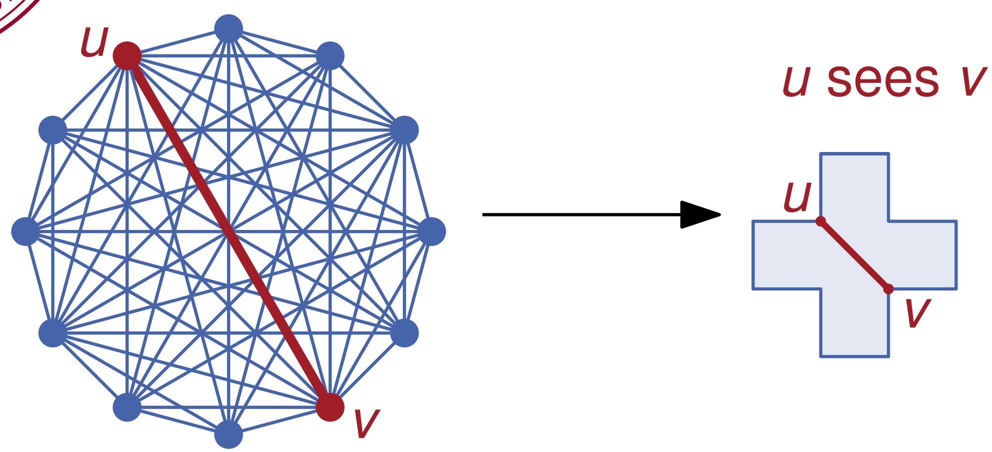
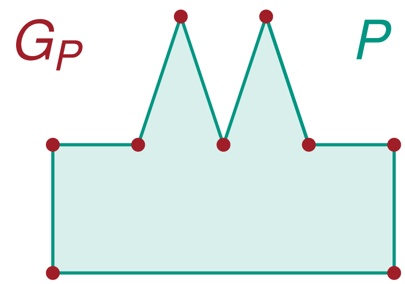
Department of Computer Science
Colgate University



Visibility graphs

Given a polygon P, we construct its *visibility graph* G_P as follows:

- vertex v in $G_P \leftrightarrow v$ is a vertex on P's boundary.
- edge $(u, v) \leftrightarrow u$ sees v. (uv does not intersect exterior of P)



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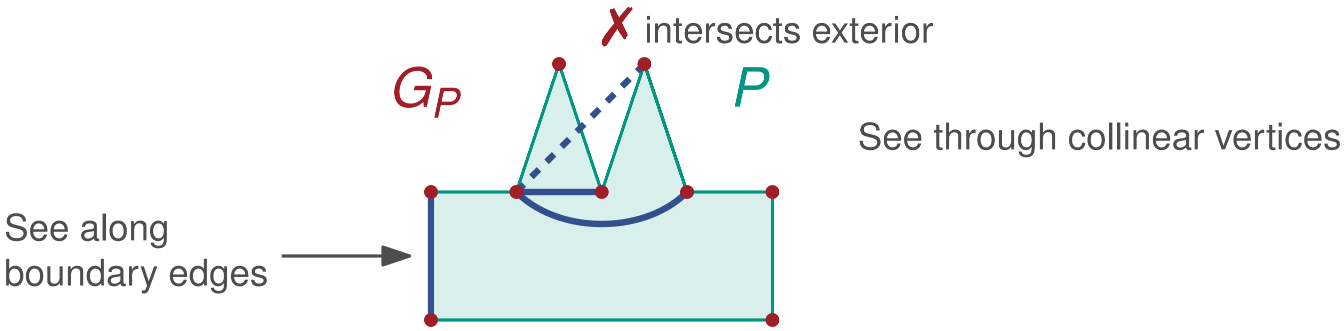

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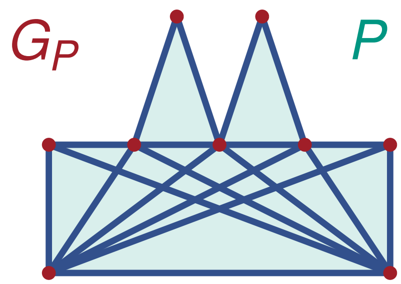

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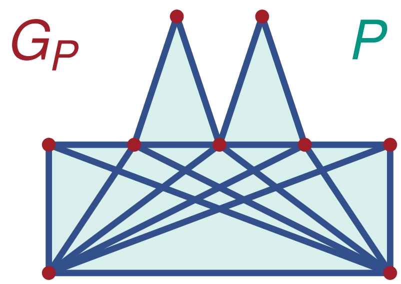
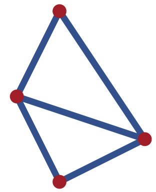


Can be computed in $O(n \log n + m)$ time [Ghosh & Mount, 1991]

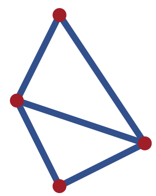
What about the reverse?

Recognition and reconstruction

Input: A graph G:

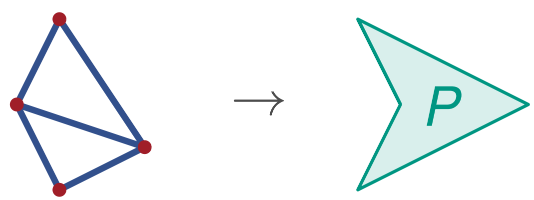


- **Recognition Problem:** Is G the visibility graph of *some* polygon?



$\rightarrow \quad \exists P?$

- **Reconstruction Problem:** Give a polygon P, which has G has its visibility graph.

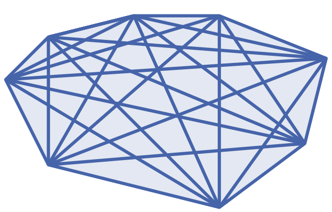


Quick: Known Results

- Recognition is in PSPACE. [Everett, 1994]
- Reconstruction complexity is open
- Special cases: limited visibility due to reflex chains

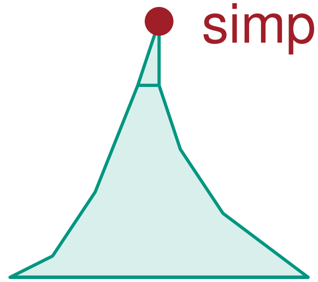
A single clique

convex



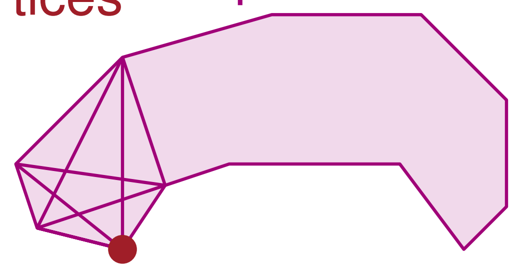
tower

simplex vertices



[Choi et al., 1995]

spiral

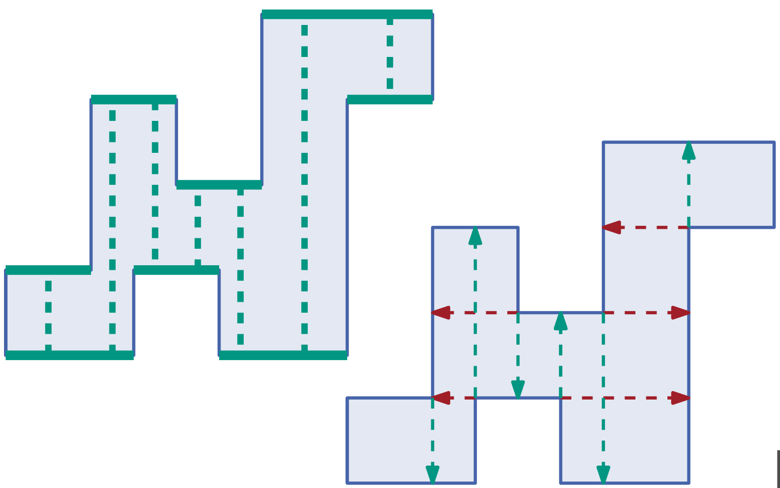


[Everett & Corneil, 1995]

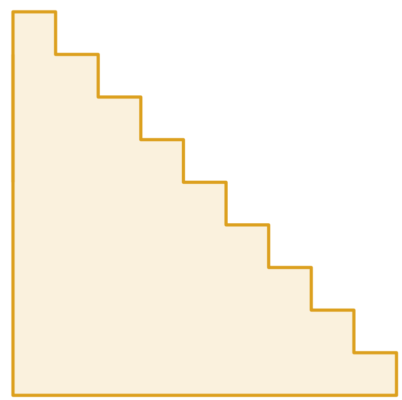
Quick: Known Results

- Recognition is in PSPACE. [Everett, 1994]
- Reconstruction complexity is open
- Special cases: limited visibility due to reflex chains
- Orthogonal polygons?

Alternative visibility definitions

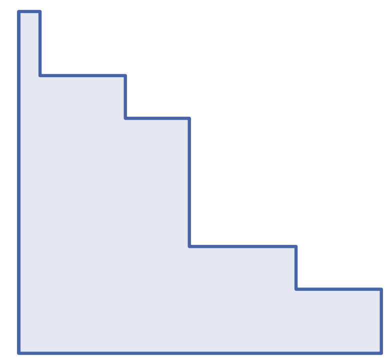


Reconstruction



[Abello & Eğecioğlu, 1993]

Recognition

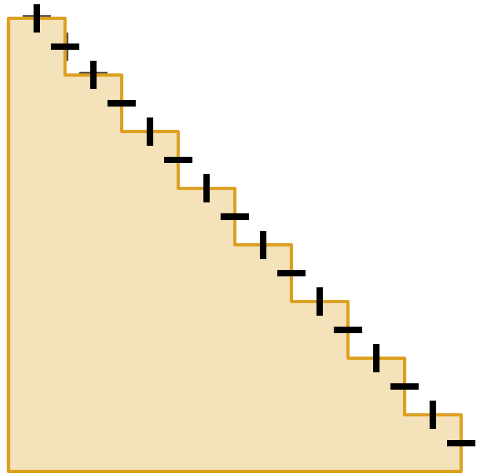


[Abello et al., 1995]

Uniform-Step Length Polygons

The *only* reconstruction result is for staircase polygons with uniform length [Abello & Eğecioğlu, 1993]

All "steps" have same length

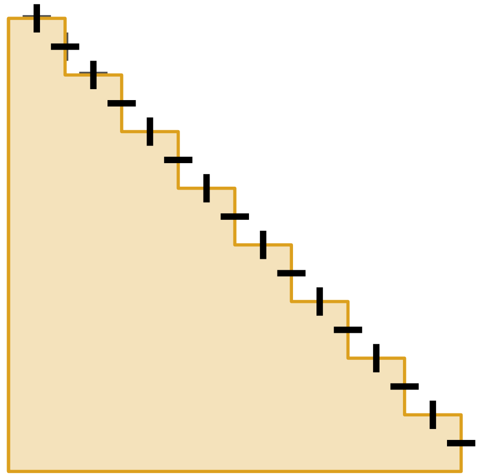


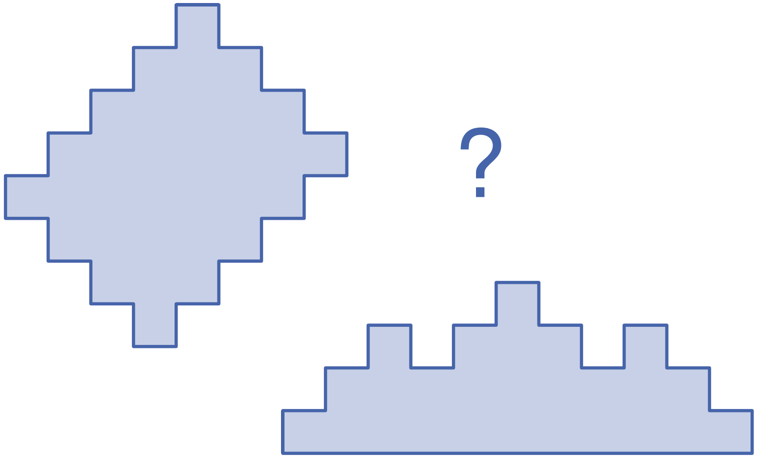
Uniform-Step Length Polygons

The *only* reconstruction result is for staircase polygons with uniform length [Abello & Eğecioğlu, 1993]

Can we *efficiently* reconstruct polygons with more staircases?

All “steps” have same length

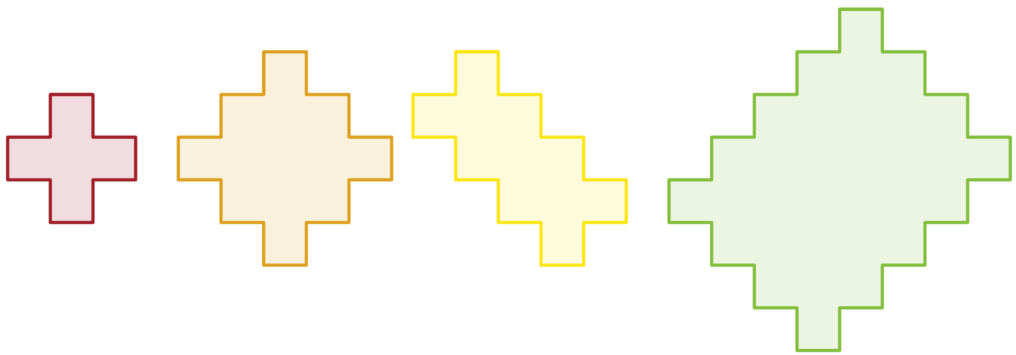




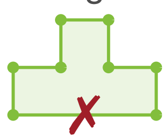
Our Results

Uniform-length orthogonally convex polygons can be reconstructed $O(n^2m)$ time. (n vertices, m edges.)

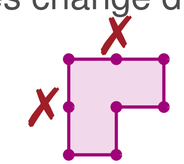
unit-length orthogonally convex



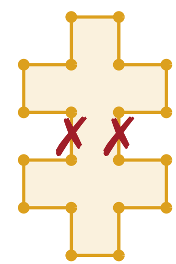
same edge lengths



edges change direction



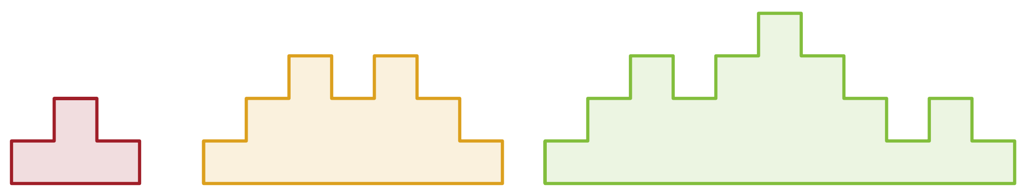
no dents

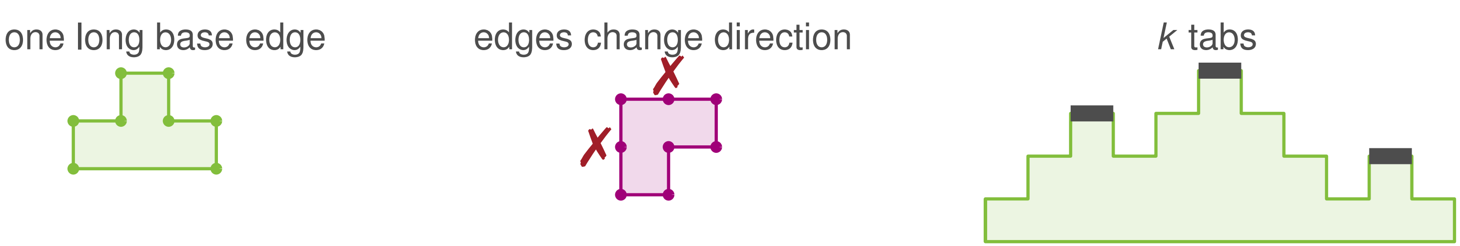


Our Results

Histogram reconstruction is fixed parameter tractable on the number of tabs k, with time $O(n^2 m + (k-2)!2^{k-2}(n \log n + m))$.

uniform-length histogram

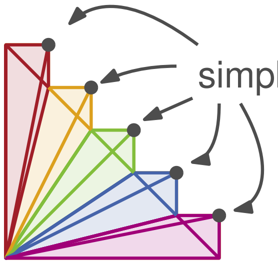




Simplicial vertices and edges

Maximal clique in $G_P \leftrightarrow$ **maximal** convex subpolygon on vertices of P.

- Staircase polygons have simplicial vertices



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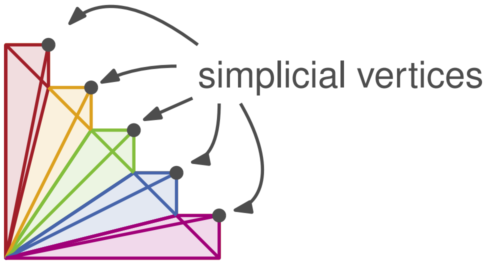
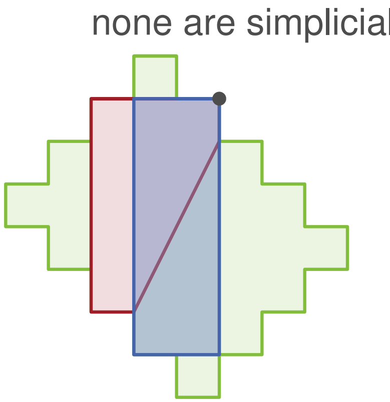


- No *simplicial vertices*? $\rightarrow$ we need new techniques



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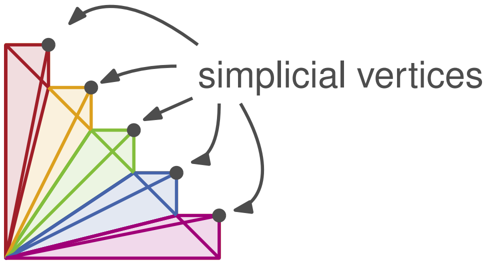

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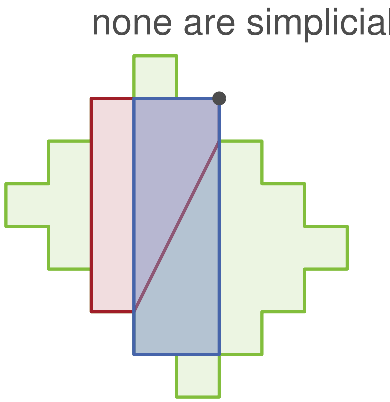
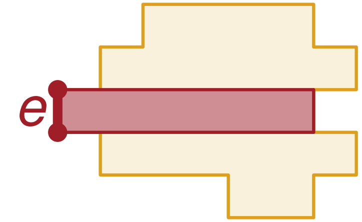


- Idea: Evaluate *edges* in only one maximal clique

  $\rightarrow$ 1-simplicial edges

  $\rightarrow$ identify convex-convex boundary edges



1-simplicial edges

Most expensive operation is testing each edge for membership in exactly 1 maximal clique.

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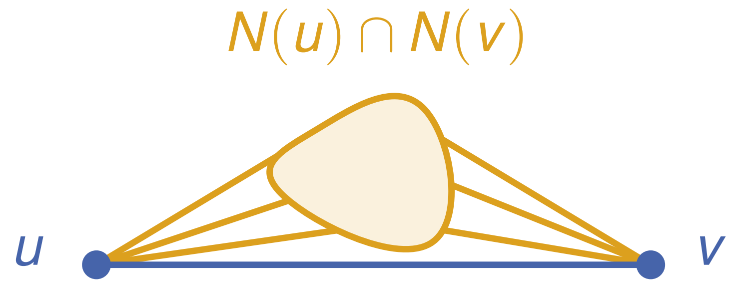
2 Steps:

$\rightarrow$ for (u, v), find common neighborhood $O(n)$ time



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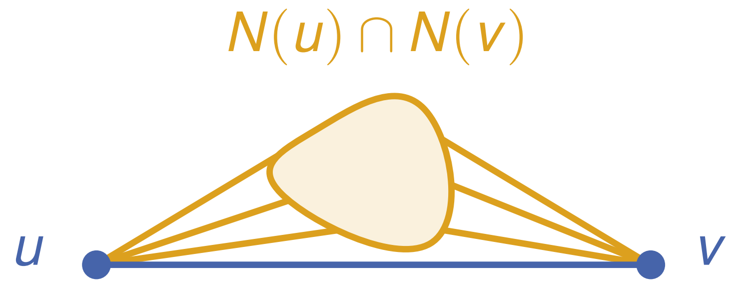
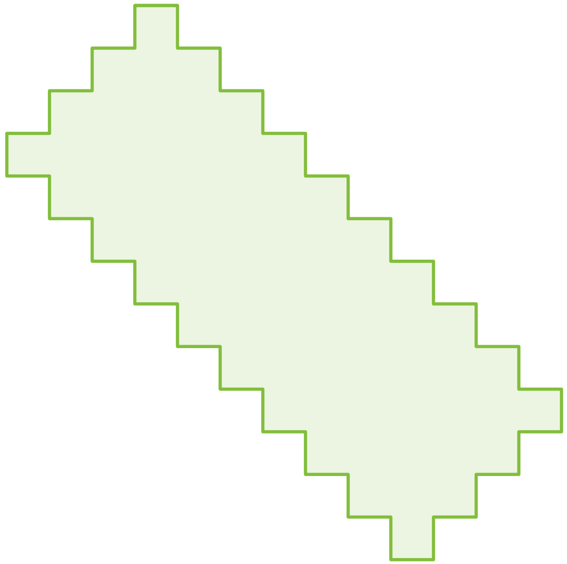


$\rightarrow$ Test if $N(u) \cap N(v)$ is a clique in $O(n^2)$ time

For all m edges, this takes $O(n^2 m)$ total time.

Convex case: the algorithm

Step 0: Determine convex and reflex vertices.



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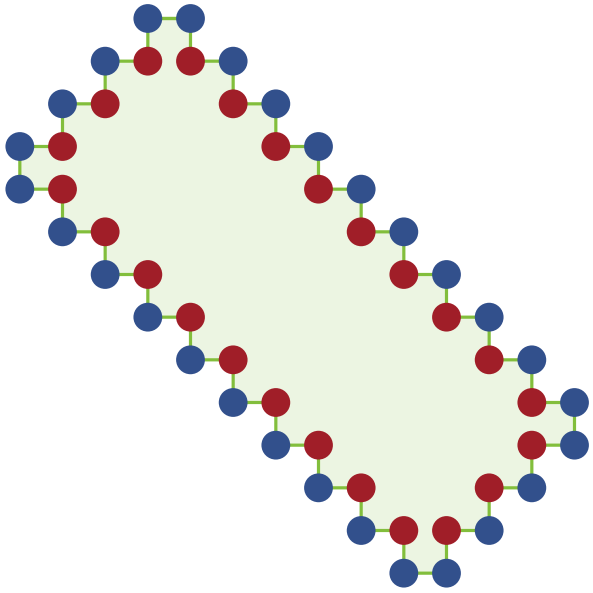
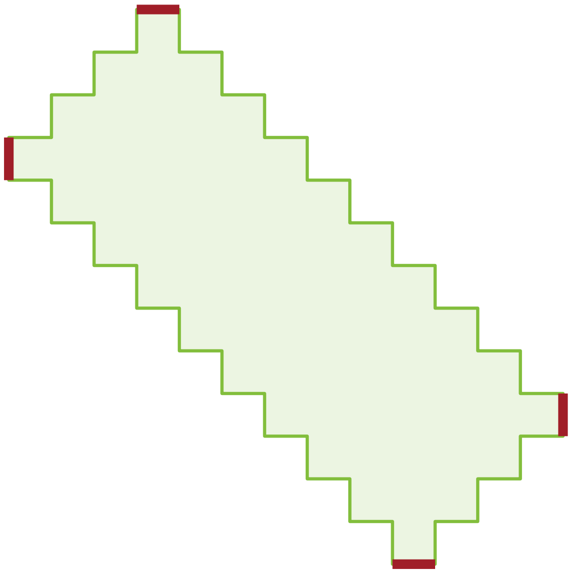


Convex case: the algorithm

Step 0: Determine convex and reflex vertices.

Step 1: Find tab edges



Convex case: the algorithm

Step 0: Determine convex and reflex vertices.

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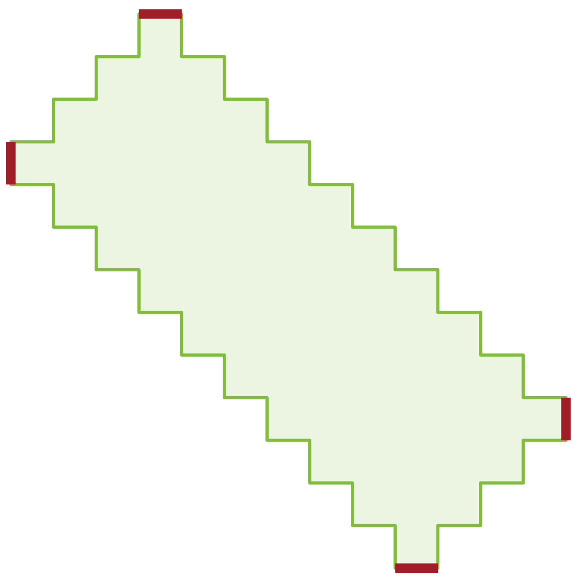
Step 2: Choose one to be the topmost



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Step 0: Determine convex and reflex vertices.

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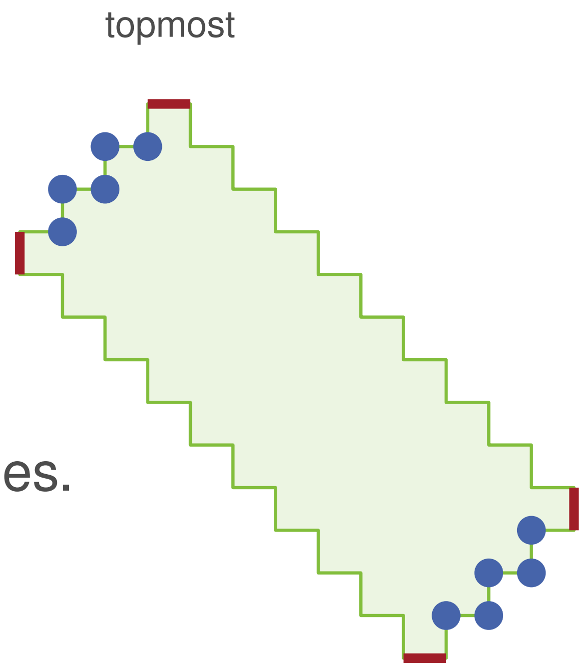
Step 3: Determine vertices on “short” staircases.



Convex case: the algorithm

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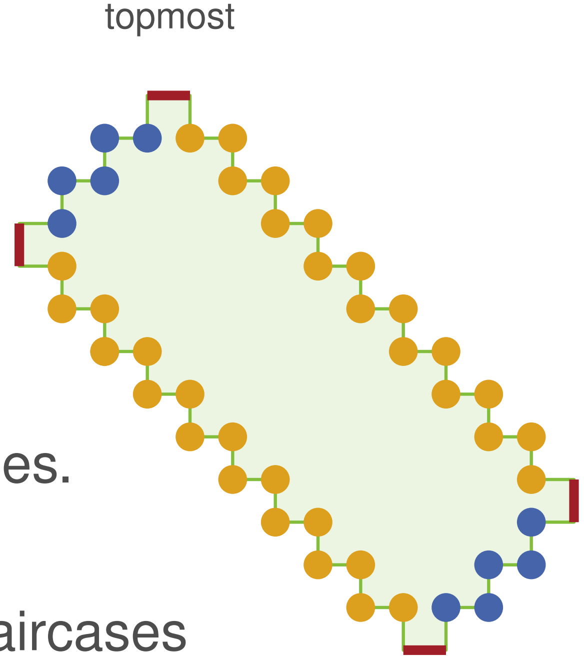
Step 3: Determine vertices on “short” staircases.

Step 4: Assign remaining vertices to “long” staircases



Convex case: the algorithm

Step 0: Determine convex and reflex vertices.

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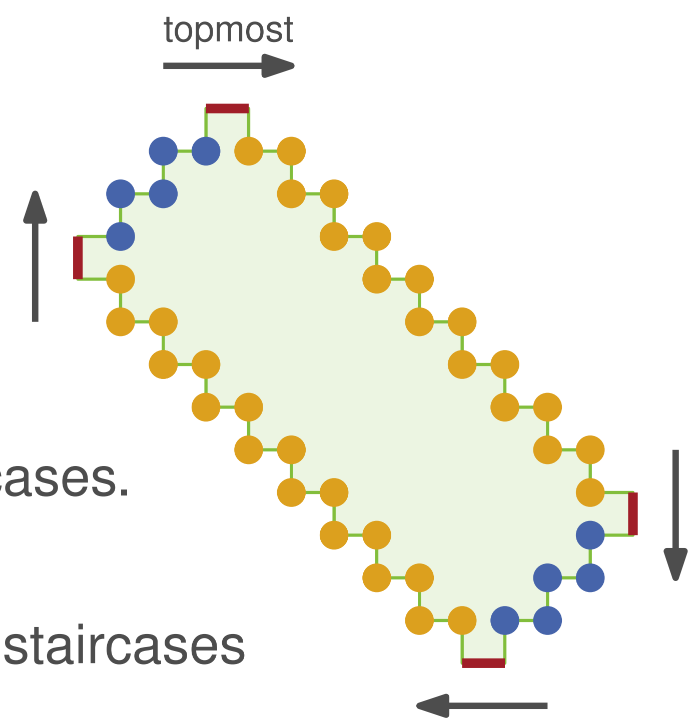
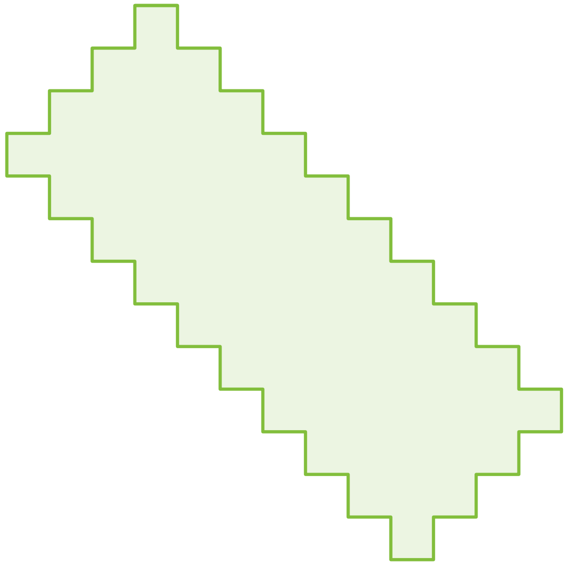
Step 5: Orient & construct the boundary



Convex/Reflex Vertices

Maximal clique in $G_P \leftrightarrow$ **maximal** convex subpolygon on vertices of P.



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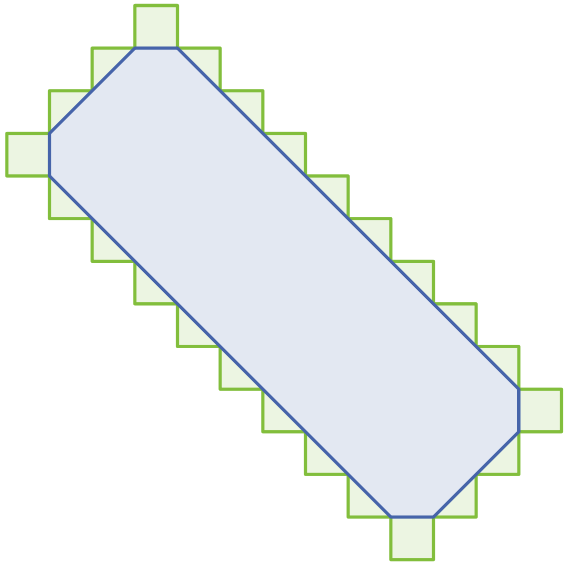

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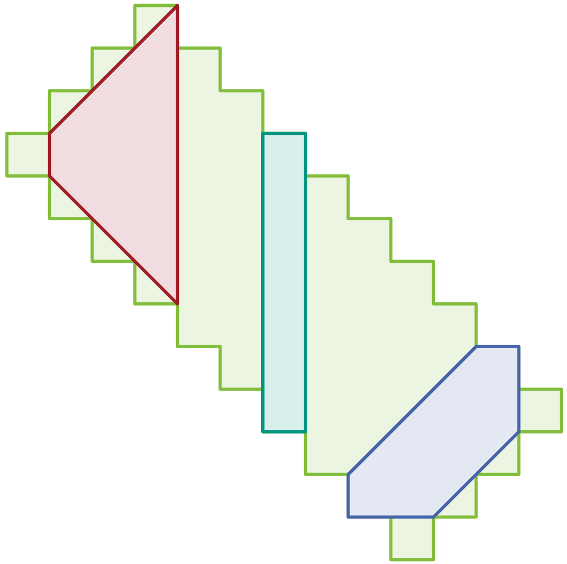

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Convex/Reflex Vertices

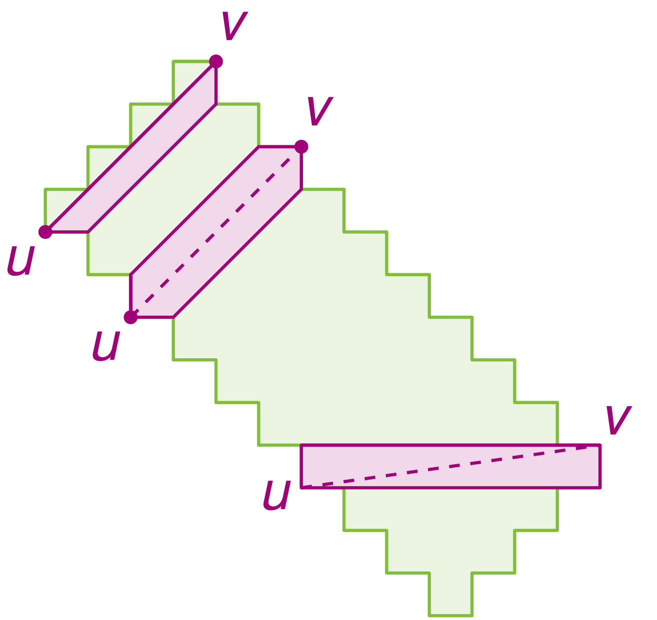
Maximal clique in $G_P \leftrightarrow$ **maximal** convex subpolygon on vertices of P.

Structural lemma:

Every convex vertex u has a convex neighbor v, such that (u, v) is in one maximal clique.

and

Edges incident to a reflex vertex are in two or more maximal cliques



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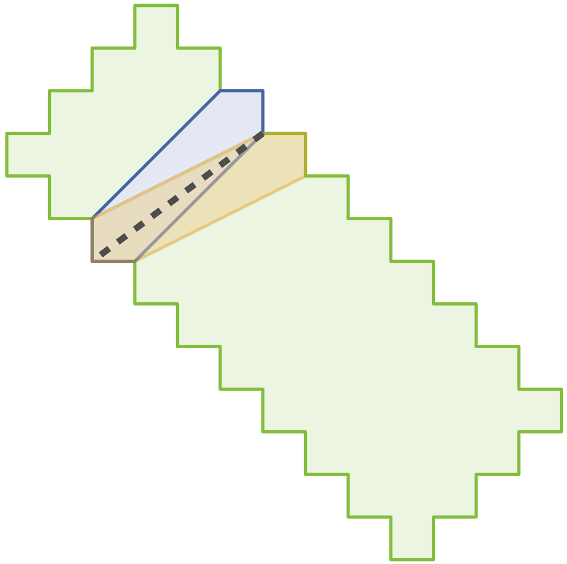

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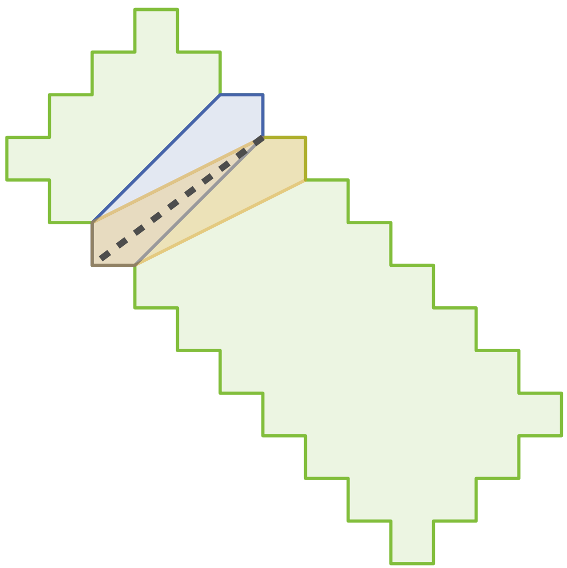


Now Simple:

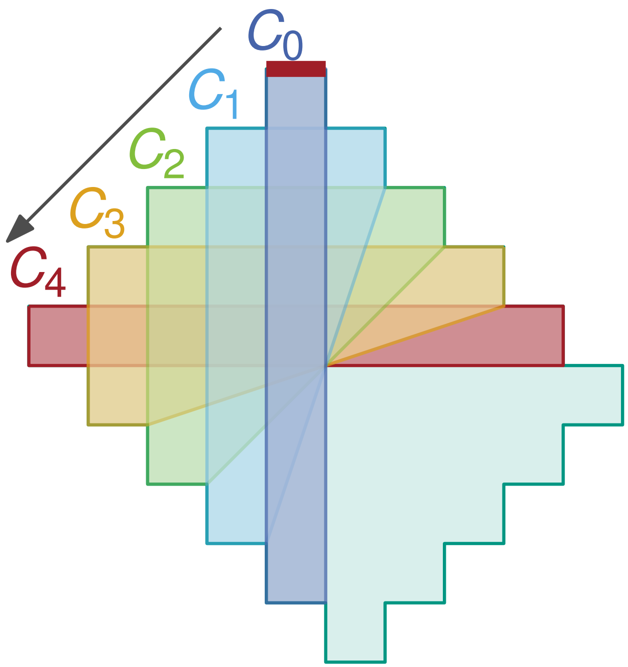
Compute all convex vertices by computing all edges contained in exactly one maximal clique.

Initial staircase reconstruction

Elementary clique:

A maximal clique containing 3 convex vertices*.

Elementary cliques can be **ordered**, starting from a tab.



Initial staircase reconstruction

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A maximal clique containing 3 convex vertices*.

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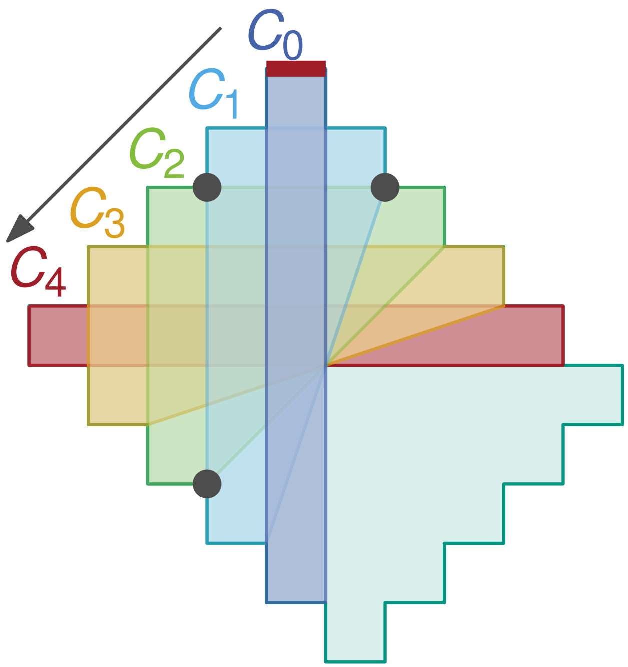
C_i shares 3 reflex vertices with C_{i-1}

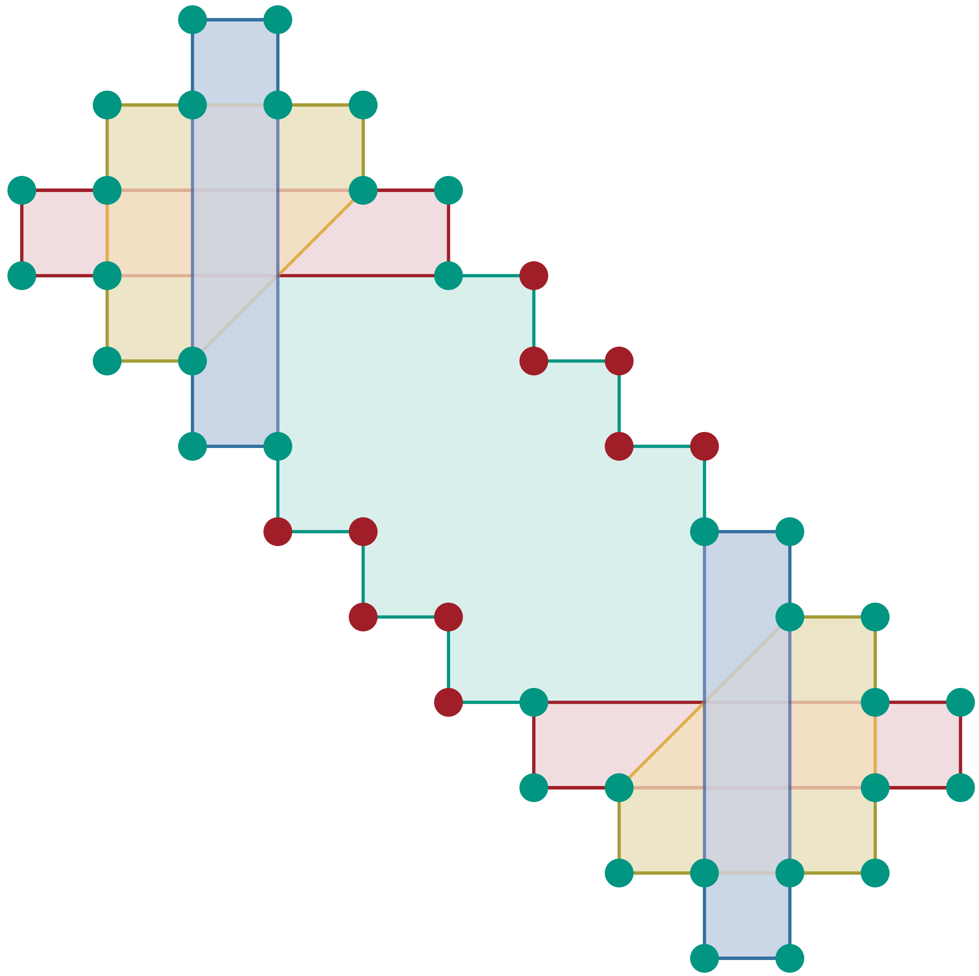
$\rightarrow$ Orders the clique vertices along **some** staircase.



Filling in remaining staircases

We know some vertices...

But some vertices remain after looking at all elementary cliques.



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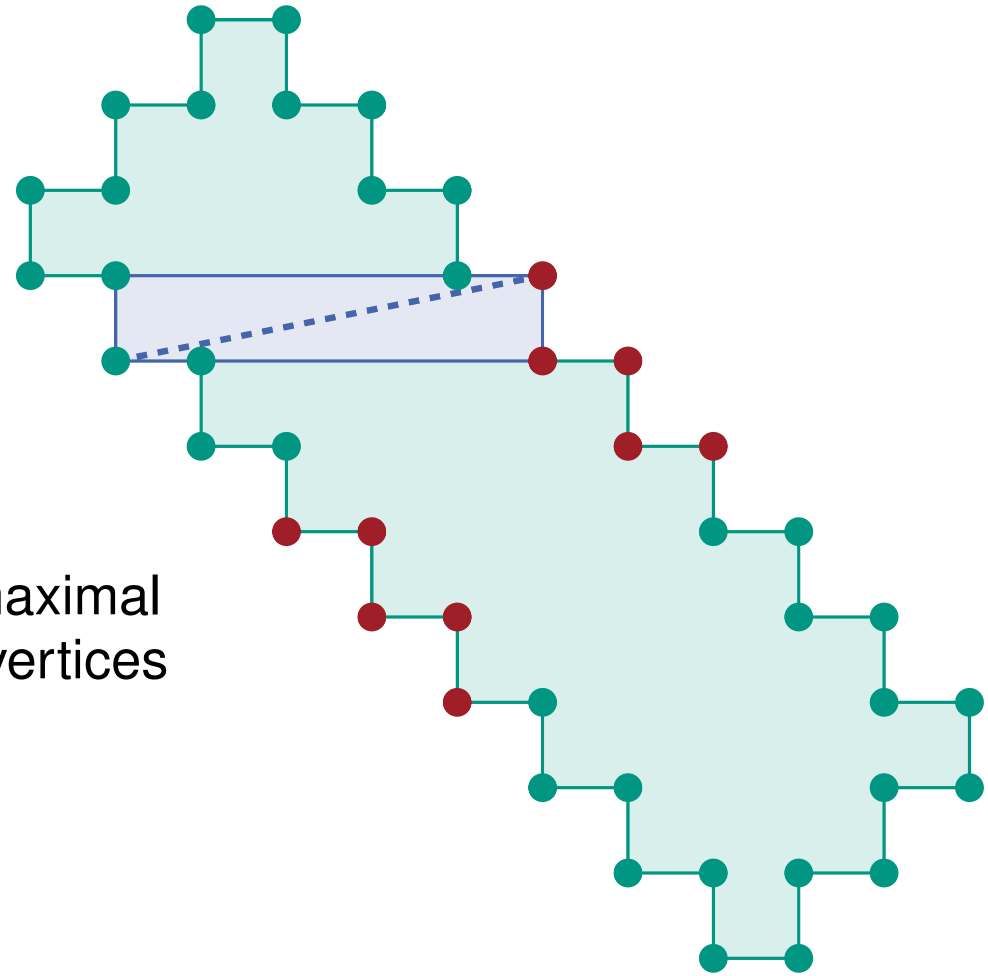
Construct via “rectangular” maximal cliques from **known** convex vertices to **unknown** convex vertices.



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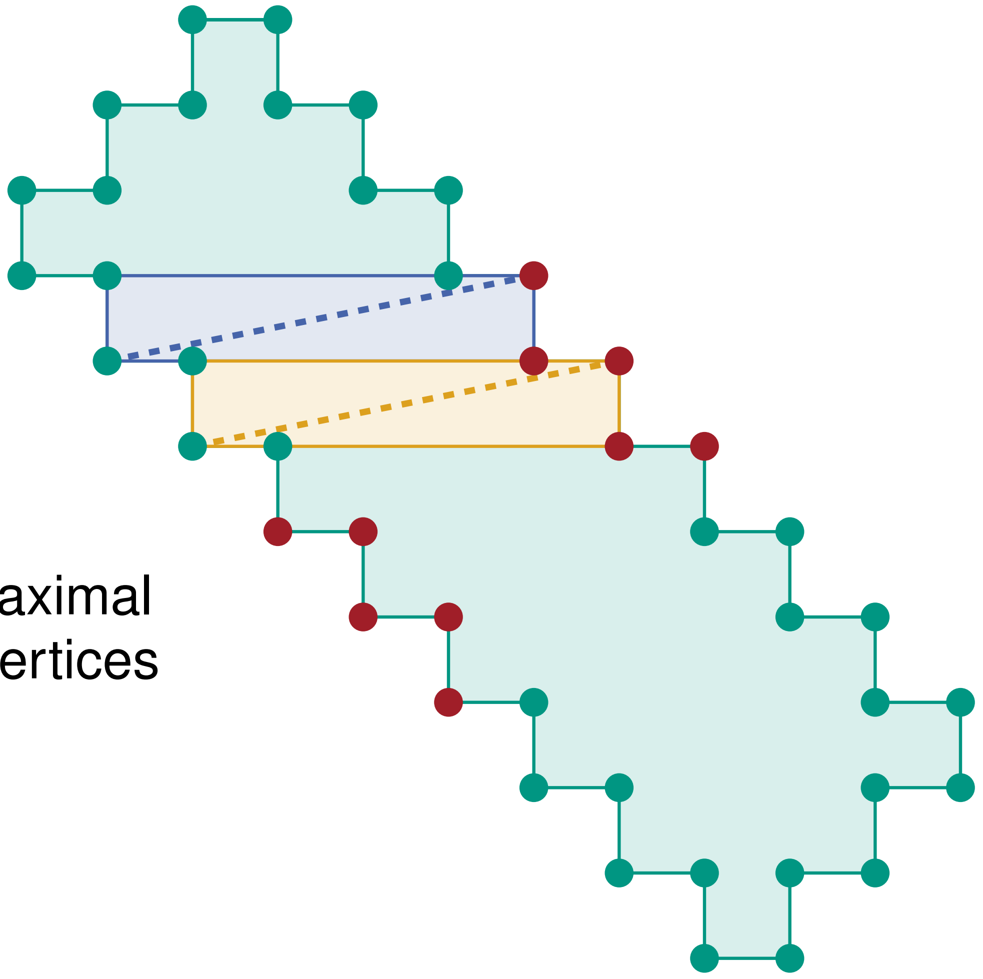

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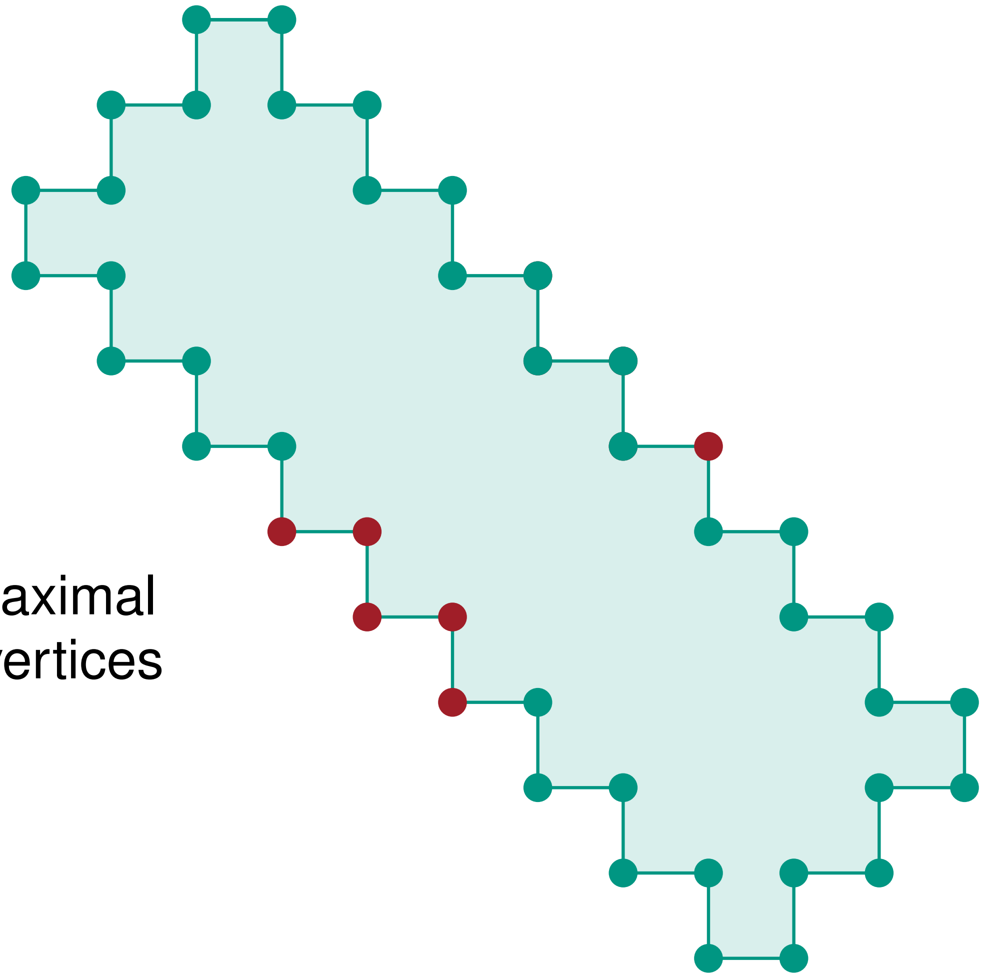

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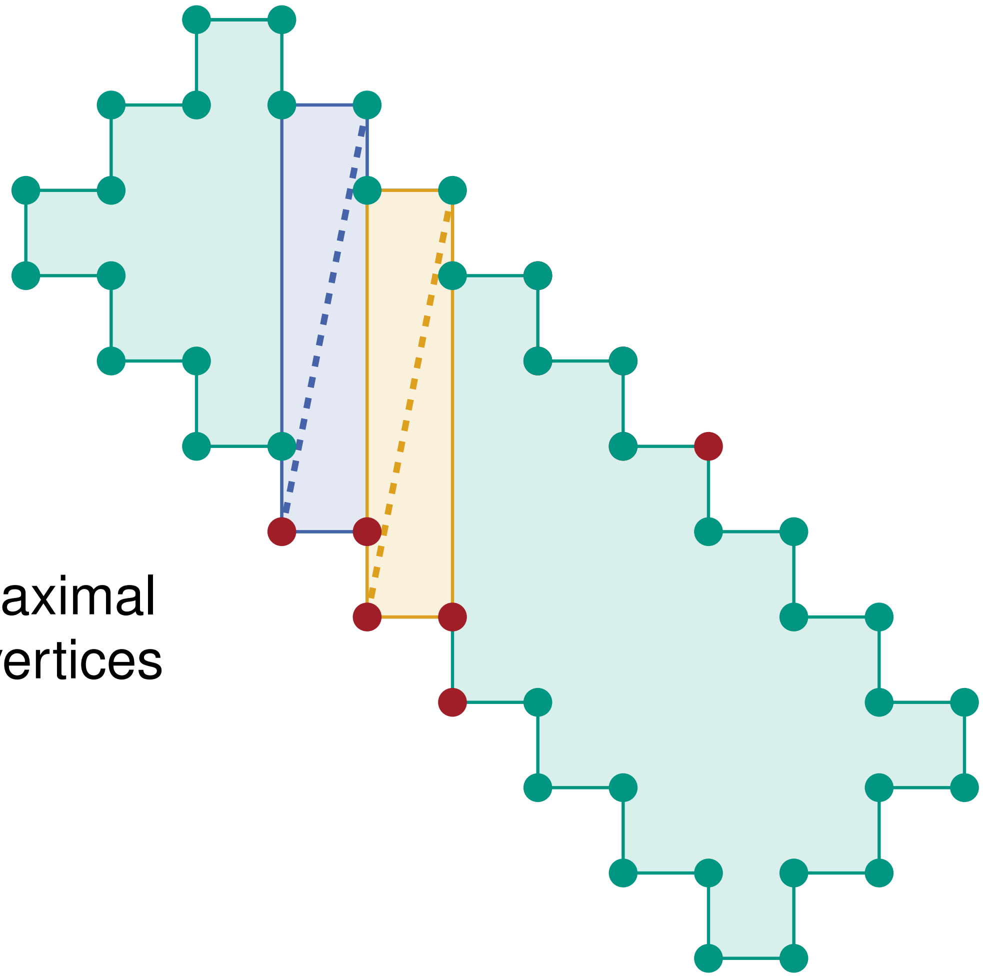

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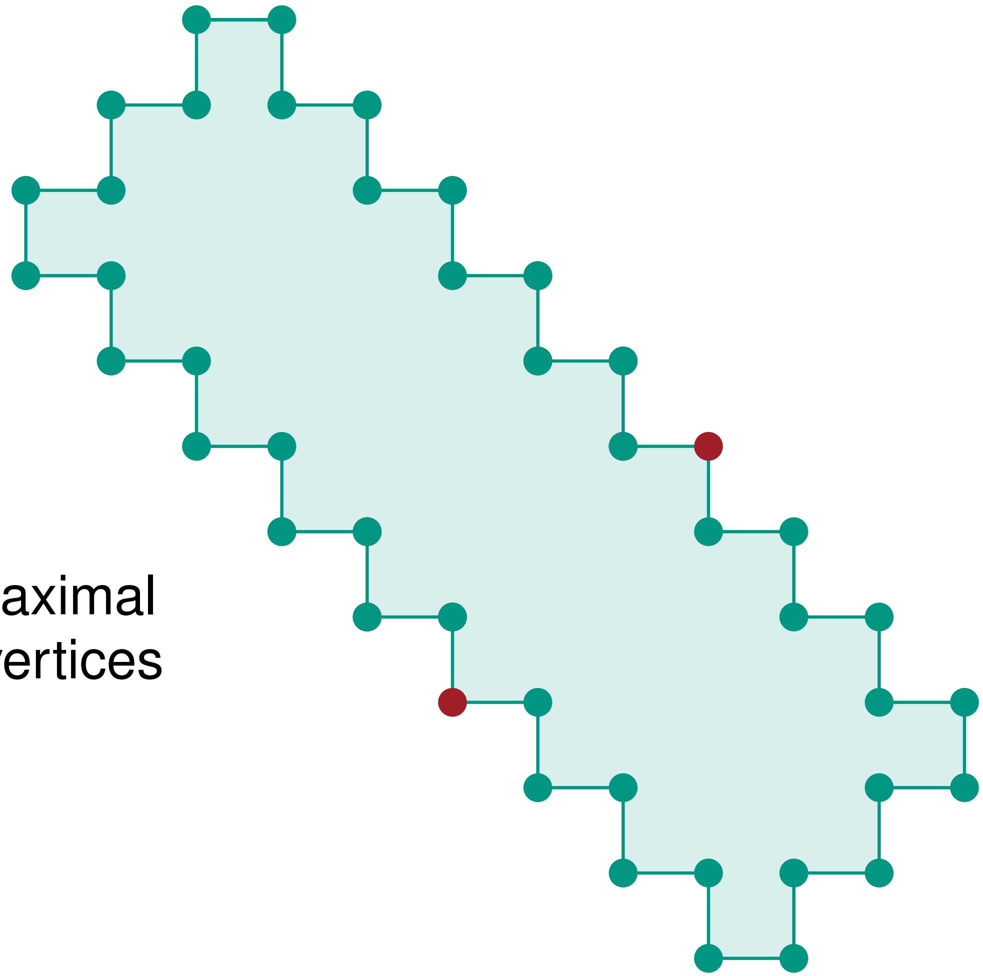

Filling in remaining staircases

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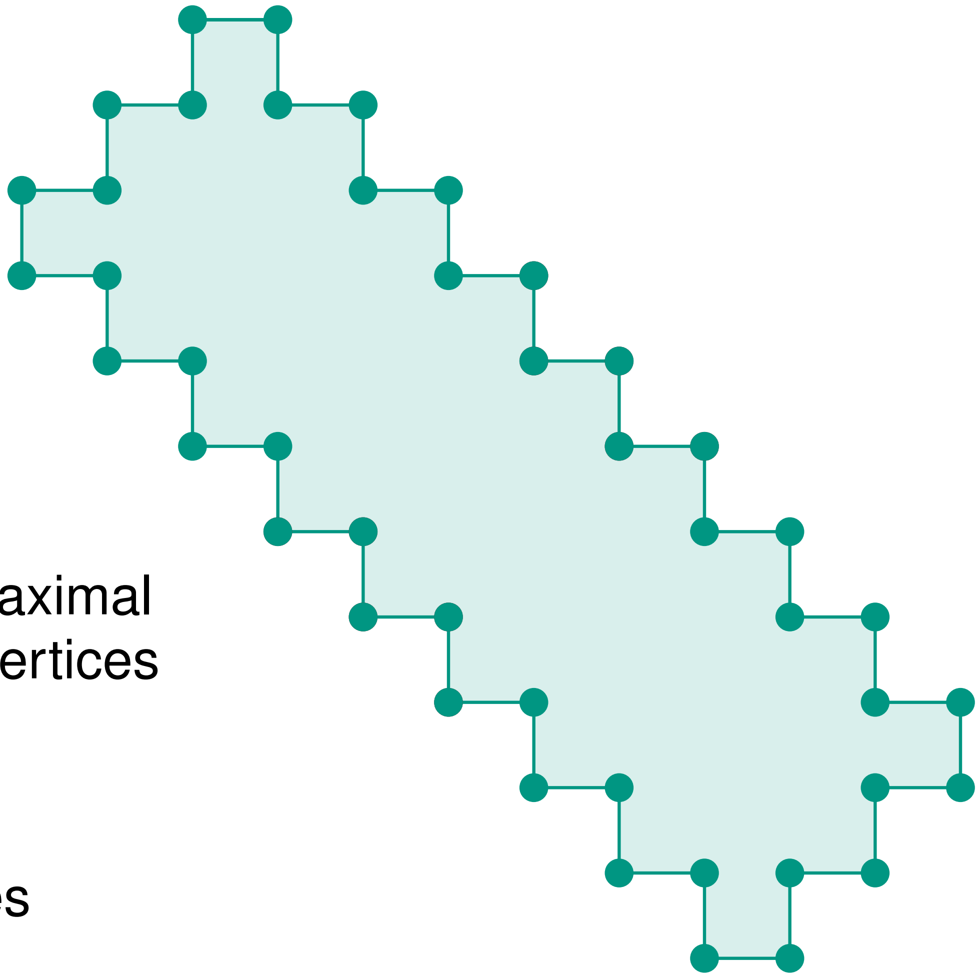
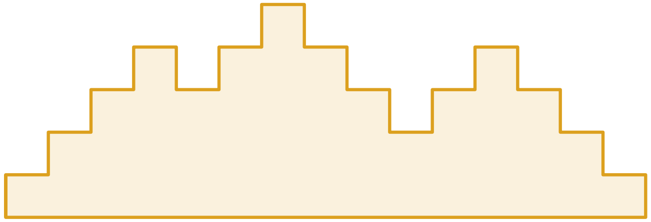
Construct via “rectangular” maximal cliques from **known** convex vertices to **unknown** convex vertices.

Gives us all remaining vertices

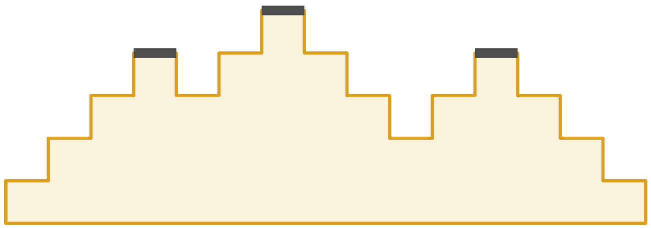


Histograms: high-level algorithm



Histograms: high-level algorithm

Step 1: Determine tabs



Histograms: high-level algorithm

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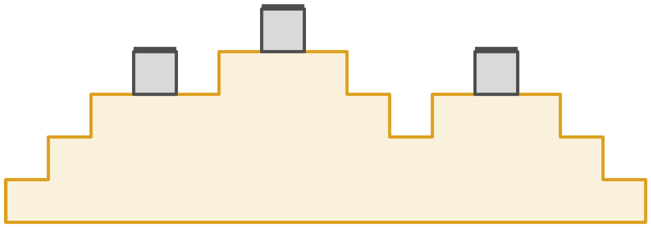
Step 2: Remove and repeat



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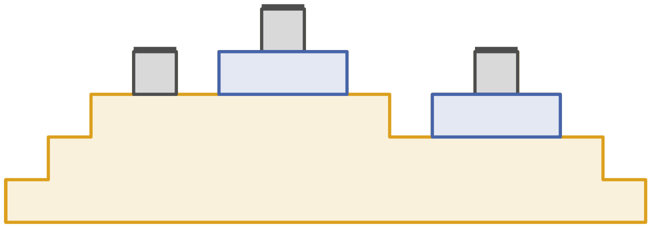

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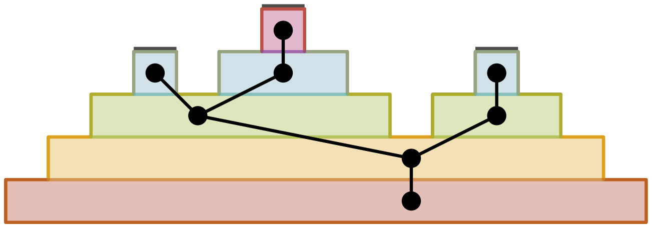

Step 2: Remove and repeat

Step 3: Construct a contract tree of all rectangles

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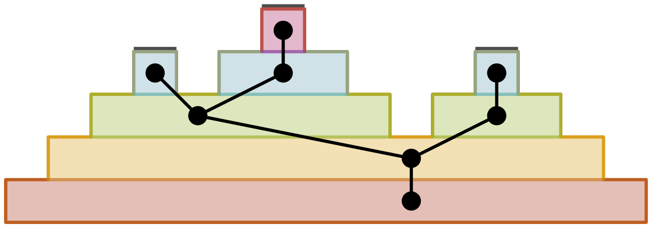

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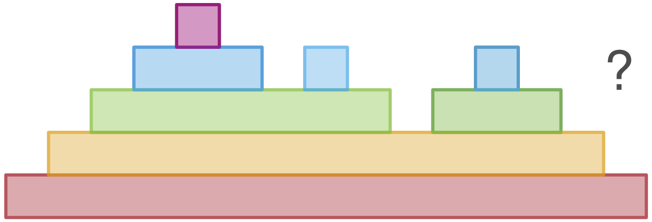
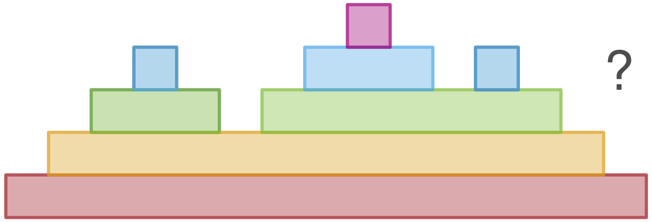
Step 3: Construct a contract tree of all rectangles

Step 4: Order the tree to construct the polygon





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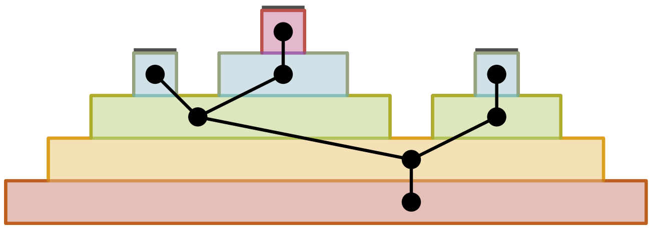

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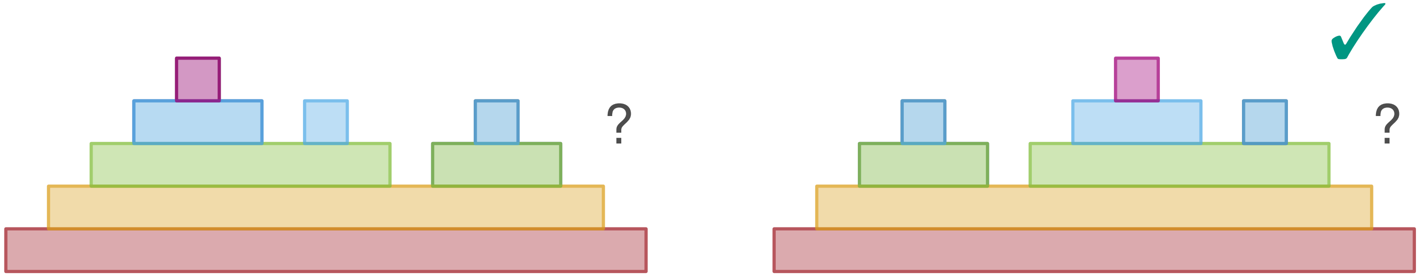
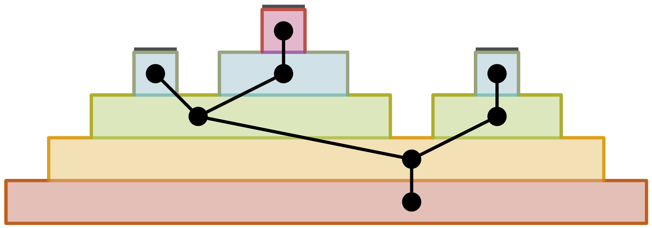
$\rightarrow$ generate polygon and check visibility graph



Histograms: Running time

An ordering of the leaves gives us an ordering of the tree

k leaves



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$\rightarrow$ and *maps* vertices to coordinates

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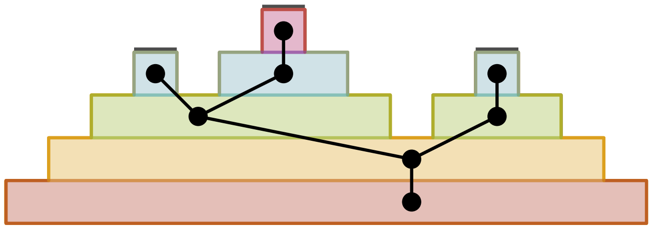

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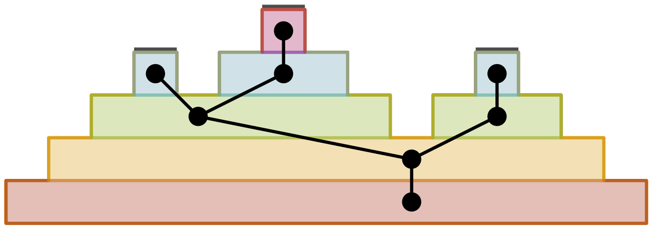

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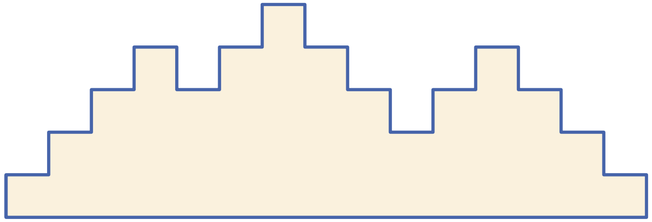

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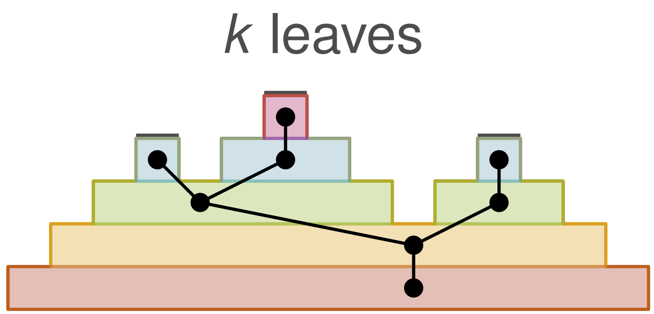

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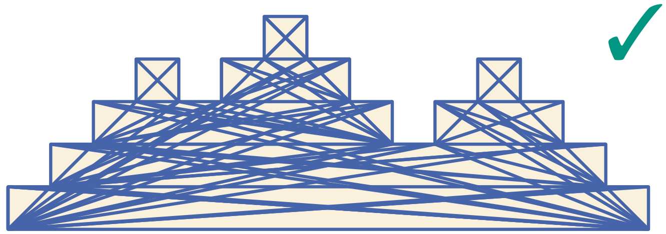

Compute its visibility graph and check for a match in $O(n \log n + m)$ time.

Histograms: Running time

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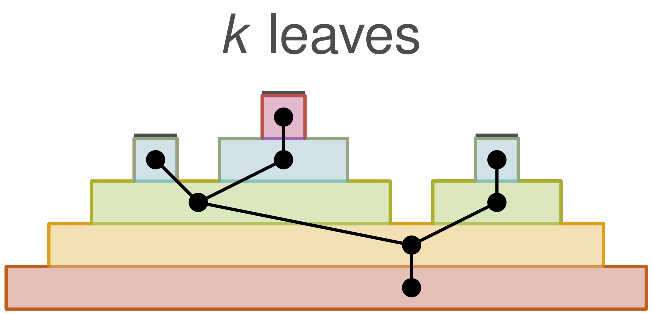

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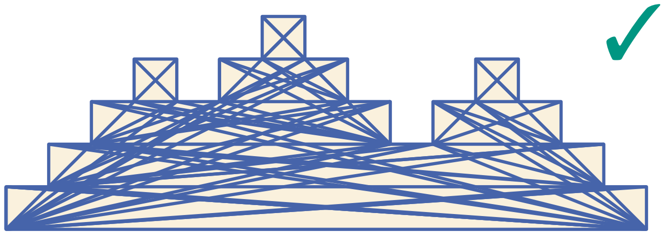


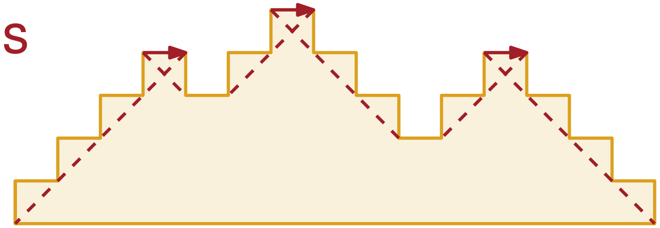
Compute its visibility graph and check for a match in $O(n \log n + m)$ time.

Seems like $O(k!(n \log n + m))$ time...

Histograms: Running time

But we also need to *orient* the tabs from left to right.

$\rightarrow$ 2^k possible orientations of leaves



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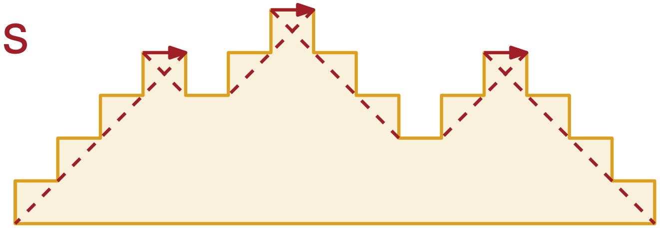

+ time to compute 1-simplicial edges

$\rightarrow$ $O(n^2 m + k!2^k(n \log n + m))$ time.

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Histograms: Running time

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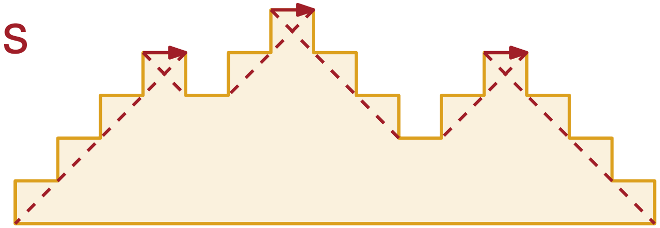

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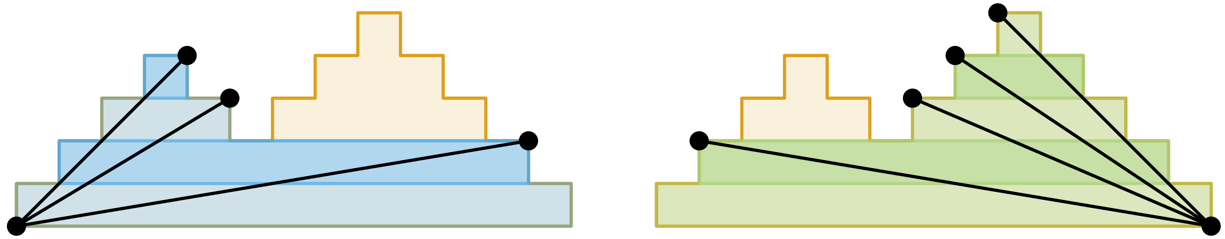

Can fix the outer two paths, leaving us with only $(k-2)!2^{k-2}$ options

$\rightarrow O(n^2 m + (k-2)!2^{k-2}(n \log n + m))$ time.

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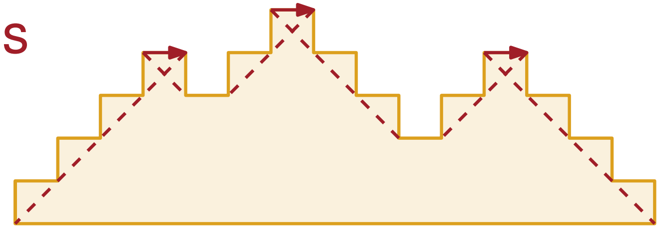

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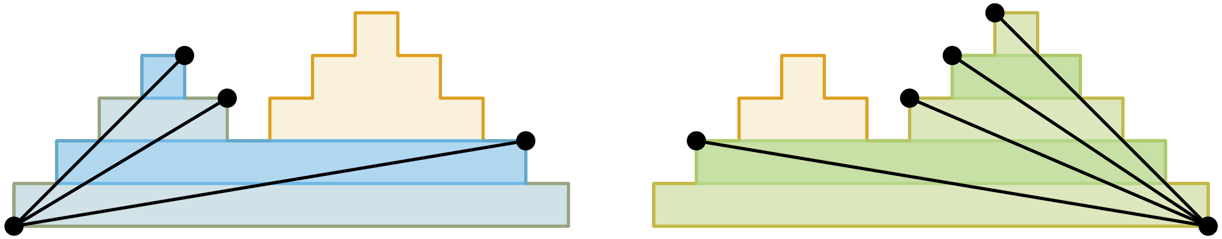

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$\rightarrow$ $O(n^2 m)$ time for binary trees (recursively fix right/left spine)

Nodari Sitchinava and Darren Strash:
Reconstructing Generalized Staircase Polygons with Uniform Step Length

Department of Computer Science
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Conclusion

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$\rightarrow$ compute coordinates and check visibility graph

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- Are there more general classes of polygons that can be recognized / reconstructed?

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Thank You!