EPG-representations with small grid-size

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Joint work with **T. Biedl, V. Dujmovic, P. Morin**.

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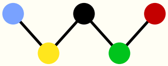
Graph: vertices, edges



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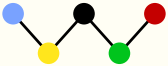
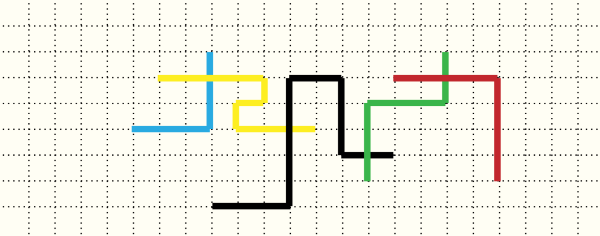


VPG-Representation (Vertex Path Grid)

vertices	=	paths in a grid
edges	=	vertex intersections between paths



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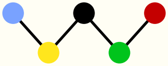

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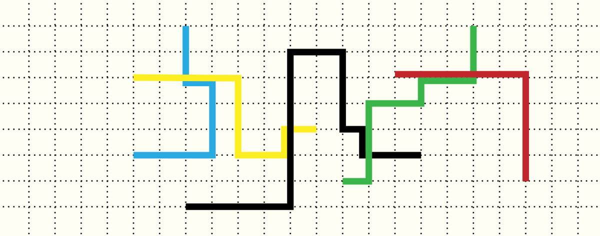

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[Golumbic et at., 2009]

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- Two directions of research:
  1. What graph classes can we represent with few bends?

     [Asinowski et al., 2009, Biedl et al., 2010, Francis + Lahiri, 2016]
  2. What can the EPG-representations be used for?

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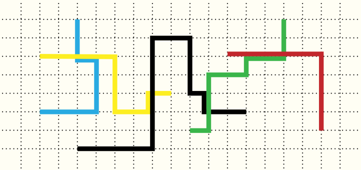
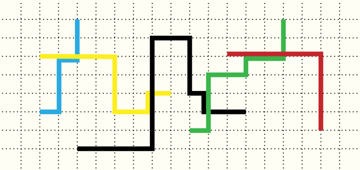
This paper:

1. What graphs can you represent in a small grid?
2. Models of EPG: general, x-monotone, xy-monotone, xy^+-monotone

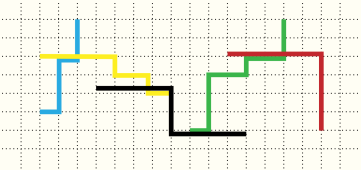
Models of EPG representations



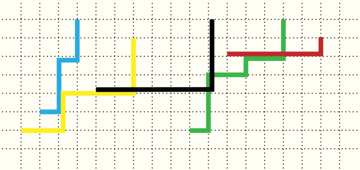
General



x-monotone



xy-monotone



xy^{+}-monotone

Constructions via VPG representations

Theorem. Every graph G with n vertices has an xy^+-monotone EPG-representation in a $3n \times 2n$-grid.

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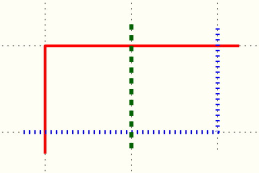
Proof.

G has an xy^+-monotone VPG-representation in a $w \times h$-grid

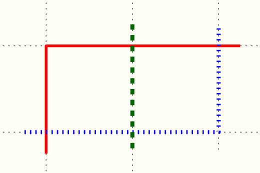
$$\Rightarrow$$

Any $G' \subseteq G$ has

1. an x-monotone EPG-representation in a $2w \times 2h$-grid.
2. an xy^+-monotone EPG-representation in a $(2w + h) \times 2h$-grid.



Start with xy^+-mon. VPG.



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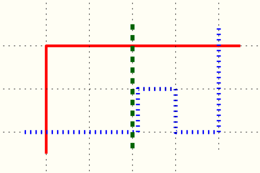
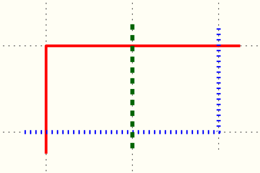


(1) Add a bump: x-mon. EPG.



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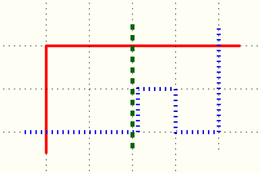

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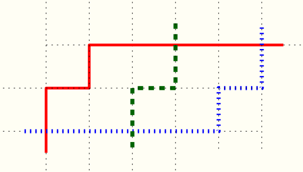
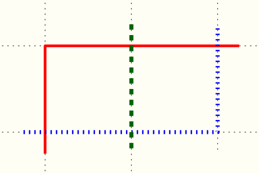


(2) Or skew…



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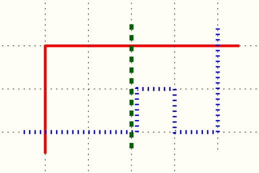

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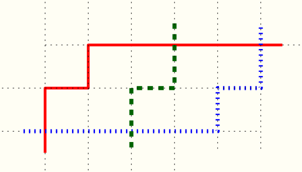

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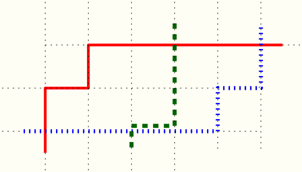
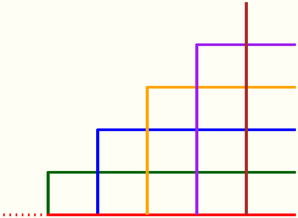


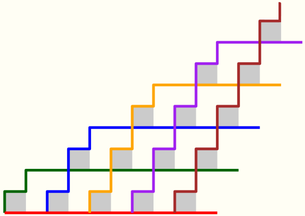
and create intersections: xy^+-mon. EPG.

Theorem. Every graph G with n vertices has an xy^+-monotone EPG-representation in a $3n \times 2n$-grid.

Proof. Use the skewing technique.



1. Represent a clique.



2. Create selected intersection.

EPG representations via pathwidth

Theorem. Every graph G of pathwidth k has an xy^{+}-monotone EPG-representation of height $8k + O(1)$ and width $O(n)$, thus with $O(kn)$ area.

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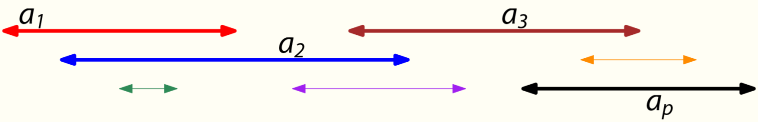
G has pathwidth k

$\Leftrightarrow$

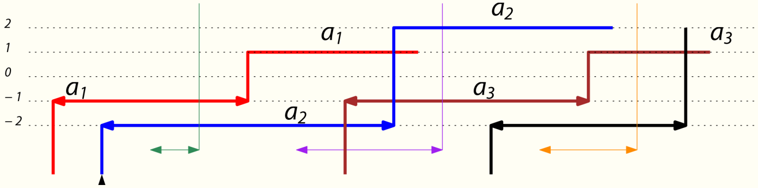
G is a subgraph of $(k+1)$-colorable interval graph



Claim. Any k-colorable interval graph has a VPG-representation such that:

- Every path xy^+-monotone and has shape ⌐¯
- The VPG representation "contains" the interval representation

in a grid of size $O(n) \times (4k + O(1))$.



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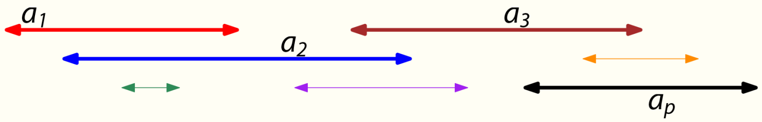
2. Use induction.

Given $(k+1)$-colorable connected interval graph G, find the farthest path $P = a_1, \ldots, a_p$.



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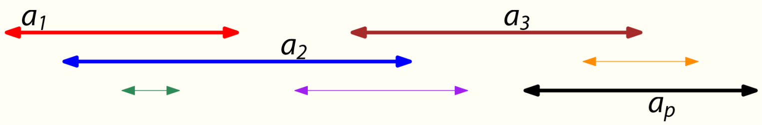
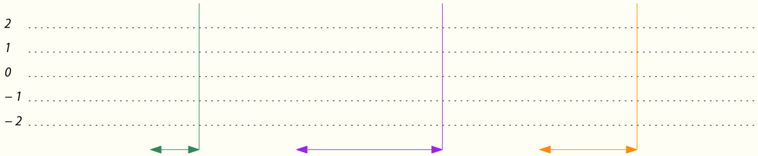


1. The farthest path P is an induced path
2. The farthest path spans the entire interval range
3. The graph $G - P$ is k-colorable (at most)

Inductive construction:



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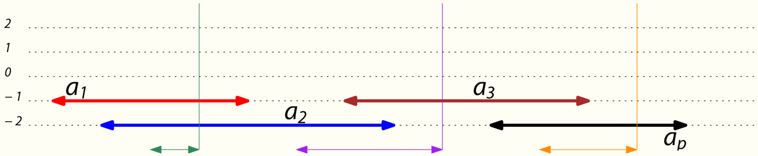

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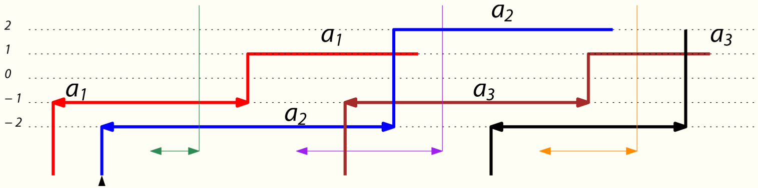
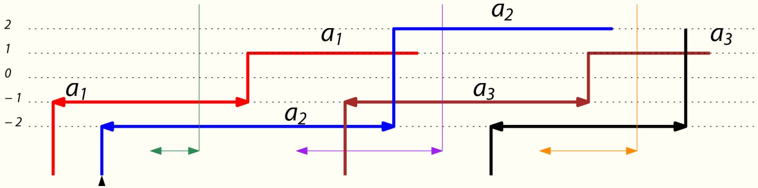


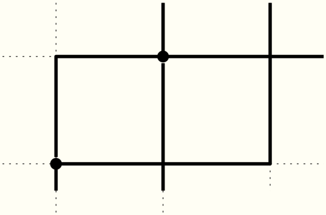


1. All the intersections are represented
2. The dimensions are $O(n) \times (4k + O(1))$
3. It is xy^+-monotone

Apply the skew technique!

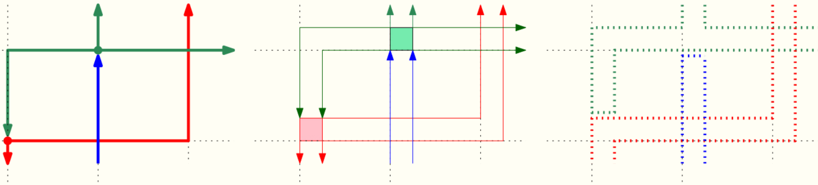
EPG-representations via orthogonal drawings

- 4-graph: all vertices have degree at most 4
- orthogonal drawing of a 4-graph:
  - grid-points $\rightarrow$ vertices
  - grid-paths $\rightarrow$ edges
  - edges are allowed to intersect



Theorem. Let G be a 4-graph that has an orthogonal drawing in a $w \times h$-grid. Then any minor of G has an EPG-representation in a $2w \times 2h$-grid

Proof:



(a) (b) (c)

Theorem. Let G be a 4-graph that has an orthogonal drawing in a $w \times h$-grid. Then any minor of G has an EPG-representation in a $2w \times 2h$-grid

Corollary. All graphs of bounded treewidth (trees, outer-planar graphs and series-parallel graphs) have an EPG-representation in $O(n)$ area. Graphs of bounded genus have an EPG-representation in $O(n \log^2 n)$ area.

Lower bounds

Theorem. Let G be a triangle-free graph with m edges. Then any EPG-representation of G uses at least m grid-edges (hence a grid of area $\Omega(m)$)

Proof. Triangle free $\Rightarrow$ no grid edge belongs to three vertex paths

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Corollaries. There are graphs that require grid sizes:

- pathwidth-k graphs: $\Omega(kn)$ (any EPG)
- triangle-free pathwidth-k graphs: $\Omega(k) \times \Omega(k)$ (any EPG)
- n-vertex graphs with $O(n)$ edges: $\Omega(n^2)$ (any EPG)

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Thank you!