

# Experimental Evaluation of Book Drawing Algorithms

GD17 · September 26, 2017

Jonathan Klawitter et al.

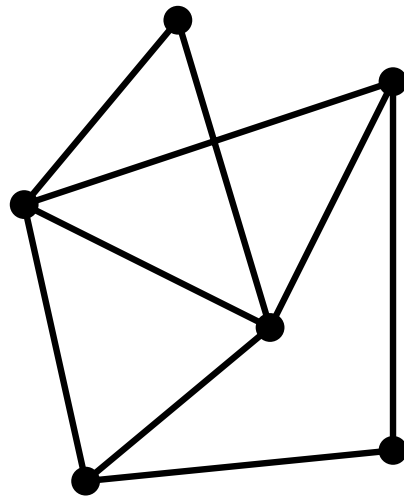
joined work with Tamara Mchedlidze and Martin Nöllenburg

GD17 · September 26, 2017



# Book drawings

# Book drawing and book embedding



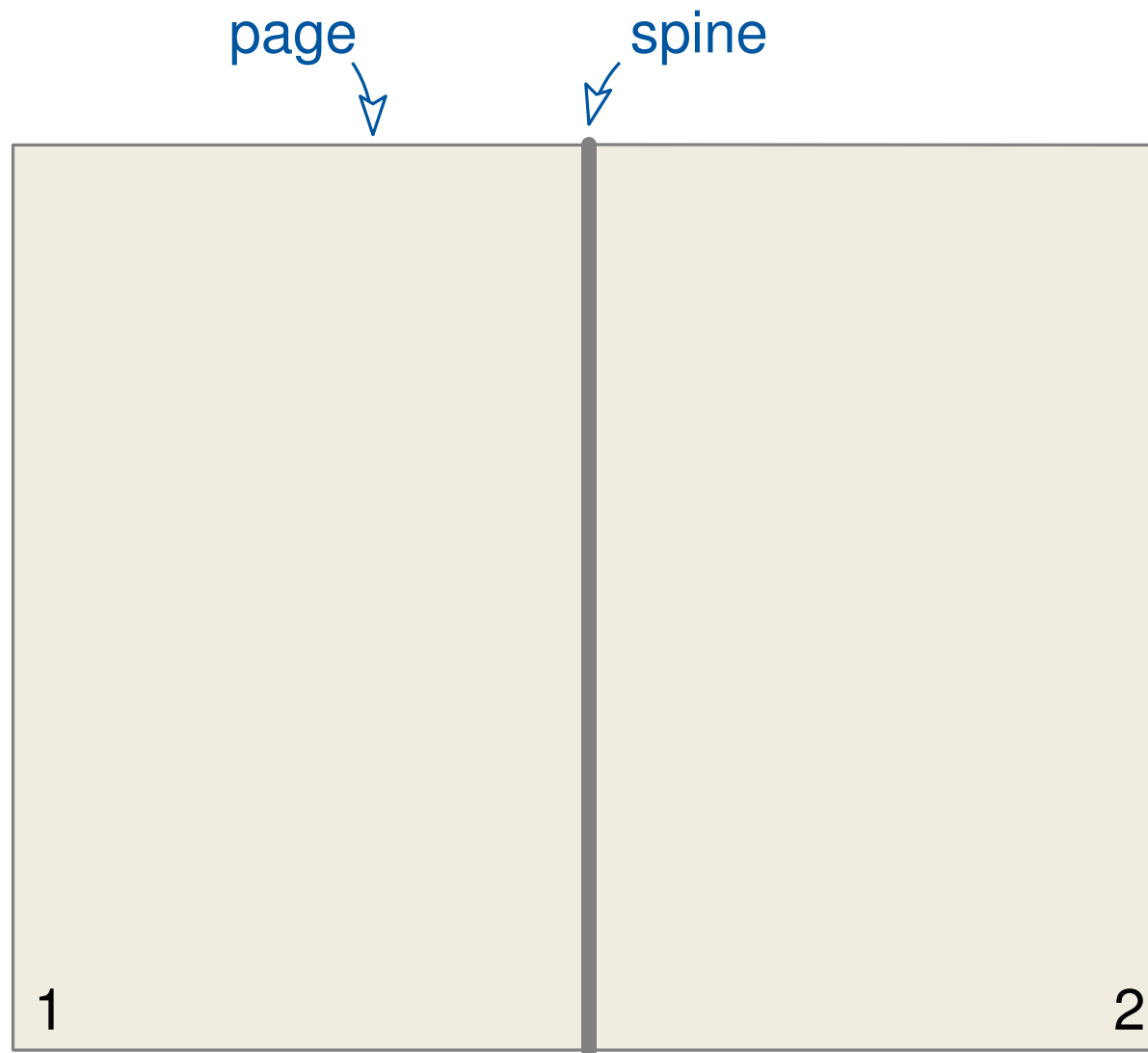
simple graph

+

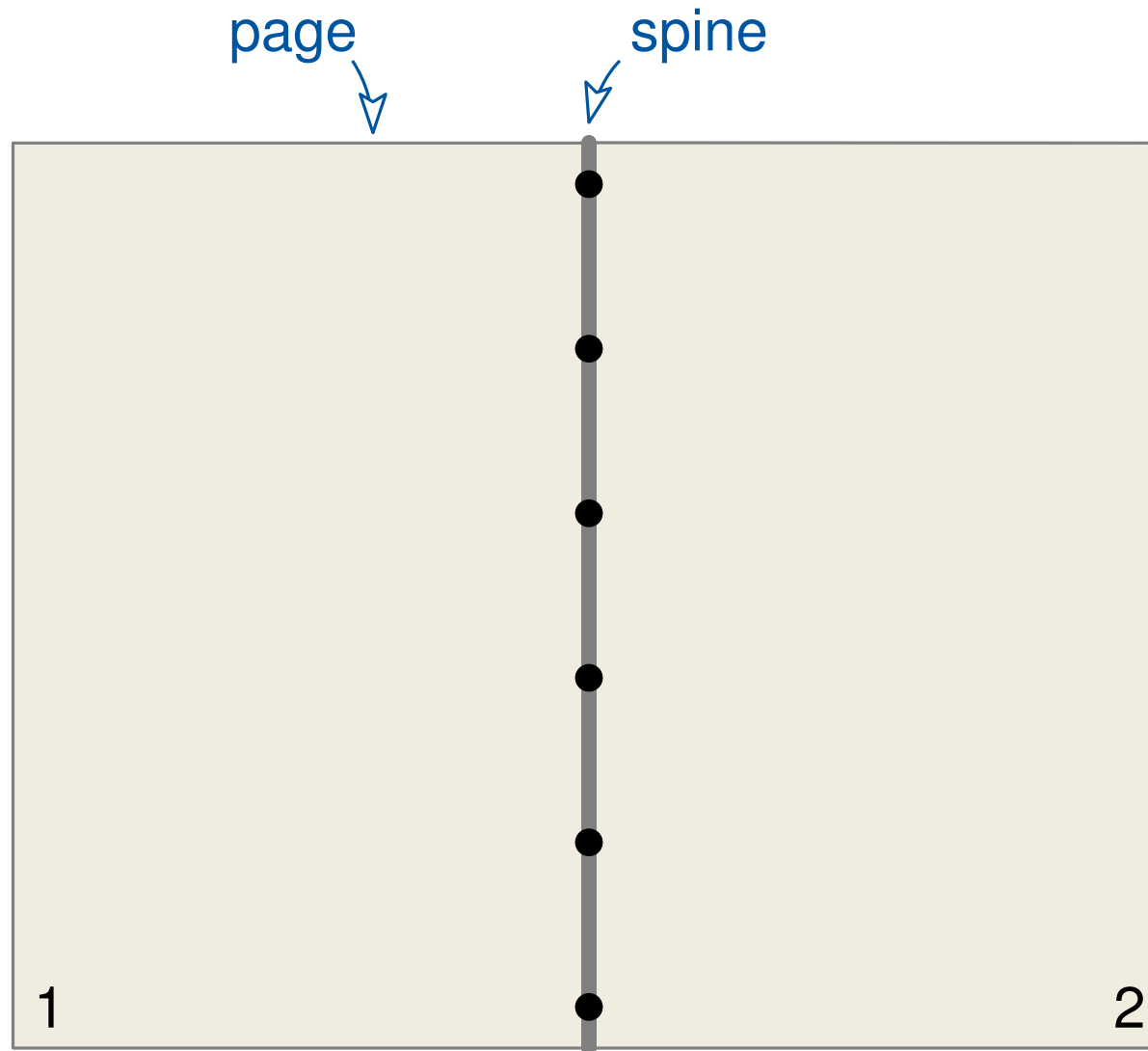


book

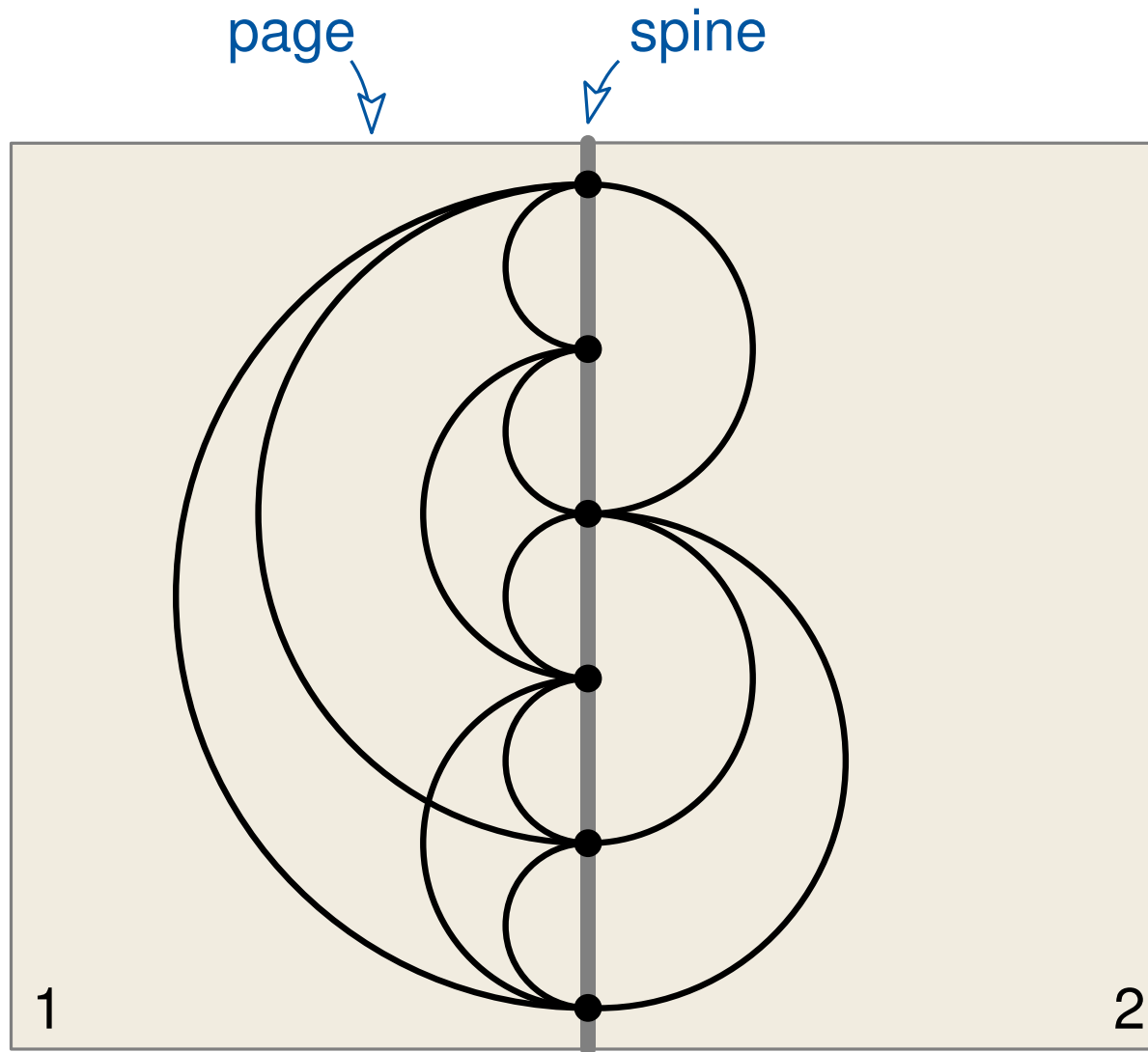
# Book drawing and book embedding



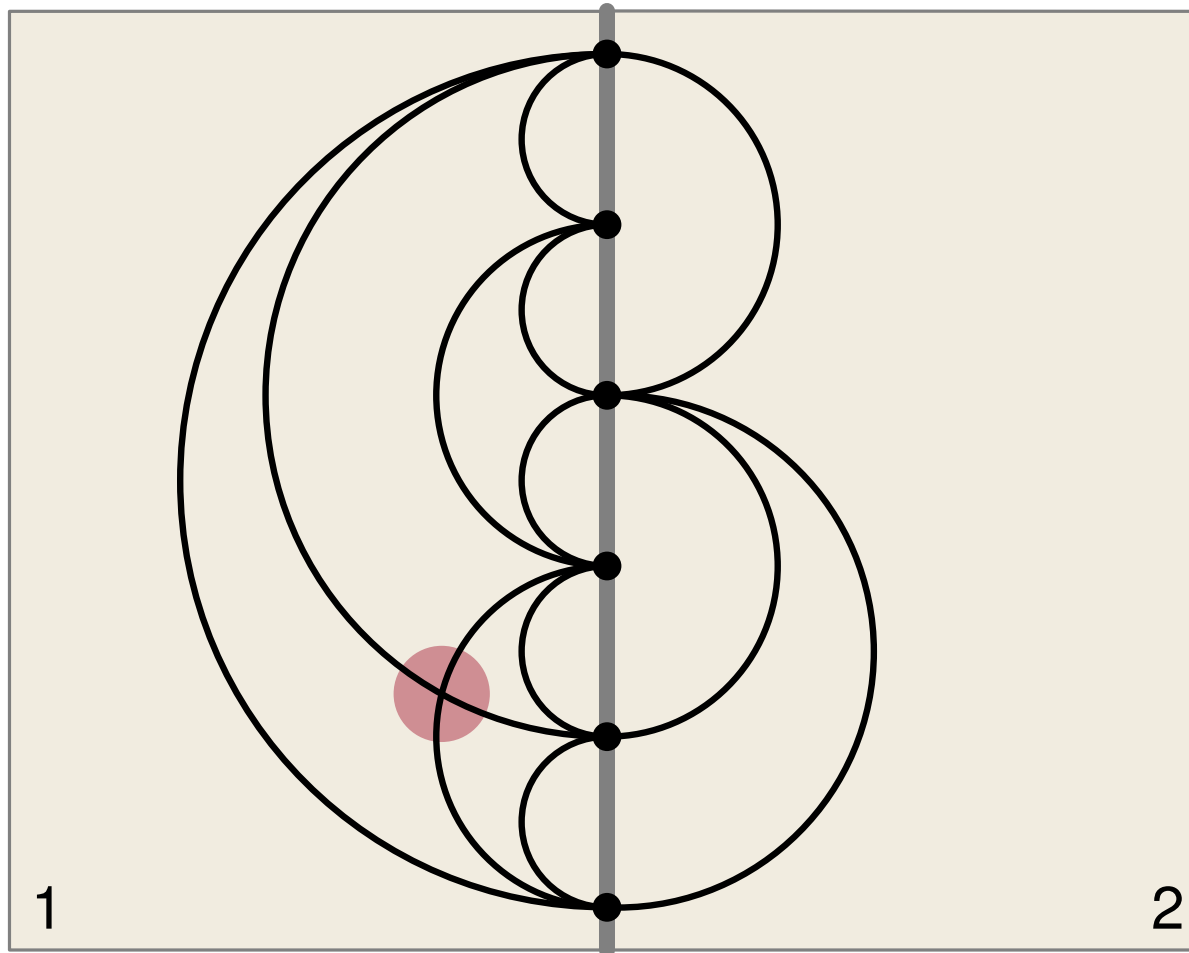
# Book drawing and book embedding



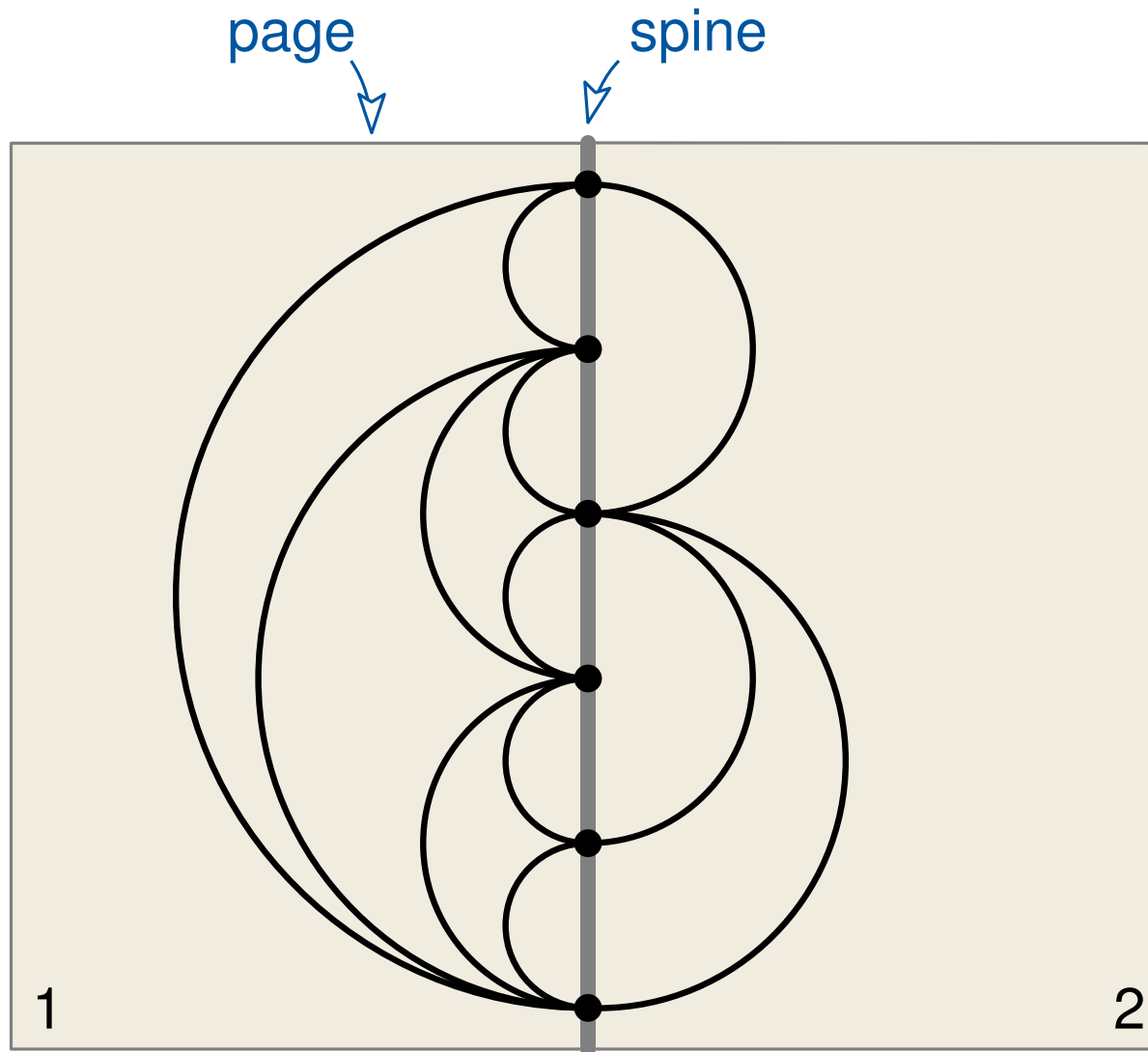
# Book drawing and book embedding



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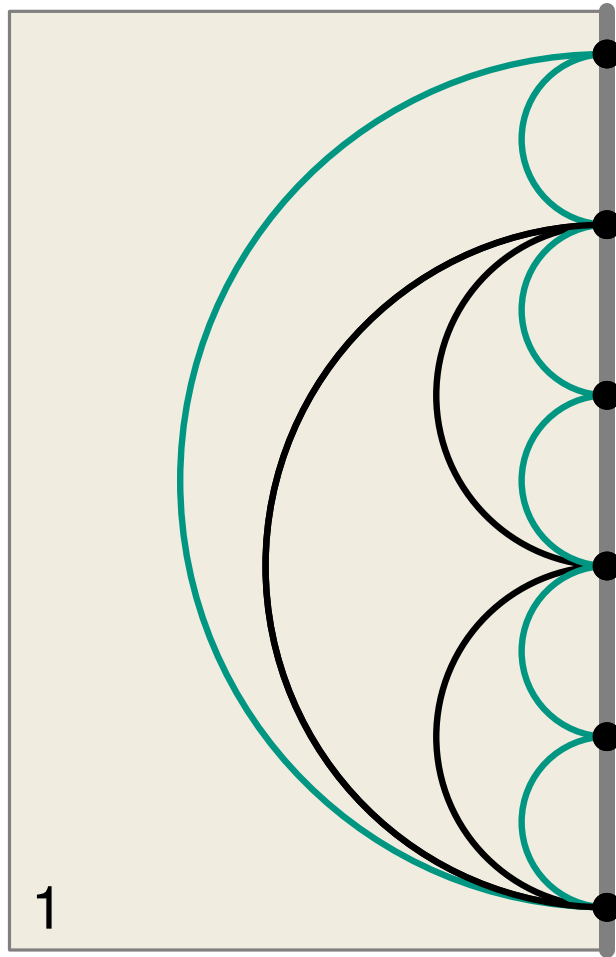


# Book drawing and **book embedding**

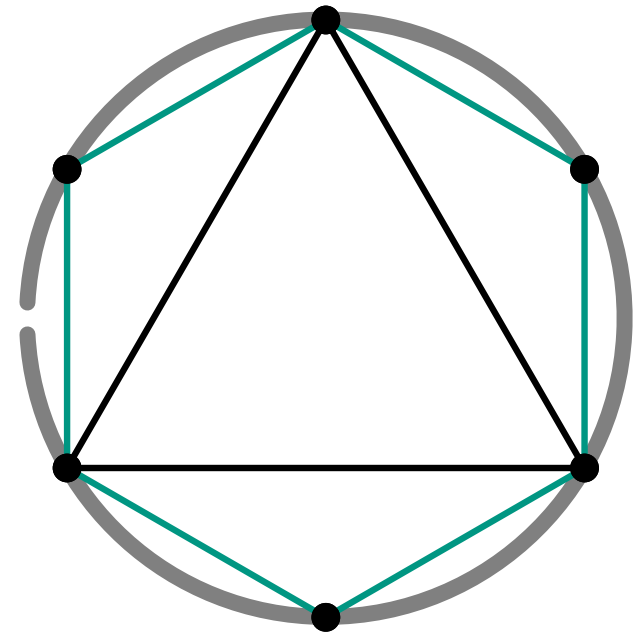
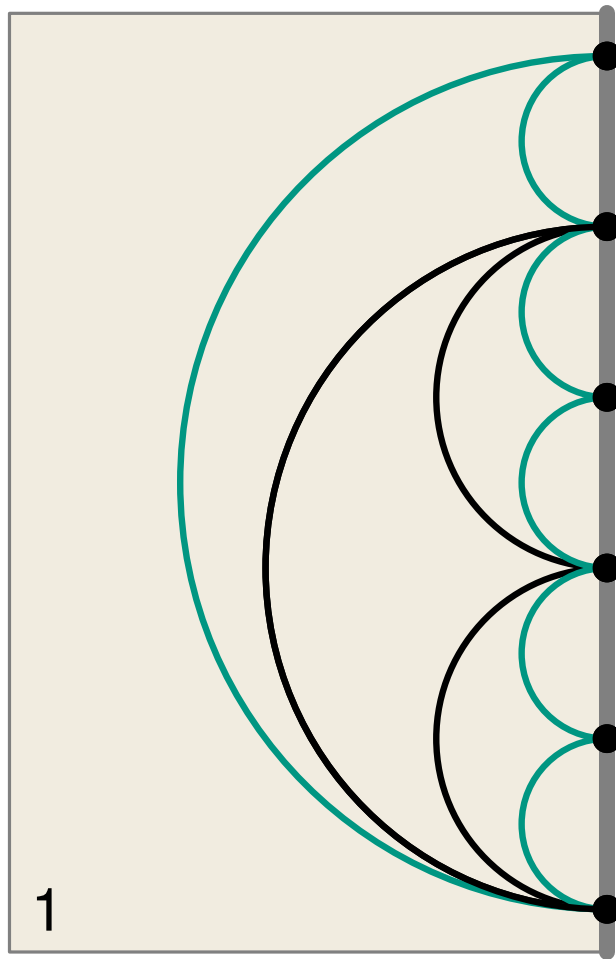




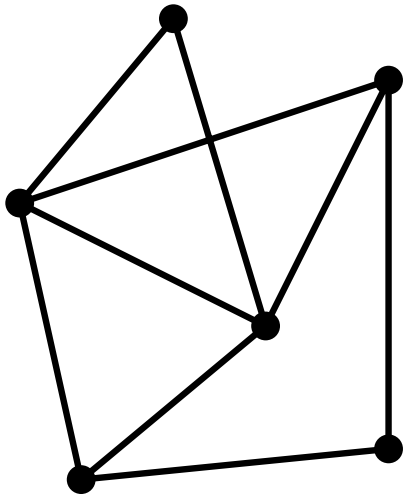
# Book drawing and book embedding



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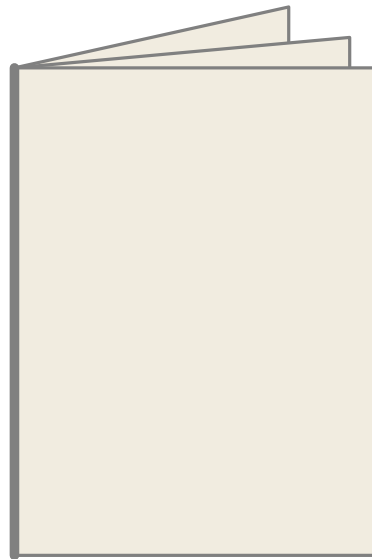


# $k$ -page crossing minimisation problem

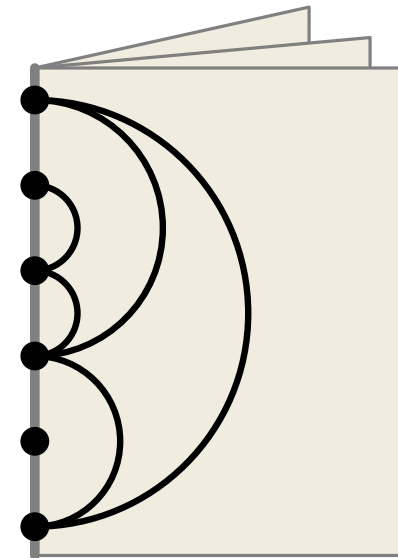


graph

+

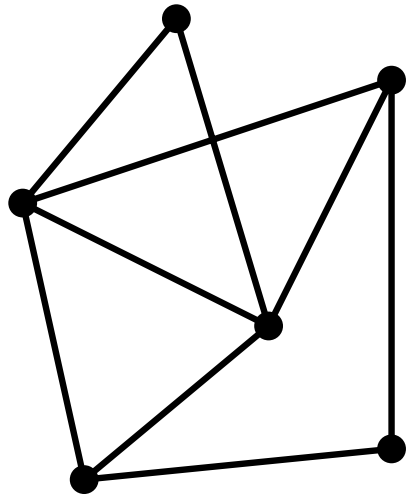


$k$  page  
book



book drawing  
with min.  
# crossings

# $k$ -page crossing minimisation problem

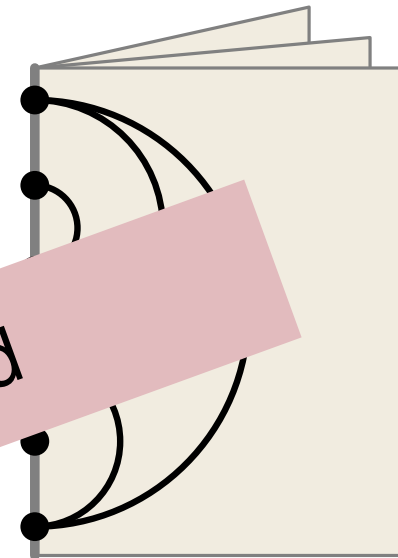


graph

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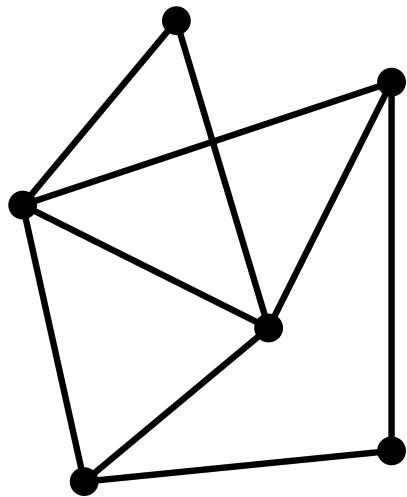


$k$  page  
book



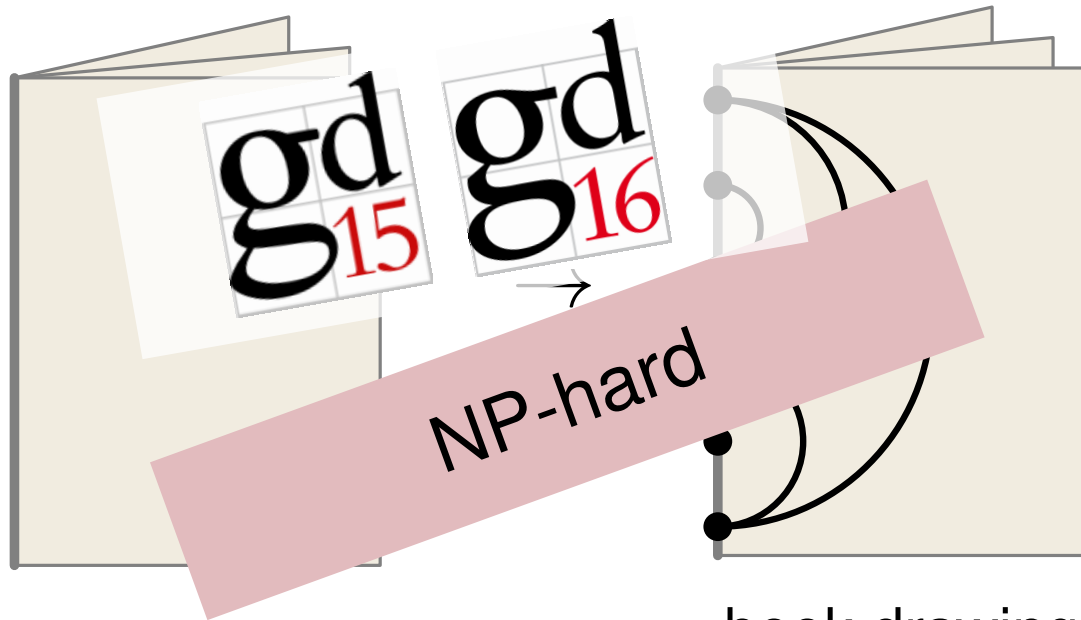
book drawing  
with min.  
# crossings

# $k$ -page crossing minimisation problem



graph

+

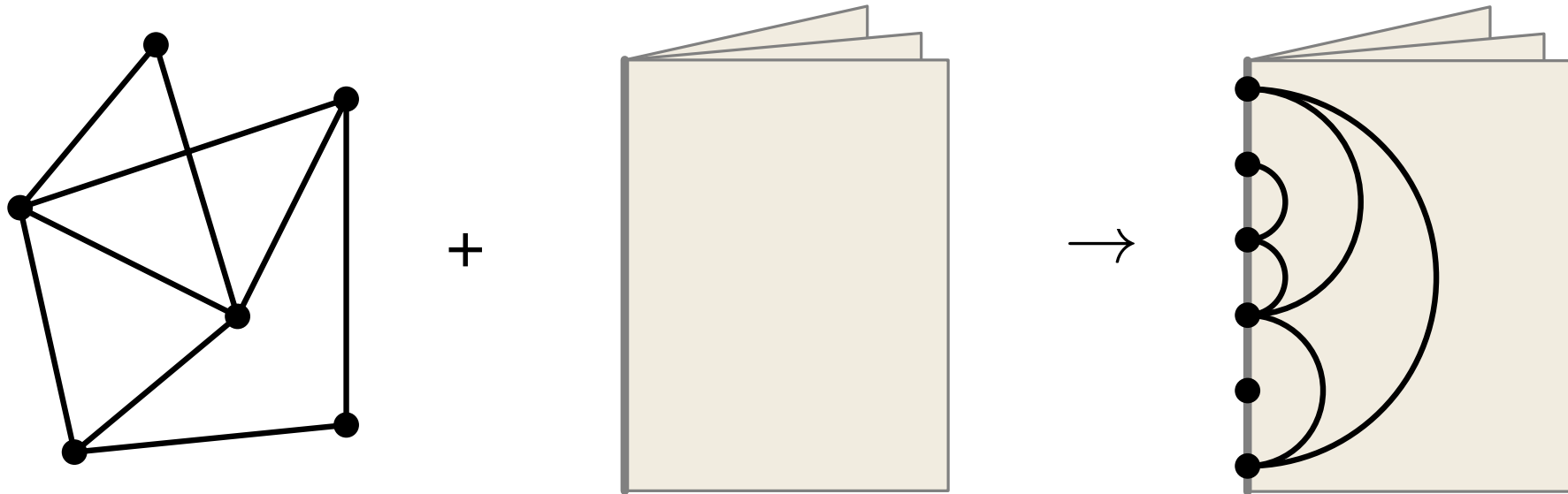


$k$  page  
book

book drawing  
with min.  
# crossings

# Algorithms

# Constructive heuristics



# Constructive heuristics - **Vertex order**

- search based:
  - rDFS · random DFS [He, Sykora '04]
  - sm1DgrDFS · smallest degree DFS [Bansal et al. '08]
  - treeBFS · BFS tree based

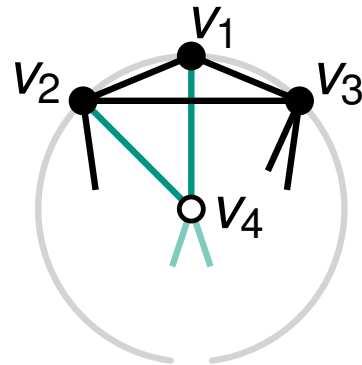
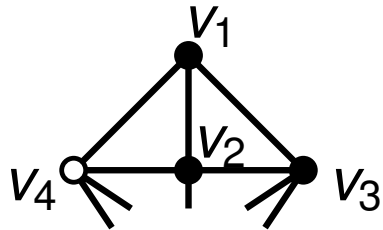


# Constructive heuristics - Vertex order

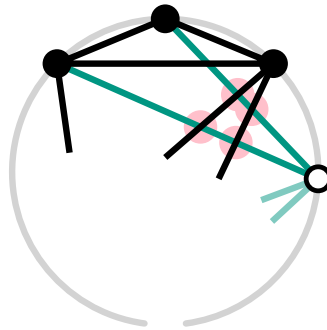
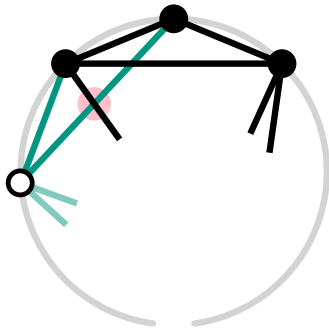
- search based:
  - rDFS · random DFS [He, Sykora '04]
  - sm1DgrDFS · smallest degree DFS [Bansal et al. '08]
  - treeBFS · BFS tree based
- connectivity based:
  - conCro [Baur, Brandes '08]
  - conGreedy

# Constructive heuristics - Vertex order conCro

## ■ vertex selection

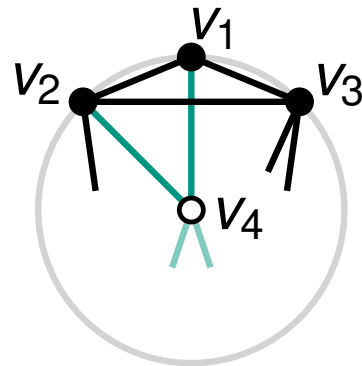
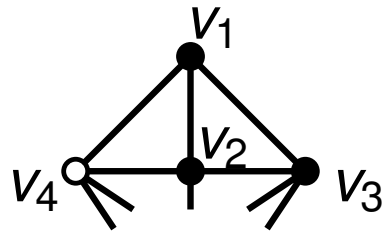


## ■ vertex placement

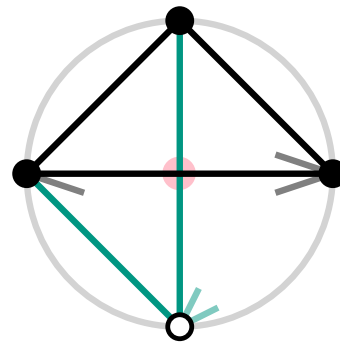
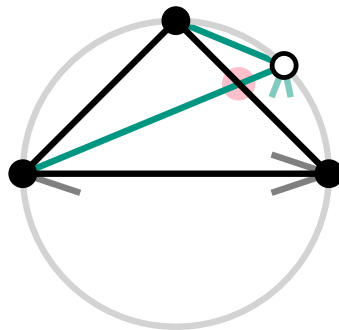
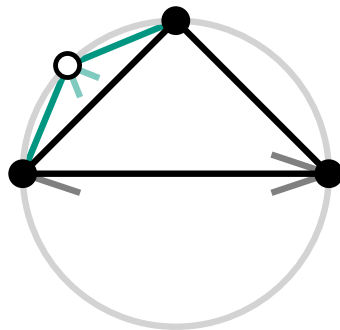


# Constructive heuristics - Vertex order conGreedy

## ■ vertex selection



## ■ vertex placement



# Constructive heuristics - Page assingment

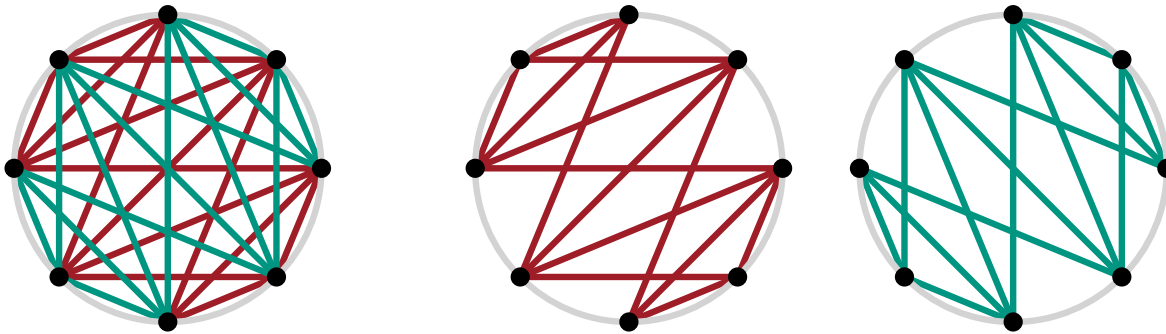
- greedy:
  - eLen [Cimikowski '02]
  - ceilFloor [Kapoor et al. '02]
  - circ [Satsangi et al. '13]

# Constructive heuristics - Page assingment

- greedy:
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  - circ [Satsangi et al. '13]
- earDecomp · ear decomposition based

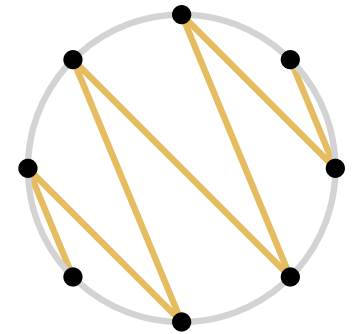
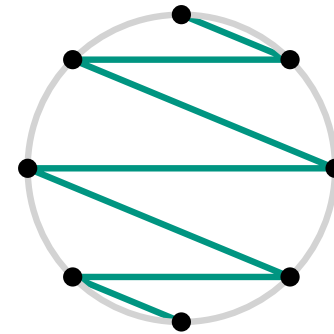
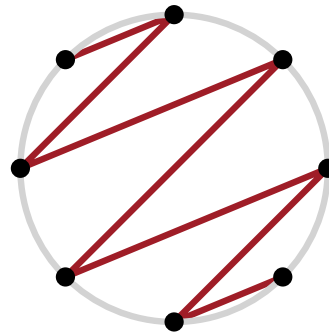
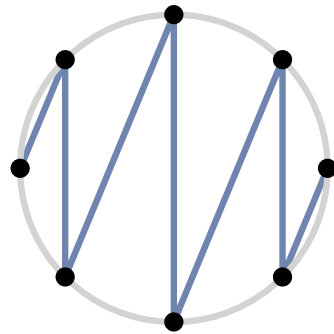
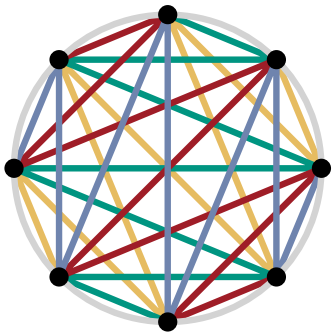
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- slope [He et al. '05]



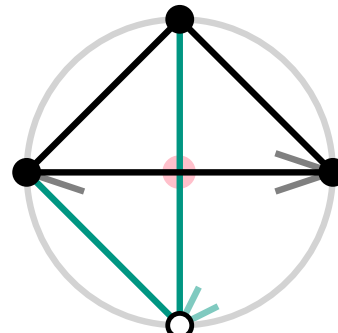
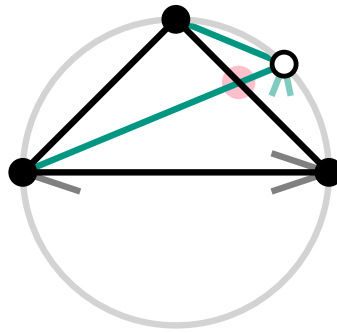
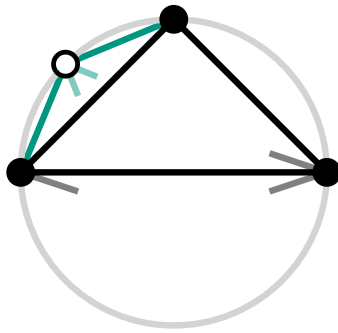
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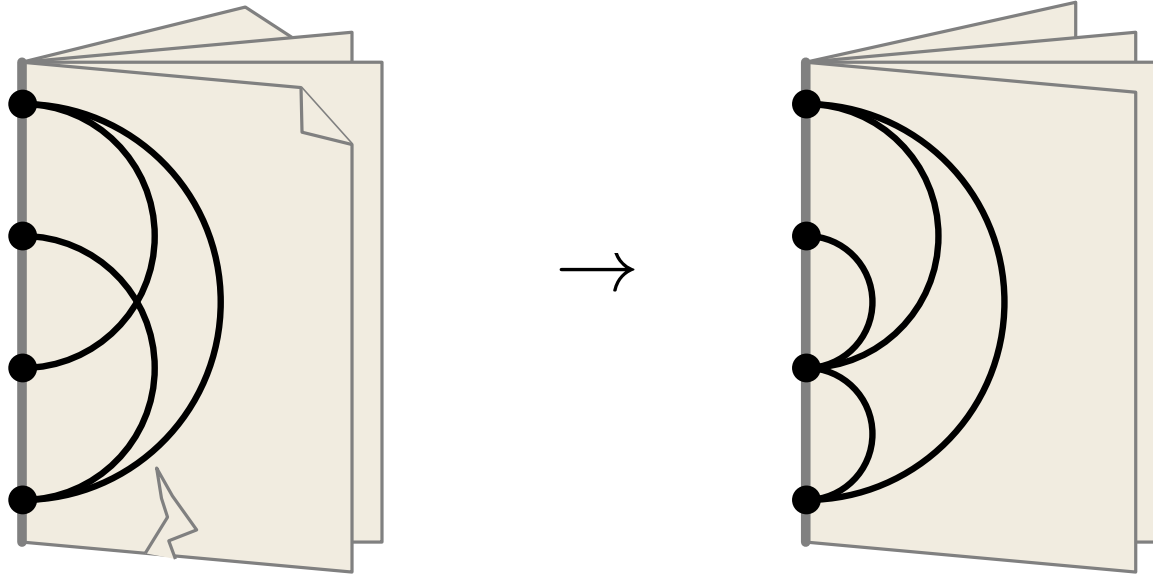
# Constructive heuristics - VO & PA combined

- first VO heuristic, then PA heuristic
- VO and PA simultaneously
  - conGreedy+

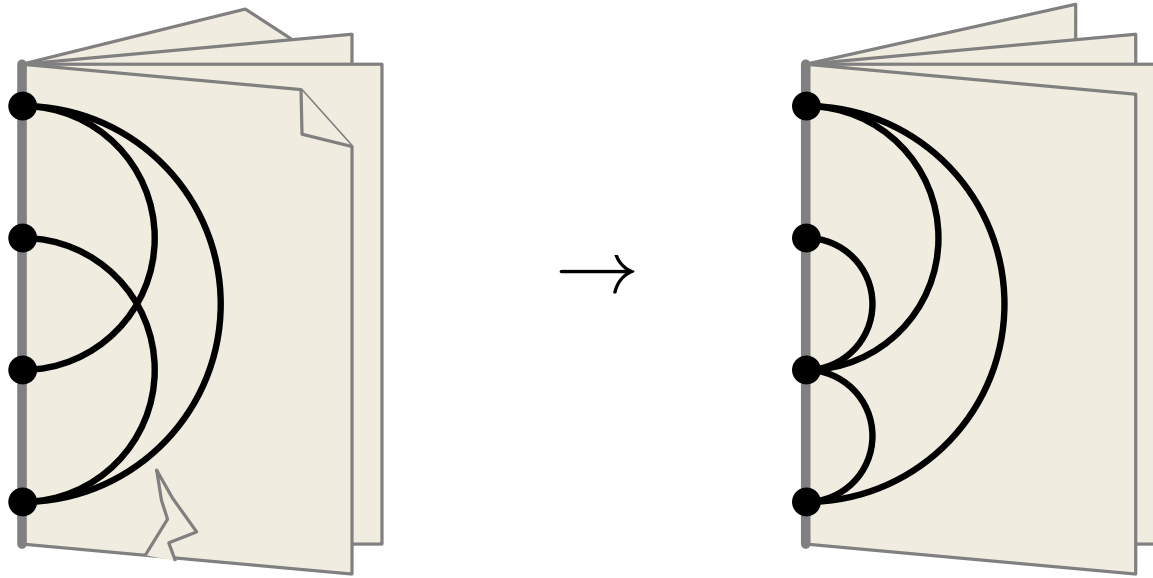




# Local search heuristics



# Local search heuristics



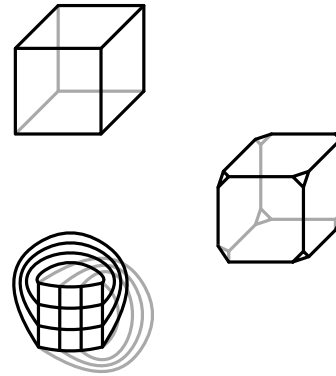
- `greedyAlt` · greedily optimise VO and PA alternately
- `greedy+` · greedily optimise VO and PA simultaneously
- `simAnn` · simulated annealing approach [Cibulka '15]

# Benchmark graphs

# Benchmark graphs

## Homogenous graphs

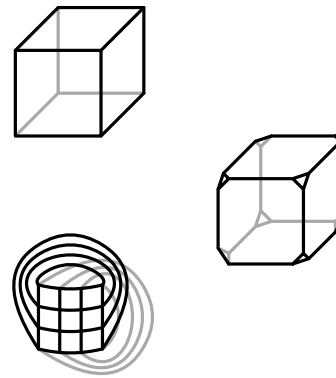
- Hypercube ·  $Q_d$
- Cube-connected cycles ·  $CCC_d$
- Toroidal graphs ·  $C_i \times C_j$
- 3-toroidal graphs ·  $C_i \times C_j \times C_l$



# Benchmark graphs

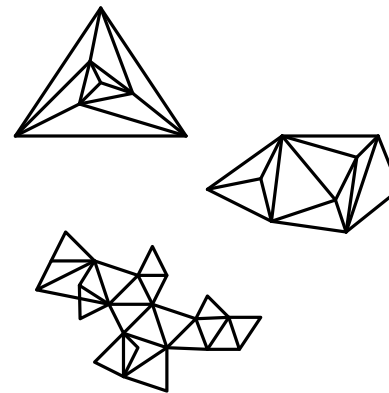
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## Structured graphs

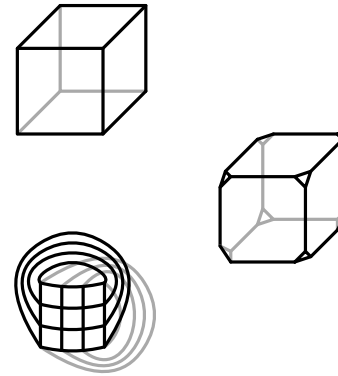
- Topological planar and 1-planar
- Geometric  $k$ -planar ( $k = 0, 1, 2, 3, 4$ )
- $k$ -trees ( $k = 3, \dots, 10$ )



# Benchmark graphs

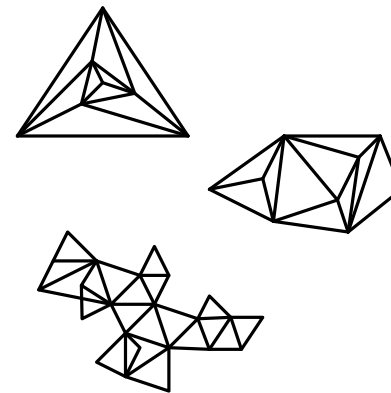
## Homogenous graphs

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- Cube-connected cycles ·  $CCC_d$
- Toroidal graphs ·  $C_i \times C_j$
- 3-toroidal graphs ·  $C_i \times C_j \times C_l$



## Structured graphs

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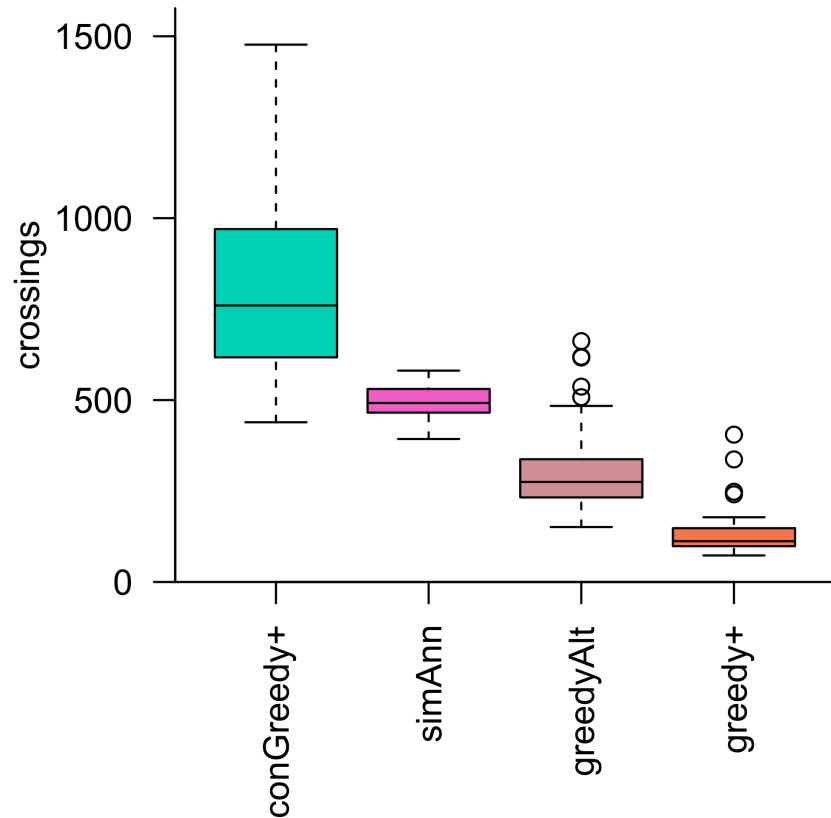


## Random graphs

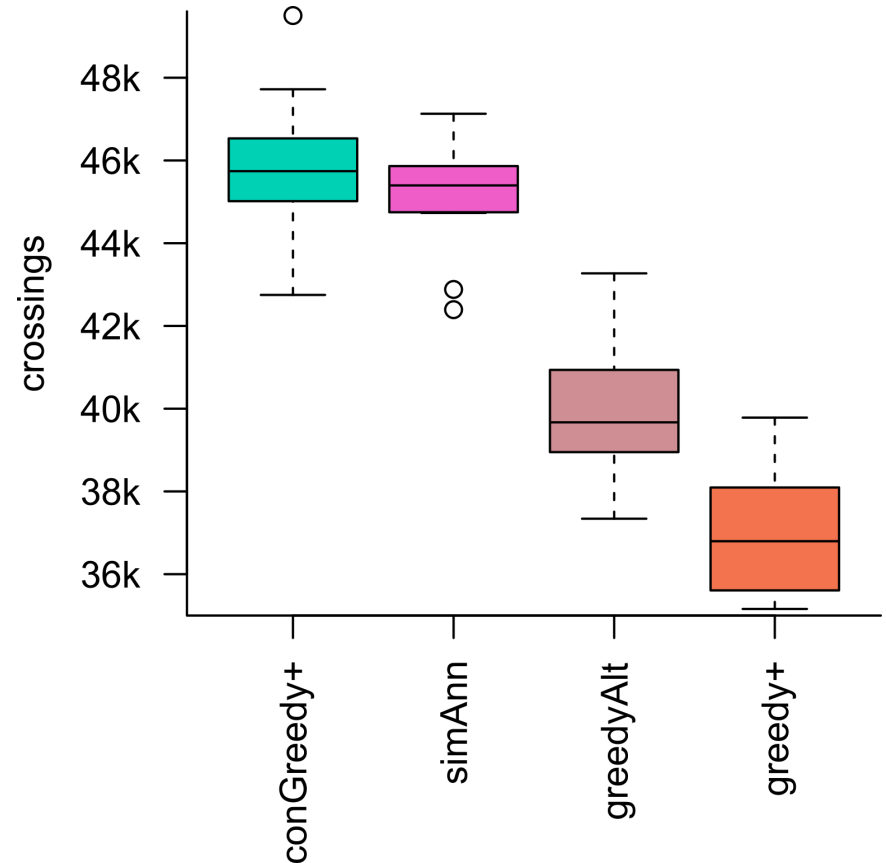
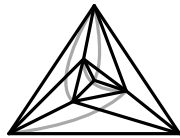
- Linear density ( $\approx a \cdot n$  edges)
- Quadratic density (edge probability  $p$ )

# Evaluation

# Local search heuristics



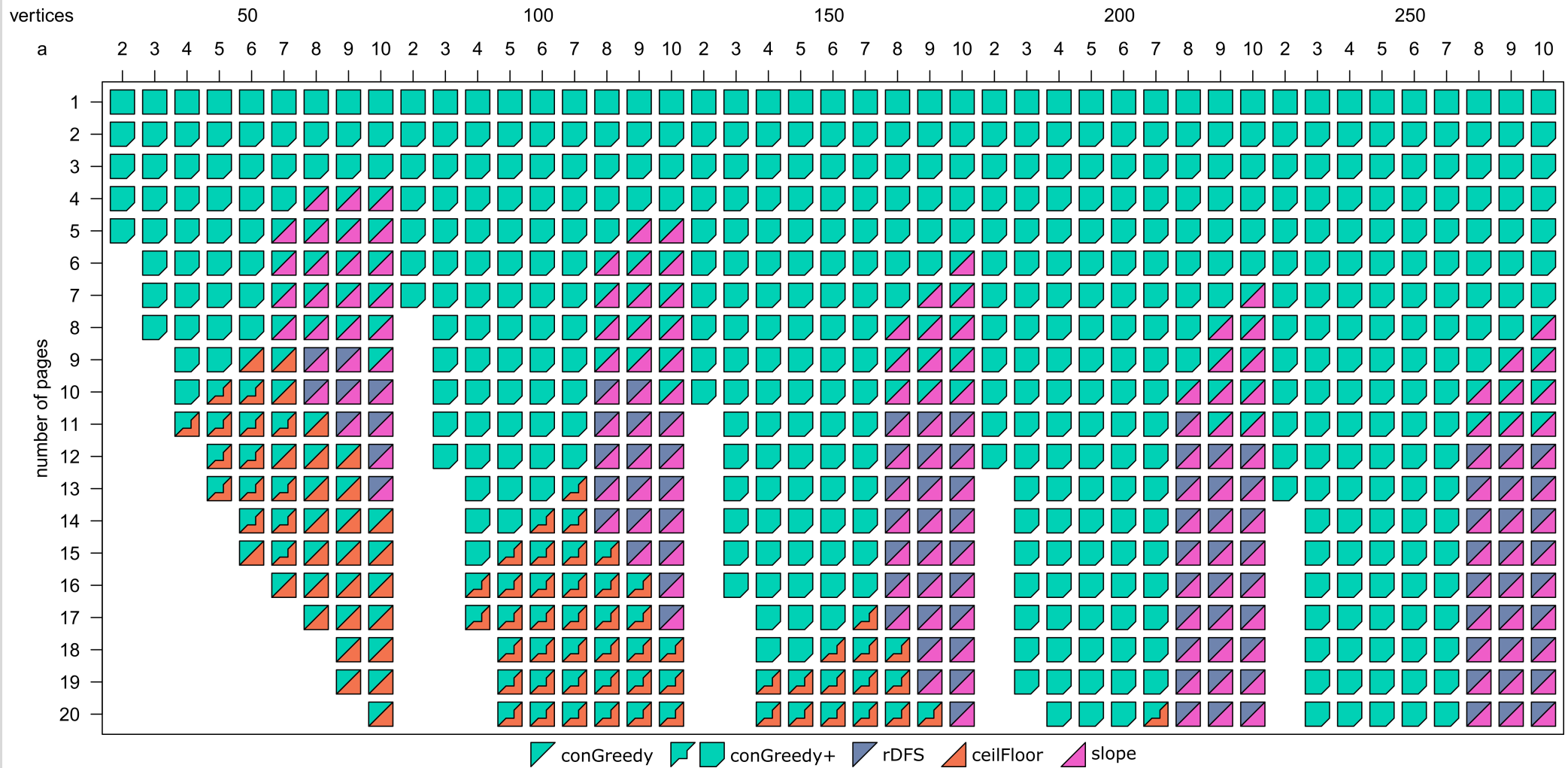
- Topological 1-planar,  $n = 250$ , 3 pages,



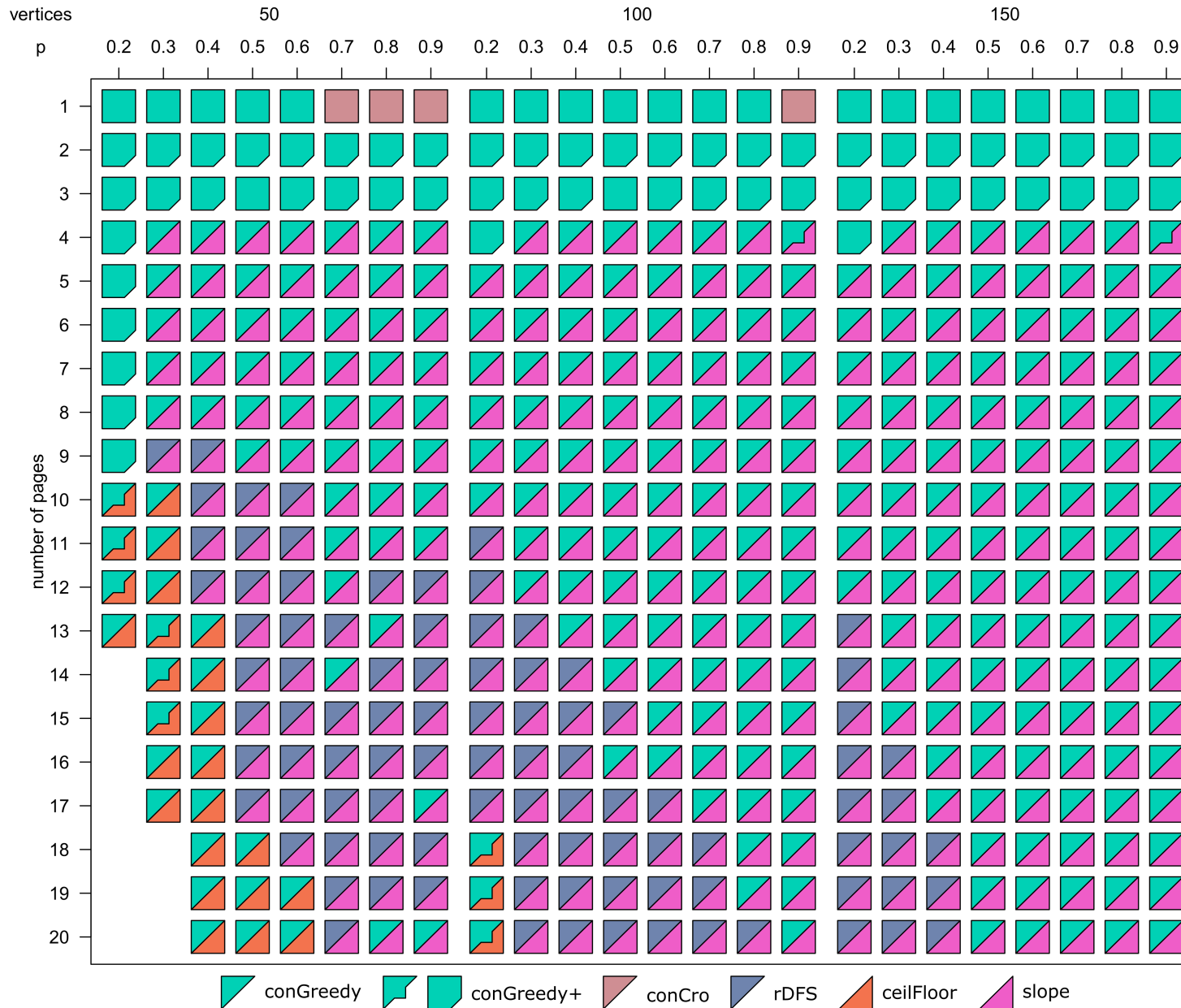
- Random (edge probability 50%),  $n = 100$ , 6 pages,



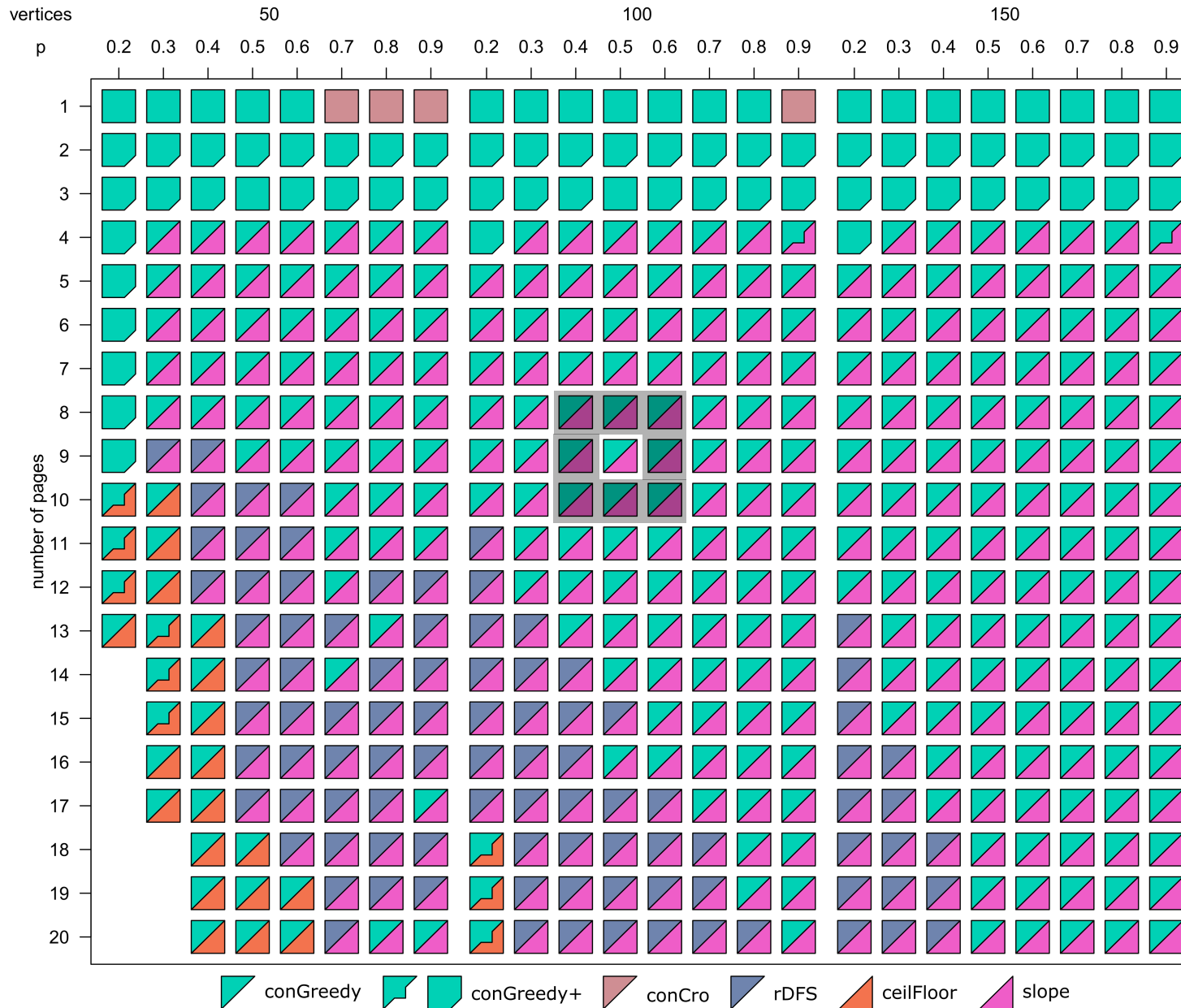
# Random graphs - linear # of edges



# Random graphs - edge probability $p$

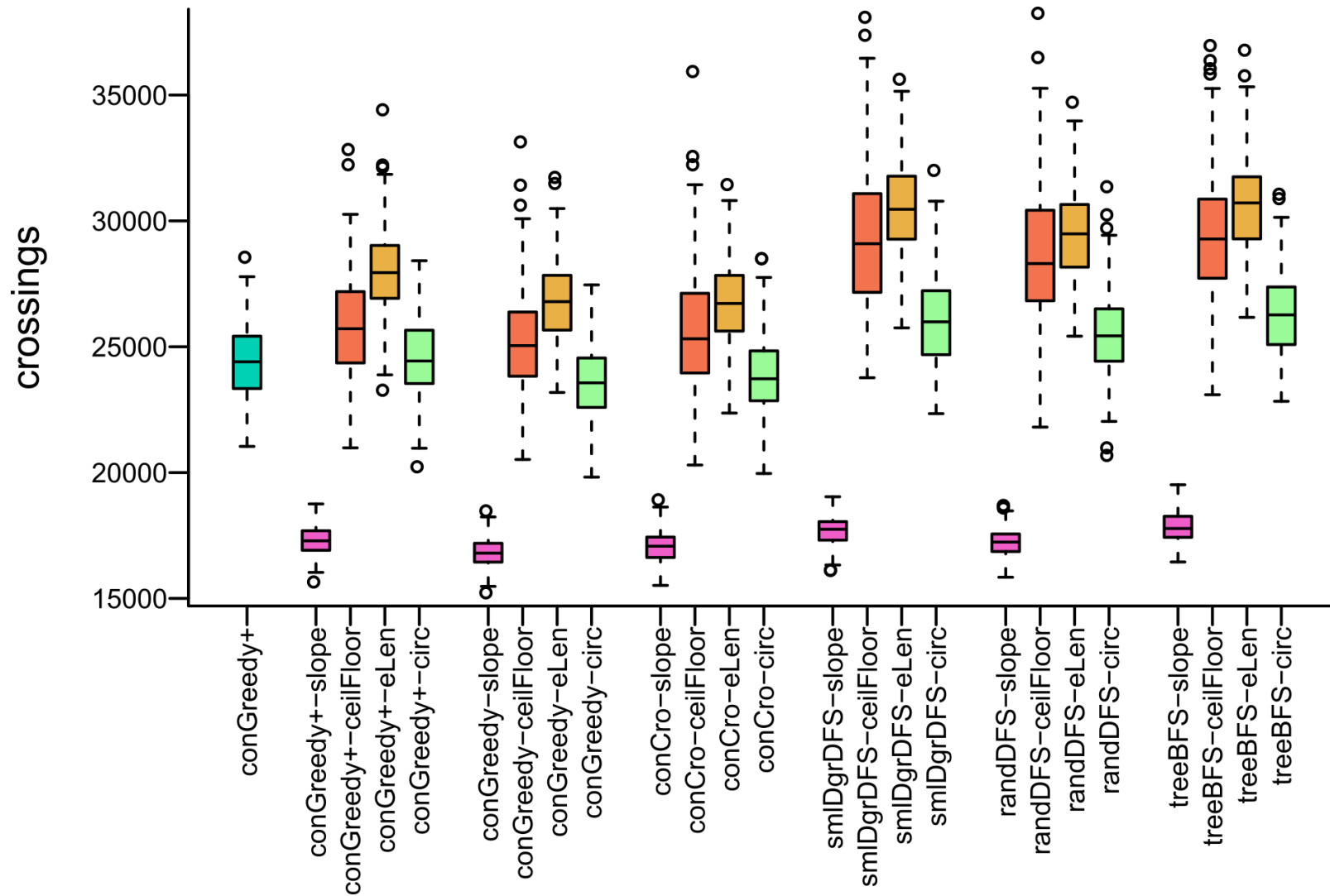


# Random graphs - edge probability $p$

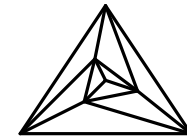
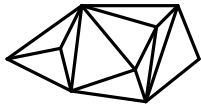


# Random graphs - edge probability $p$

- Edge probability  $p = 0.5$ ,  $n = 100$ , 9 pages

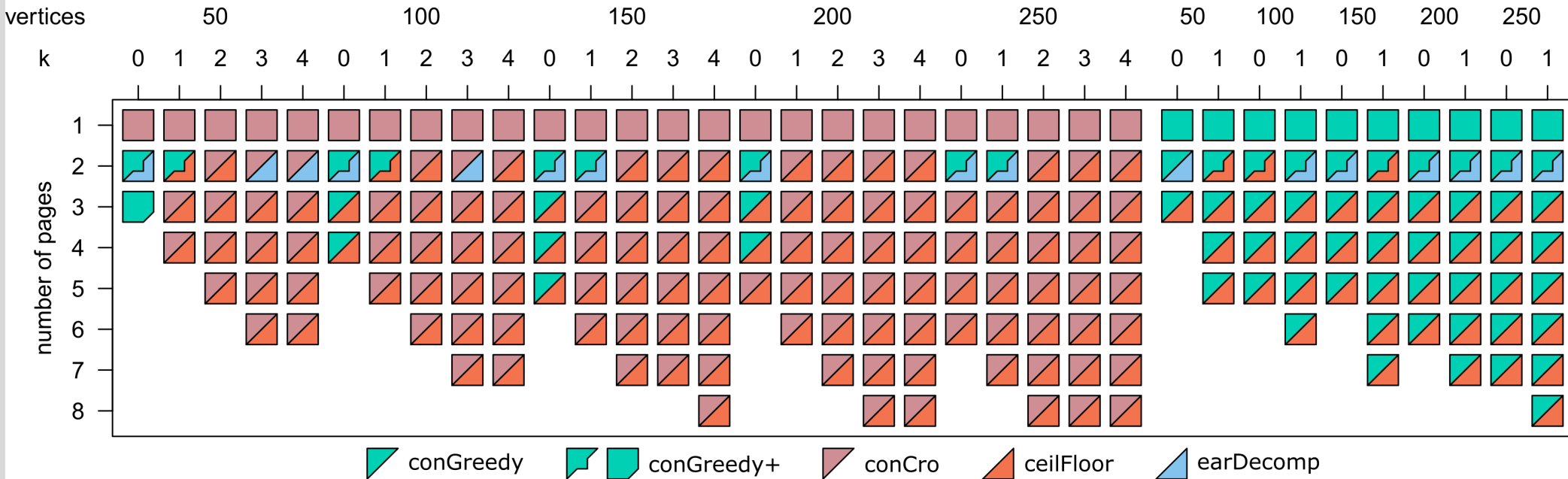


# Structured graphs - $k$ -planar

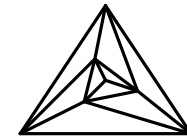
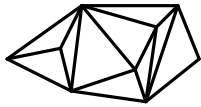


■ Geometric  $k$ -planar

■ Topological 0/1-planar

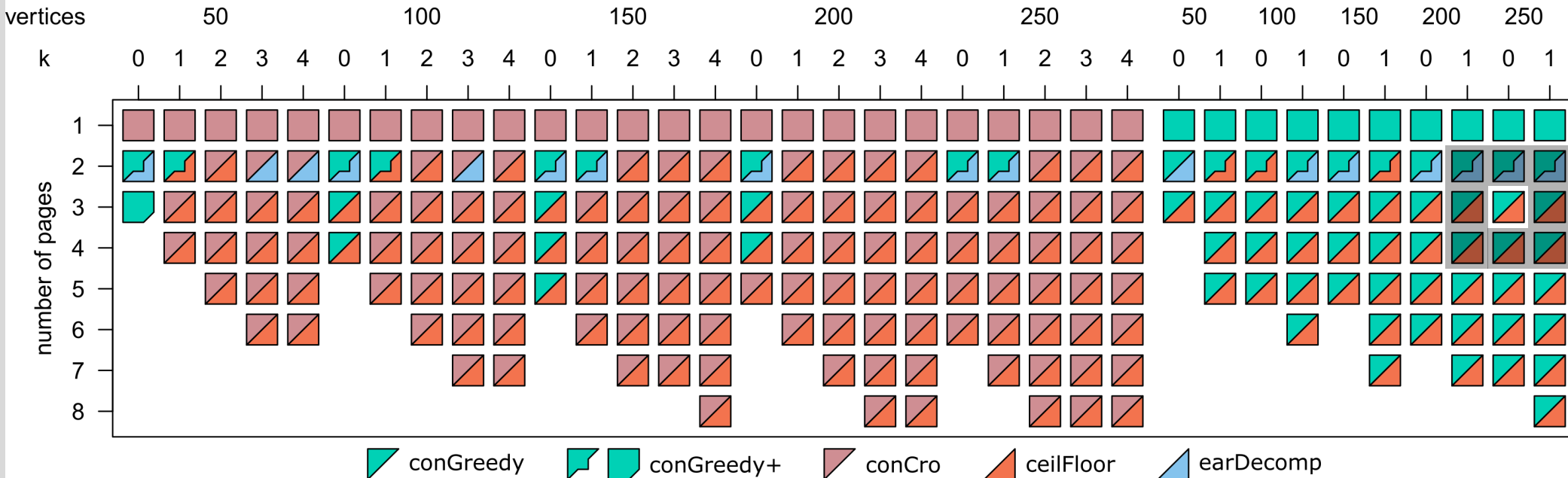


# Structured graphs - $k$ -planar

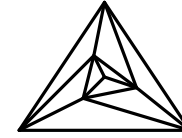


■ Geometric  $k$ -planar

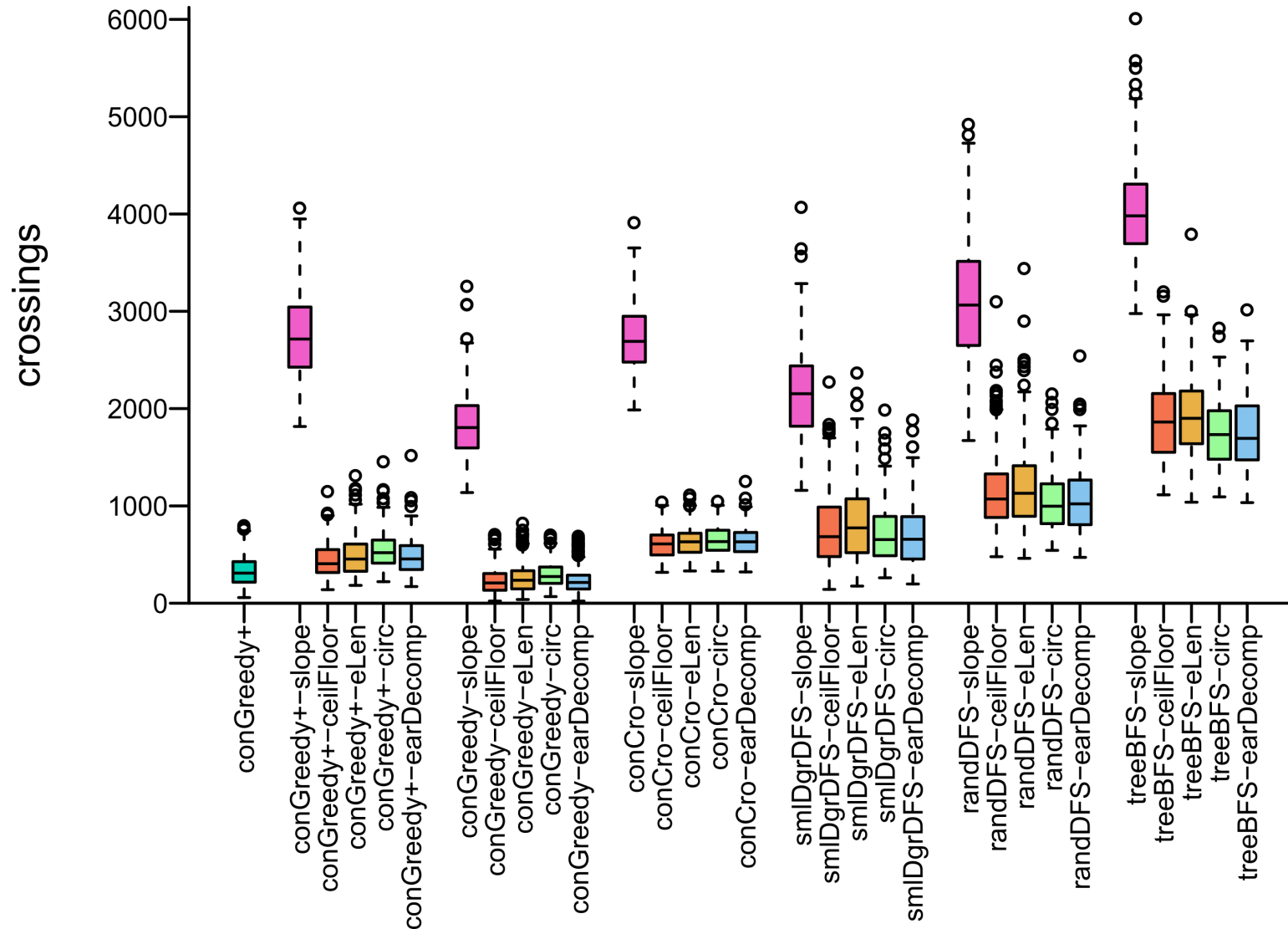
■ Topological 0/1-planar



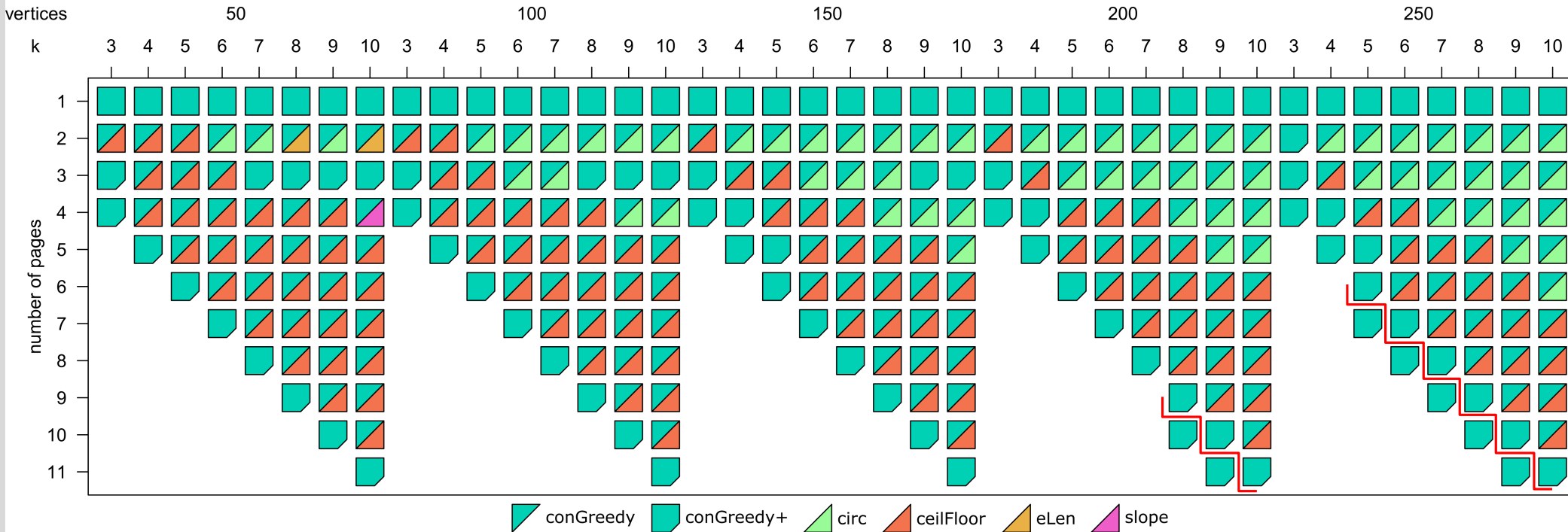
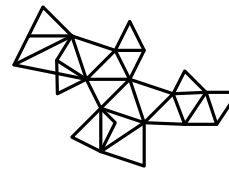
# Structured graphs - Topological planar



■  $n = 250$ , 3 pages

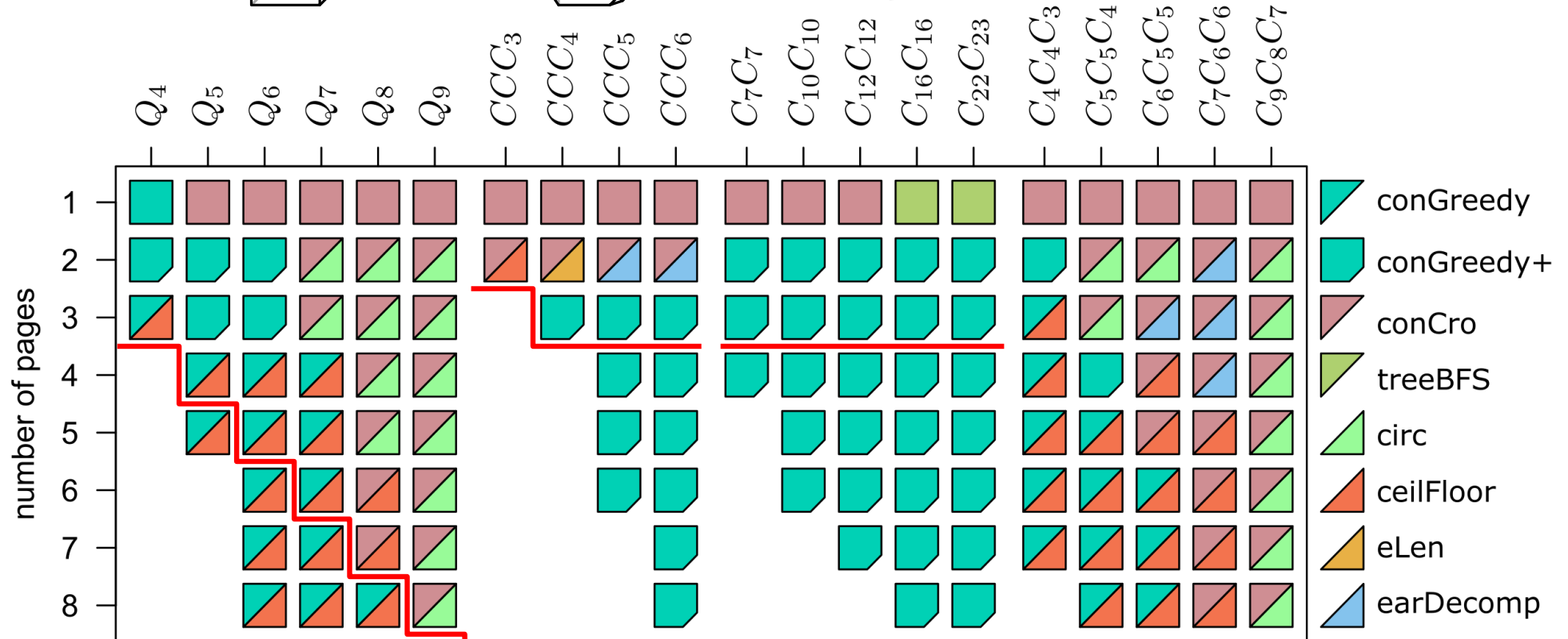
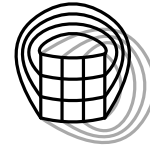
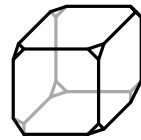
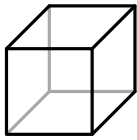


# Structured graphs - *k*-trees

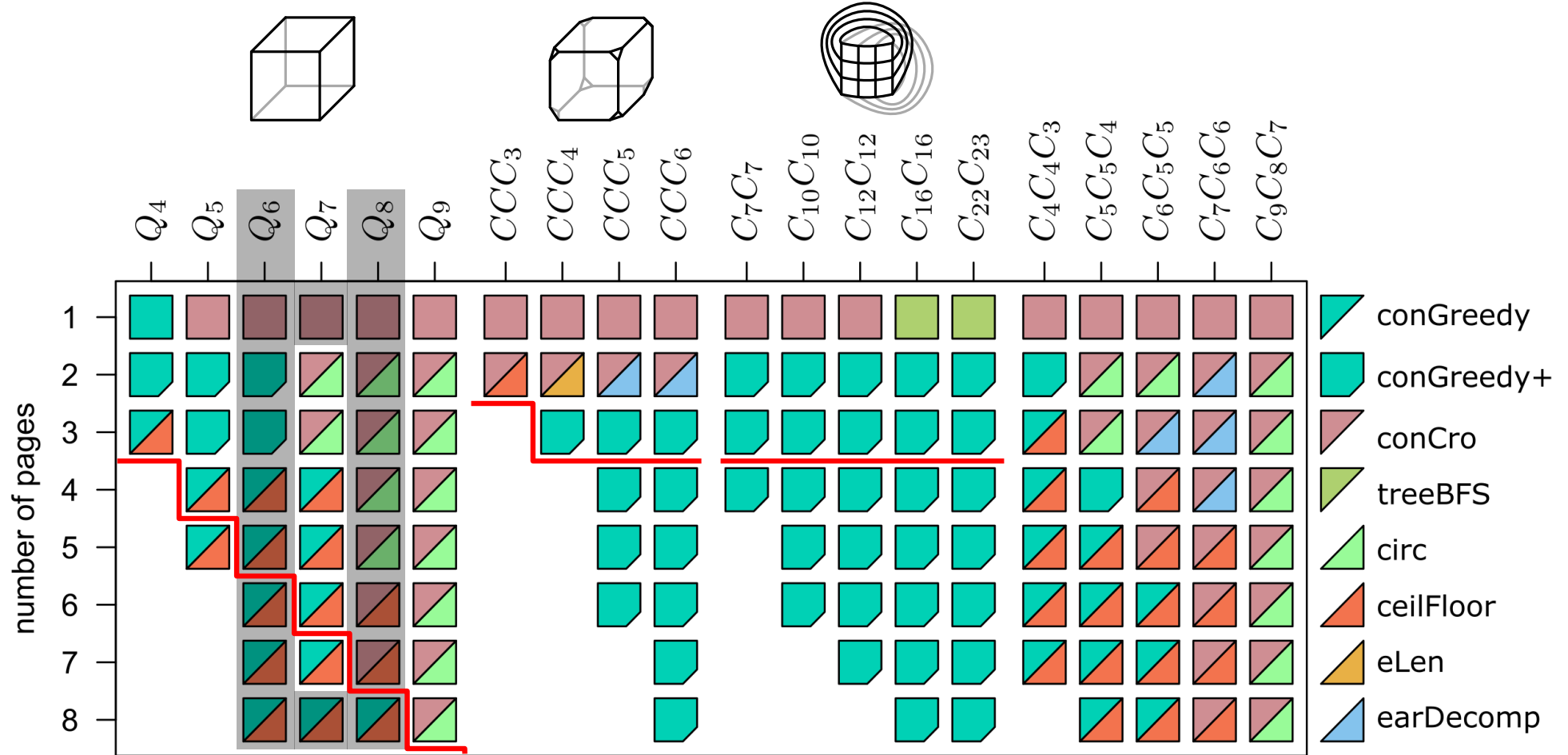




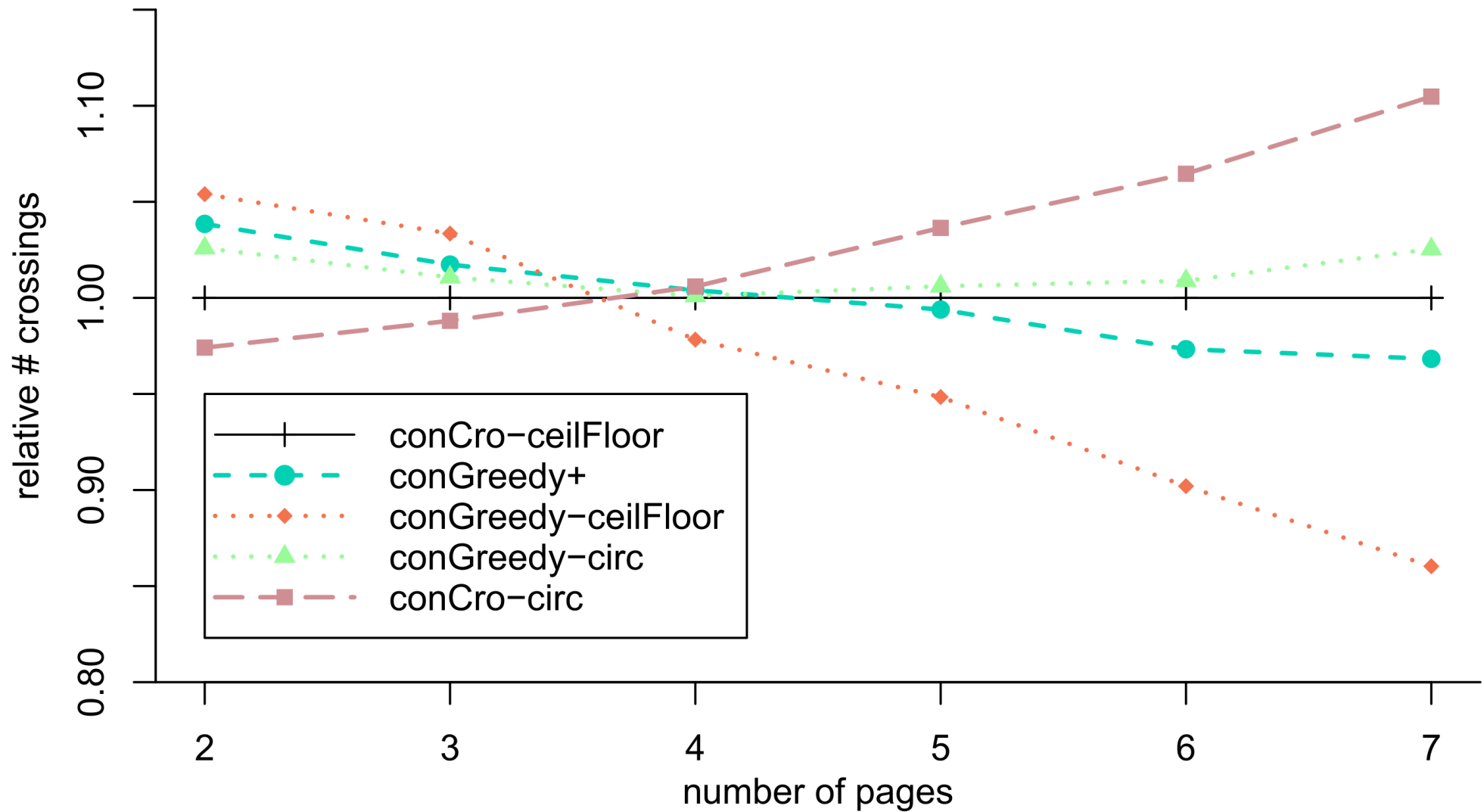
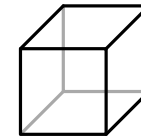
# Homogenous graphs



# Homogenous graphs



# Homogenous graphs - hypercube $Q_7$



# Summary

- Introduction of benchmark set
- Broad study of cases
  - Size, structurdeness and density of graphs matter
  - Number of pages matter
- Merging VO and PA problems
  - conGreedy+ performs well
  - greedy+ performs well
- Other new heuristics perform well in special cases
- slope best for high density
  
- Code for algorithms and graph generation on Github

Thanks