Drawing Trees on Fixed Points with L-shaped Edges

Martin Derka
mderka@uwaterloo.ca

David R. Cheriton School of Computer Science
University of Waterloo

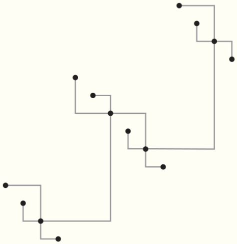
GD 2017

September 26, 2017

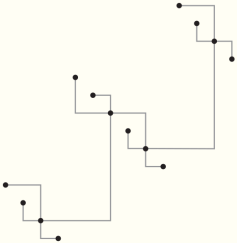
Joint work with **T. Biedl, T. Chan, K. Jain, A. Lubiw**.

UNIVERSITY OF WATERLOO

Goal: Draw a tree on a fixed set of points in general position so that every edge has the shape of an L.



Goal: Draw a tree on a fixed set of points in general position so that every edge has the shape of an L.



How many points do you need so that any tree can be drawn this way?

- $O(n^2)$ points always suffice for planar graphs

[de Fraysseix et al., Schnyder, 1990]

- $O(n^2)$ points always suffice for planar graphs

  [de Fraysseix et al., Schnyder, 1990]

- Lower bound $c \cdot n$ for some $c > 1$ for planar graphs

  [Chrobak + Karloff, 1989]

- $O(n^2)$ points always suffice for planar graphs

  [de Fraysseix et al., Schnyder, 1990]

- Lower bound $c \cdot n$ for some $c > 1$ for planar graphs

  [Chrobak + Karloff, 1989]

- Maximum degree 4 graphs studied first in 2010

  [Katz et al., 2010]

- Any tree of maximum degree 4 can be embedded on $n^2 - 2n + 2$ points

  [Di Giacomo et al., 2013]

- $O(n^2)$ points always suffice for planar graphs

  [de Fraysseix et al., Schnyder, 1990]

- Lower bound $c \cdot n$ for some $c > 1$ for planar graphs

  [Chrobak + Karloff, 1989]

- Maximum degree 4 graphs studied first in 2010

  [Katz et al., 2010]

- Any tree of maximum degree 4 can be embedded on $n^2 - 2n + 2$ points

  [Di Giacomo et al., 2013]

- $O(n^{\log_2 3 \approx 1.585})$ points suffice for any maximum degree 4 tree

  [Aichholzer et al., 2016]

- $O(n^2)$ points always suffice for planar graphs

  [de Fraysseix et al., Schnyder, 1990]

- Lower bound $c \cdot n$ for some $c > 1$ for planar graphs

  [Chrobak + Karloff, 1989]

- Maximum degree 4 graphs studied first in 2010

  [Katz et al., 2010]

- Any tree of maximum degree 4 can be embedded on $n^2 - 2n + 2$ points

  [Di Giacomo et al., 2013]

- $O(n^{\log_2 3 \approx 1.585})$ points suffice for any maximum degree 4 tree

  [Aichholzer et al., 2016]

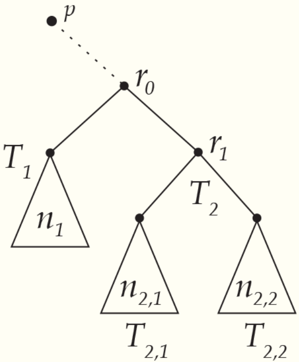
- Nothing better known for maximum degree 3 trees

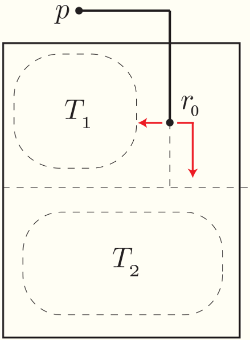
Our paper: Improvements on pointsets sufficient to draw maximum degree 3 and 4 trees

This talk: Maximum degree 3 trees (binary trees)

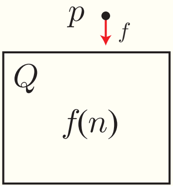
Main idea:

- Draw the tree inside a rectangle.
- Partition the tree inside subrectangles and draw the subtrees recursively inside those.

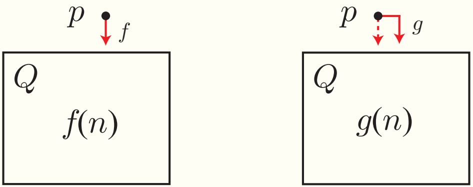




1. f-configuration: downward ray into Q is reserved
   $f(n)$: the number of points sufficient to draw a tree



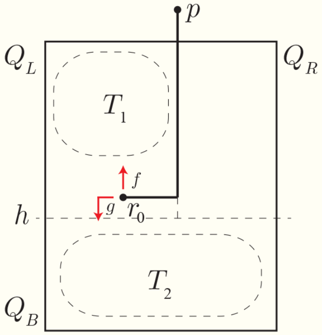
1. f-configuration: downward ray into Q is reserved
   $f(n)$: the number of points sufficient to draw a tree
2. g-configuration: rightward ray with bend into Q is reserved, and possibly also downward if the position of p allows
   $g(n)$: the number of points sufficient to draw a tree



Drawing method f-draw-1

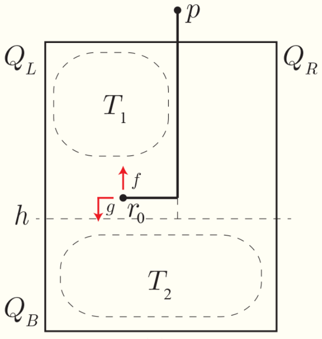
Assume that Q has $f(n)$ points.

- h: the highest half-grid line s.t. Q_L or Q_R has $f(n_1)$ points
- r_0: the bottommost point of Q_L



Observe that:

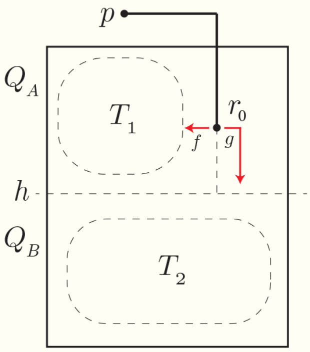
- $f(n) \leq 2f(n_1) + g(n_2)$
- $f(n) \leq 2g(n_1) + f(n_2)$ by swapping f and g



Drawing method *g*-draw

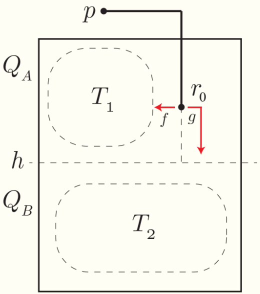
Assume that Q has $g(n)$ points.

- h: the highest half-grid line s.t. top rectangle Q_A has $f(n_1)$ points
- r_0: the rightmost point of Q_A



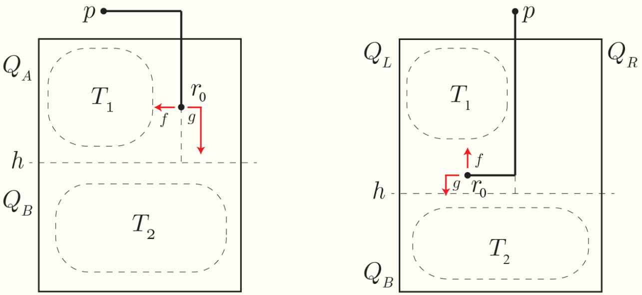
Observe:

- if a point right of p exists: $g(n) \leq f(n_1) + g(n_2)$



Observe:

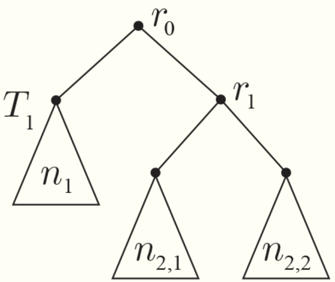
- if a point right of p exists: $g(n) \leq f(n_1) + g(n_2)$
- if no point right of p exists: use f-draw-1 with Q_R empty, so: $g(n) \leq f(n_1) + g(n_2)$

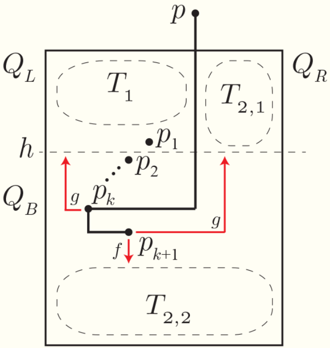


Drawing method *f*-draw-2

Assume that Q has $f(n)$ points.

- h: the highest half-grid line s.t. Q_L or Q_R has $g(n_1)$ points
- two cases depending on size of Q_R

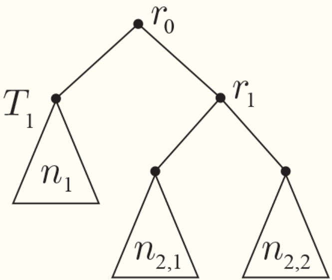


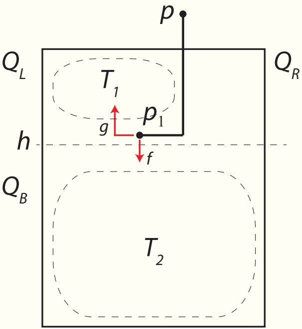


Drawing method *f*-draw-2

Assume that Q has $f(n)$ points.

- h: the highest half-grid line s.t. Q_L or Q_R has $g(n_1)$ points
- Case 1: $|Q_R| < g(n_{2,1})$: Use $g(n_1) + g(n_{2,1}) + f(n_2) - 1$ points

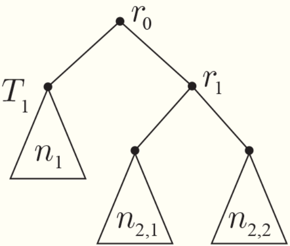


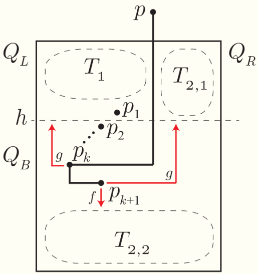


Drawing method *f*-draw-2

Assume that Q has $f(n)$ points.

- h: the highest half-grid line s.t. Q_L or Q_R has $g(n_1)$ points
- Case 2: $|Q_R| \geq g(n_{2,1})$, $k \geq n$
- Di Giacomo et al.: any tree of can be embedded on a diagonal point set with n points

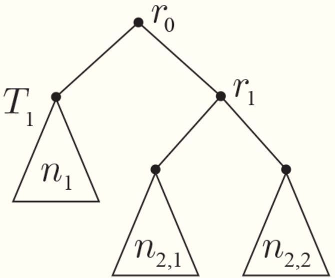


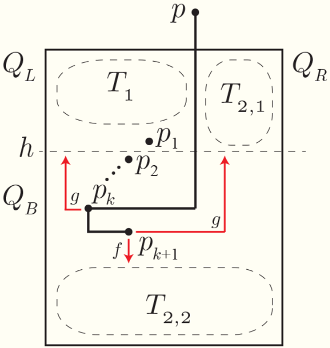


Drawing method *f*-draw-2

Assume that Q has $f(n)$ points.

- h: the highest half-grid line s.t. Q_L or Q_R has $g(n_1)$ points
- Case 2: $|Q_R| \geq g(n_{2,1})$, $k < n$: $2g(n_1) + n + f(n_{2,2}) - 1$ points





Putting everything together:

1. $f(n) \leq 2f(n_1) + g(n_2)$
2. $f(n) \leq 2g(n_1) + f(n_2)$
3. $g(n) \leq f(n_1) + g(n_2)$
4. $f(n) \leq \max\{g(n_1) + g(n_{2,1}) + f(n_2), 2g(n_1) + f(n_{2,2}) + n\}$

Putting everything together:

1. $f(n) \leq 2f(n_1) + g(n_2)$
2. $f(n) \leq 2g(n_1) + f(n_2)$
3. $g(n) \leq f(n_1) + g(n_2)$
4. $f(n) \leq \max\{g(n_1) + g(n_{2,1}) + f(n_2), 2g(n_1) + f(n_{2,2}) + n\}$

Theorem: Any perfect binary tree with n nodes has an L-shaped drawing on any point set of size $c \cdot n^{1.142}$ for some constant c.

Putting everything together:

1. $f(n) \leq 2f(n_1) + g(n_2)$
2. $f(n) \leq 2g(n_1) + f(n_2)$
3. $g(n) \leq f(n_1) + g(n_2)$
4. $f(n) \leq \max\{g(n_1) + g(n_{2,1}) + f(n_2), 2g(n_1) + f(n_{2,2}) + n\}$

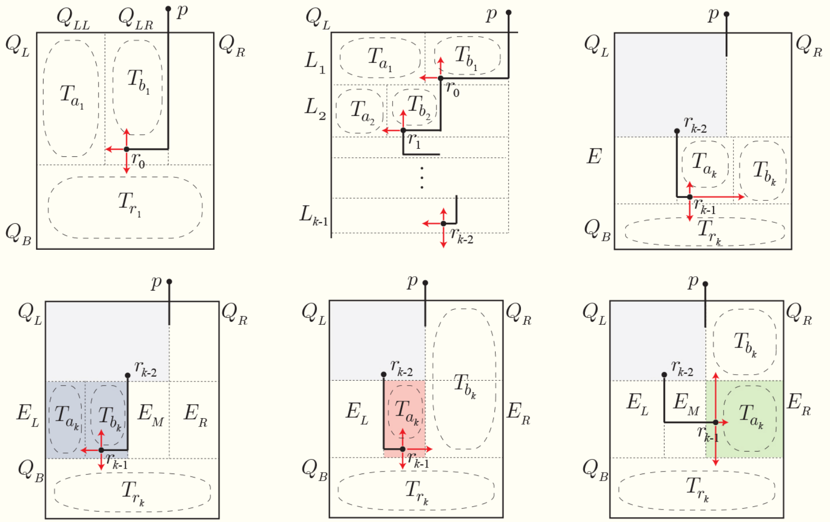
Theorem: Any perfect binary tree with n nodes has an L-shaped drawing on any point set of size $c \cdot n^{1.142}$ for some constant c.

Theorem: Any binary tree with n nodes has an L-shaped drawing on any point set of size $c \cdot n^{1.22}$ for some constant c.

Summary of the results

The follwing number of points are sufficient for L-drawing of any with n points:

	previous	new
deg 3 perfect	$n^{1.585}$	$n^{1.142}$
deg 3 general	$n^{1.585}$	$n^{1.22}$
deg 4 perfect	$n^{1.4651}$	
deg 4 general	$n^{1.585}$	$n^{1.55}$


Q_{LL} Q_{LR} p
Q_L Q_R
T_{a_1} T_{b_1}
r_0
T_{r_1}
Q_B
Q_L p
L_1 T_{a_1} T_{b_1}
r_0
L_2 T_{a_2} T_{b_2}
r_1
$L_{k\text{-}1}$
$r_{k\text{-}2}$
p
Q_L Q_R
$r_{k\text{-}2}$
E T_{a_k} T_{b_k}
$r_{k\text{-}1}$
Q_B T_{r_k}
p
Q_L Q_R
$r_{k\text{-}2}$
E_L T_{a_k} T_{b_k} E_M E_R
$r_{k\text{-}1}$
Q_B T_{r_k}
p
Q_L Q_R
$r_{k\text{-}2}$ T_{b_k}
E_L T_{a_k} E_R
$r_{k\text{-}1}$
Q_B T_{r_k}
p
Q_L Q_R
T_{b_k}
$r_{k\text{-}2}$
E_L E_M T_{a_k} E_R
$r_{k\text{-}1}$
Q_B T_{r_k}

Summary of the results

The follwing number of points are sufficient for L-drawing of any with n points:

	previous	new
deg 3 perfect	$n^{1.585}$	$n^{1.142}$
deg 3 general	$n^{1.585}$	$n^{1.22}$
deg 4 perfect	$n^{1.4652}$	
deg 4 general	$n^{1.585}$	$n^{1.55}$

Summary of the results

The follwing number of points are sufficient for L-drawing of any with n points:

	previous	new
deg 3 perfect	$n^{1.585}$	$n^{1.142}$
deg 3 general	$n^{1.585}$	$n^{1.22}$
deg 4 perfect	$n^{1.4652}$	
deg 4 general	$n^{1.585}$	$n^{1.55}$

Ordered caterpillars: $c \cdot n \log n$ points for some constant c

Summary of the results

The follwing number of points are sufficient for L-drawing of any with n points:

	previous	new
deg 3 perfect	$n^{1.585}$	$n^{1.142}$
deg 3 general	$n^{1.585}$	$n^{1.22}$
deg 4 perfect	$n^{1.4652}$	
deg 4 general	$n^{1.585}$	$n^{1.55}$

Ordered caterpillars: $c \cdot n \log n$ points for some constant c

Thank you!