On Upward Drawings of Trees on a Given Grid

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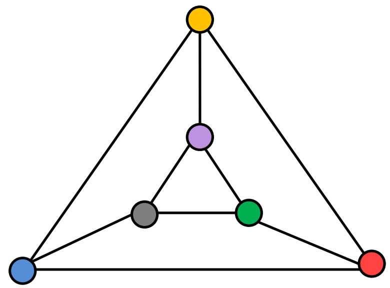
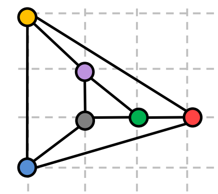
Area Minimization in Planar Straight-Line Drawings



G



A planar drawing
on a 4×4 grid.

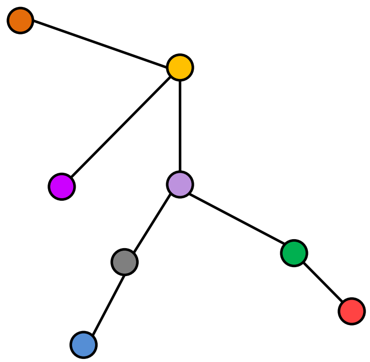
[Krug and Wagner 2008, Biedl 2014]

NP-hard for
- Arbitrary planar graphs
- Planar graphs with bounded pathwidth
- Outerplanar graphs

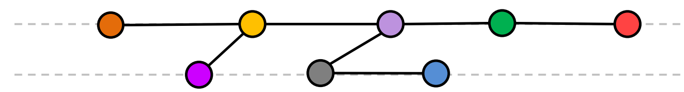
[M et al. 2008, Biedl 2014]

Polynomial time for
- Planar 3-trees
- Planar graphs with both bounded treewidth and bounded face-degrees

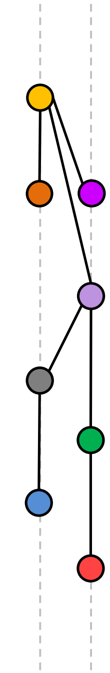
Are Trees Easy?



T



Minimizing
Height



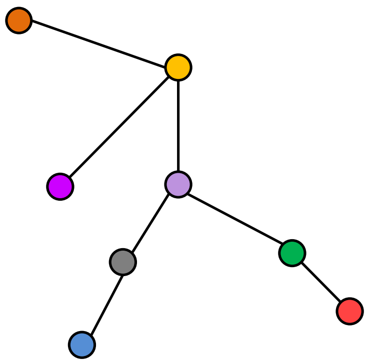
Minimizing
Width

[Alam et al. 2008, M et al 2011, Biedl 2015]

Minimizing one dimension

- Upward drawings (rooted trees)
- Strictly-upward drawings (rooted trees)
- Non-upward drawings, height (unrooted trees)

Area Minimization for Trees?



T

[Supowit and Reingold 1982]

NP-hard for

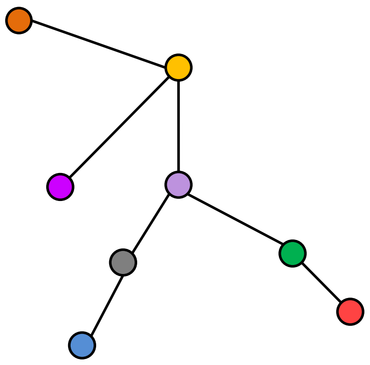
- Ordered trees - under constraints such as isomorphic subtrees must be drawn identically, left and right child must be placed on the left and right of their parent

[Bhatt and Cosmadakis 1987, Gregori 1989, Brunner and Matzeder 2010, Bachmaier and Matzeder 2013]

NP-hard for

- Drawing ordered/unordered trees on a k-grid, $k \in \{4, 6, 8\}$, with unit edge length

Area Minimization for Trees?



T

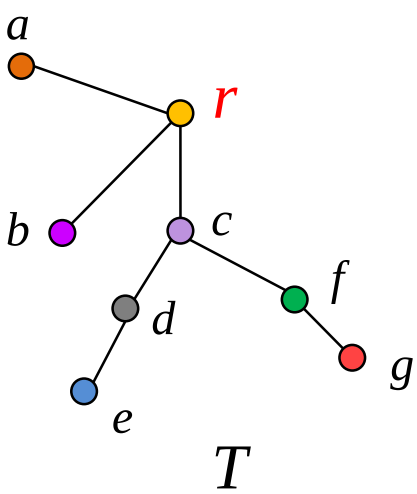
Straight-line drawings of Trees?

Upward straight-line drawings of **rooted** Trees?

Strictly upward straight-line drawings of **rooted** Trees?

Strictly upward straight-line drawings of **ordered rooted** Trees?

Strictly-Upward Drawing on a Given Grid is NP-hard

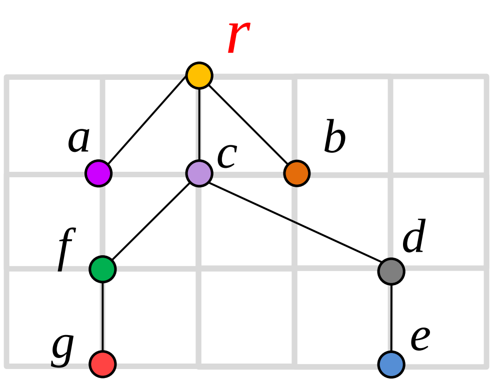


A given grid

Does T admit a strictly-upward drawing on the given grid?

A strictly-upward drawing of T

- Straight-line planar drawing
- Every child is drawn strictly below to its parent
- The ordering of the children can be chosen



Strictly-Upward Drawing on a Given Grid is NP-hard

A reduction from Numerical 3-Dimensional Matching (N3DM)

- **Instance**: Positive integers r_i, g_i, b_i, where $1 \leq i \leq k$, and an integer B such that $\Sigma_i (r_i + b_i + g_i) = k \cdot B$.
- **Question**: Do there exist permutations π and π' of $\{1, \ldots, k\}$ such that $r_{\pi(i)} + b_i + g_{\pi'(i)} = B$ for all $1 \leq i \leq k$?

r_i	b_i	g_i
5	5	3
2	7	2
4	7	7

r_i		b_i		g_i		
2	+	5	+	7	=	14
5	+	7	+	2	=	14
4	+	7	+	3	=	14

$$\Sigma_i (r_i + b_i + g_i) = k \cdot B$$

$$42 = 3 \cdot 14$$

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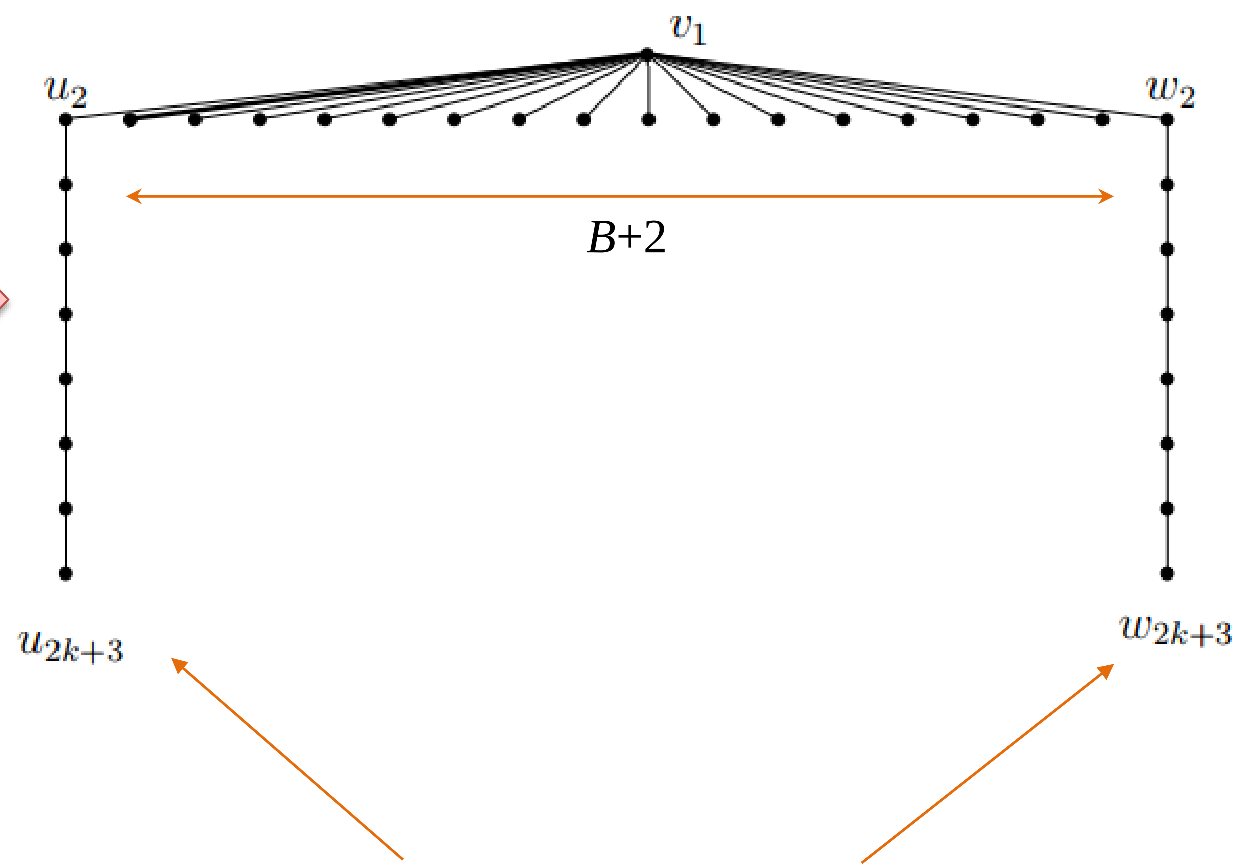
Remains NP-hard under the following restrictions:

- g_is are huge $\in O(k^{4c})$
- b_is are odd and large $\in O(k^{2c})$
- r_is are small $\in O(k^{c})$

Strictly-Upward Drawing on a Given Grid is NP-hard

r_i	b_i	g_i
5	5	3
2	7	2
4	7	7



Supporting paths

Strictly-Upward Drawing on a Given Grid is NP-hard

r_i	b_i	g_i
5	5	3
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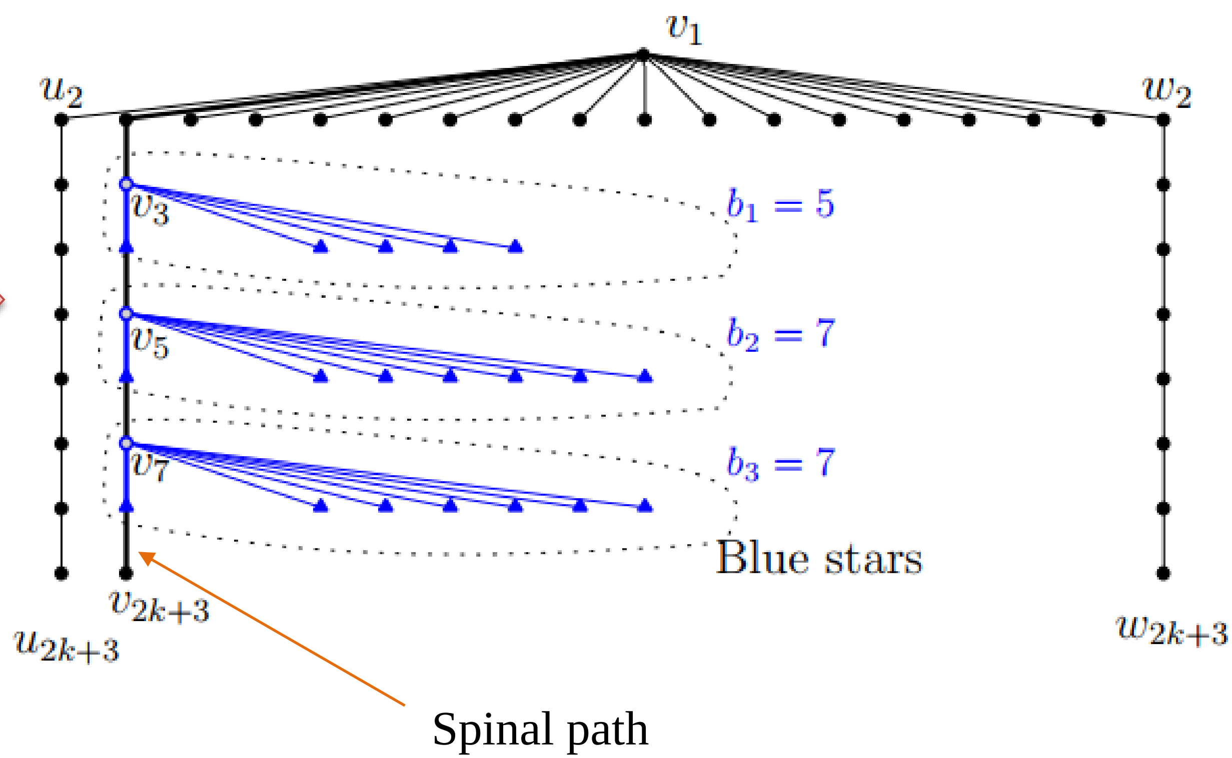

Strictly-Upward Drawing on a Given Grid is NP-hard

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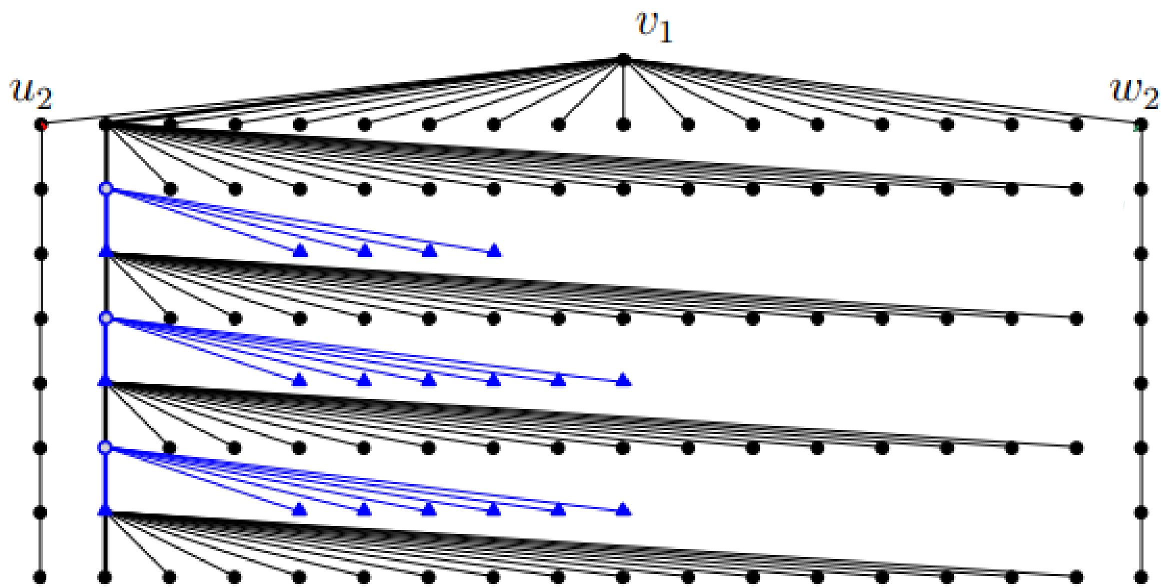
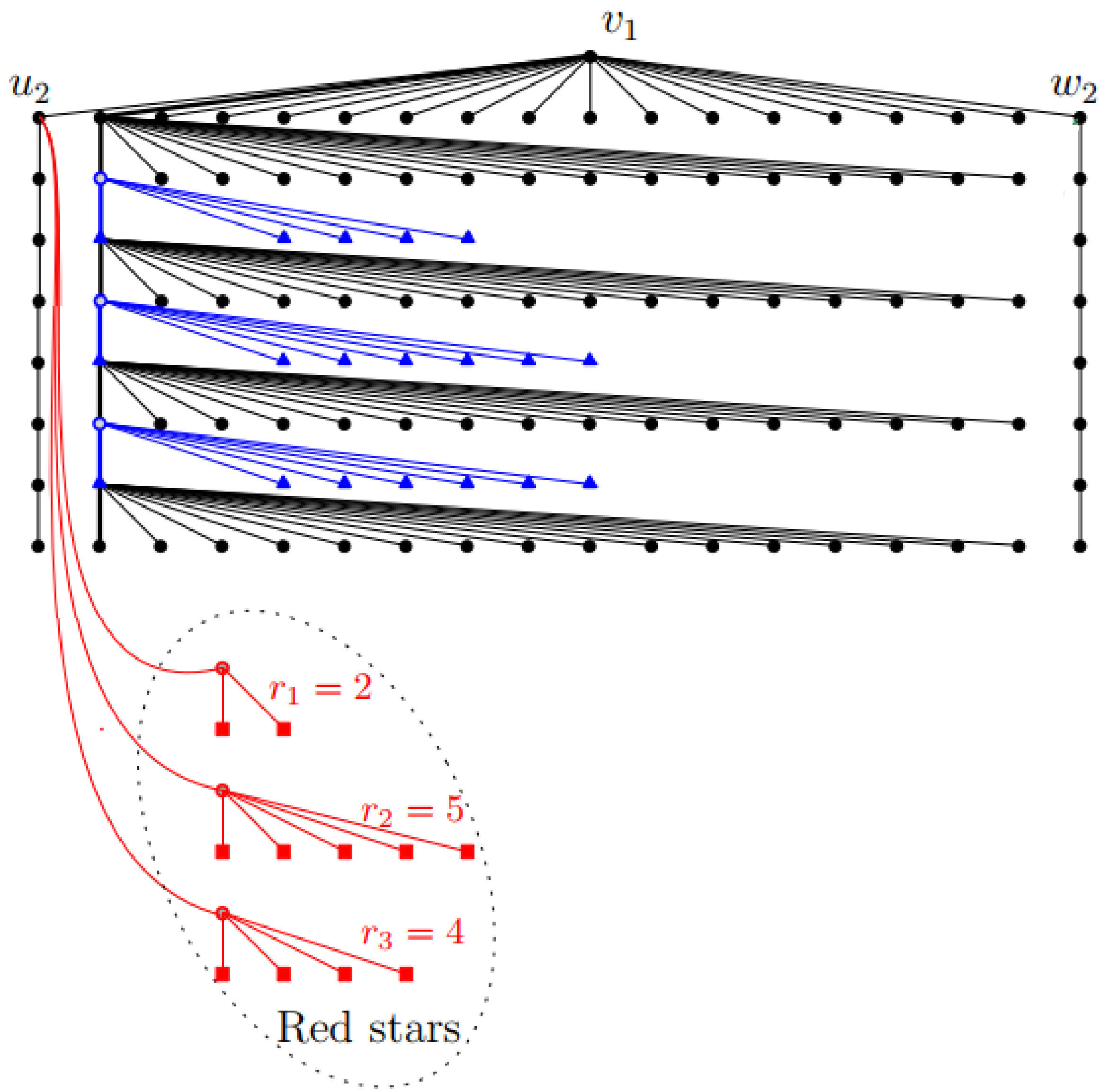


Add wall vertices

Strictly-Upward Drawing on a Given Grid is NP-hard

r_i	b_i	g_i
5	5	3
2	7	2
4	7	7



Strictly-Upward Drawing on a Given Grid is NP-hard

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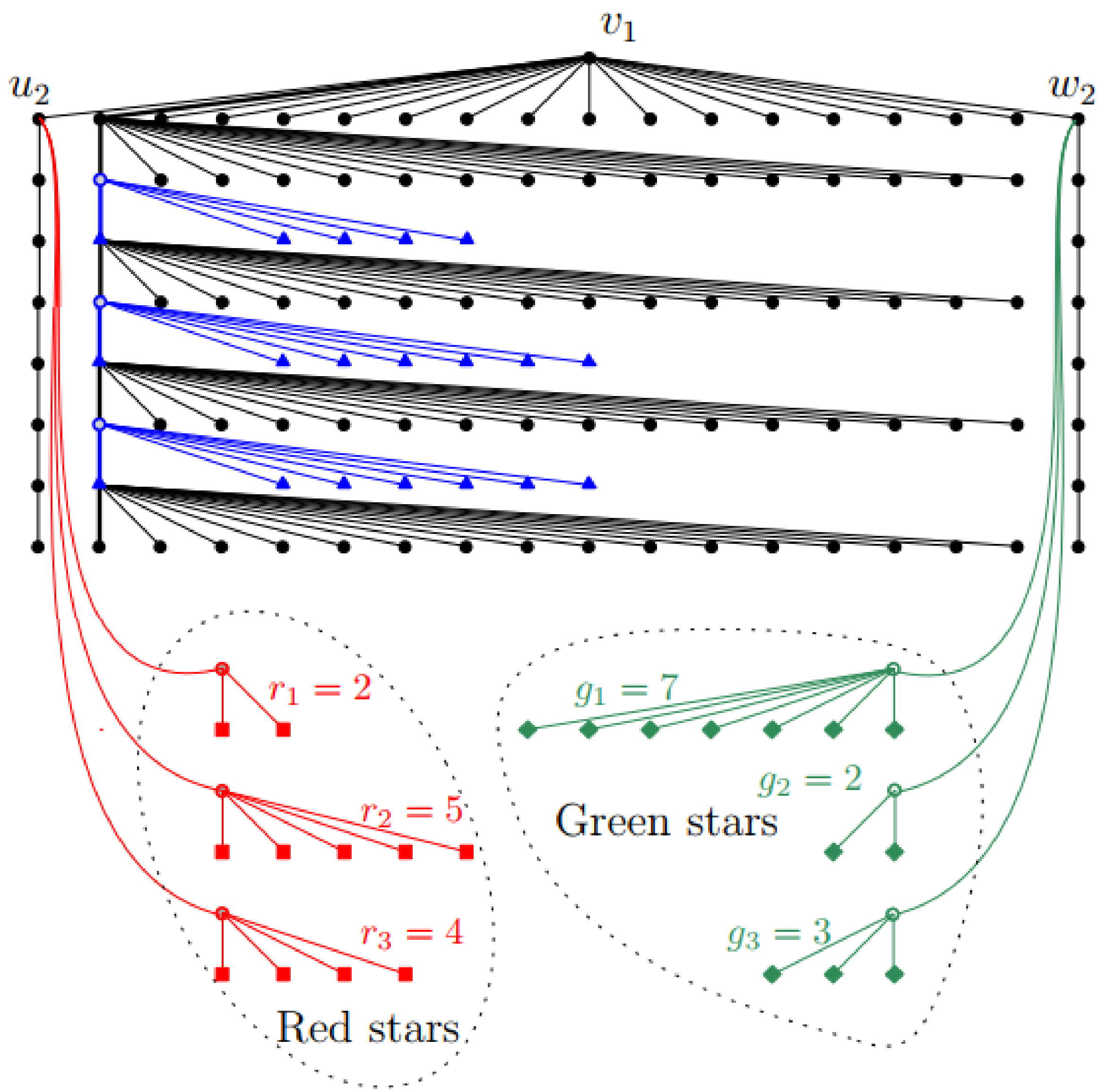

Strictly-Upward Drawing on a Given Grid is NP-hard

r_i	b_i	g_i
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2	7	2
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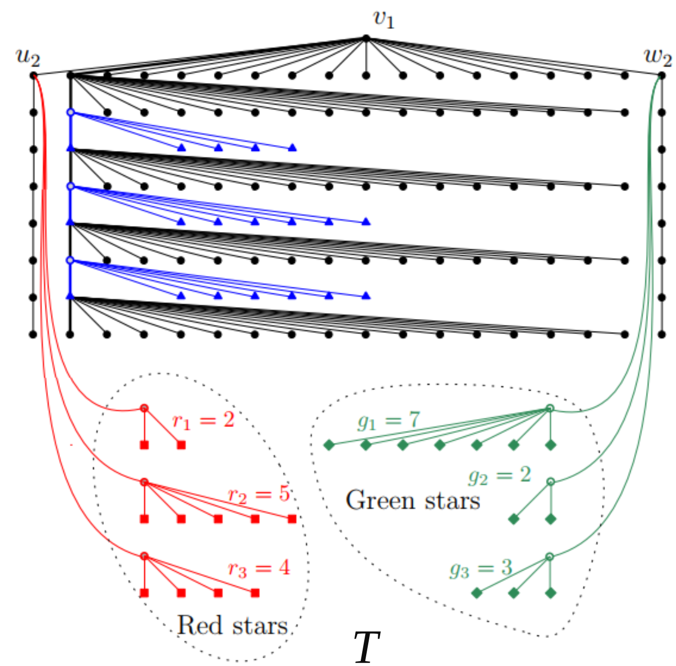
I



T

I has an affirmative solution if and only if
T admits a drawing on a $(B+4)\times(2k+3)$ grid

From N3DM to Tree Drawing

r_i	b_i	g_i
5	5	3
2	7	2
4	7	7

r_i		b_i		g_i		B
2	+	5	+	7	=	14
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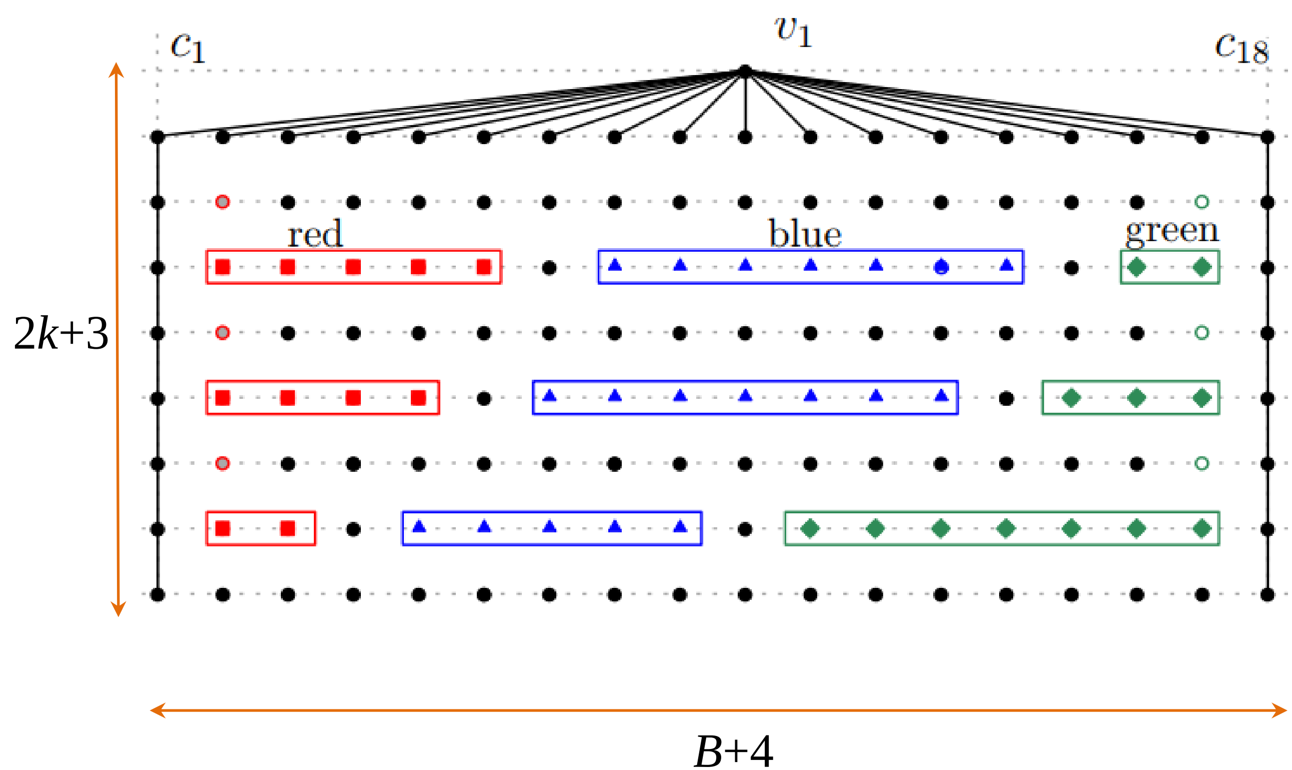

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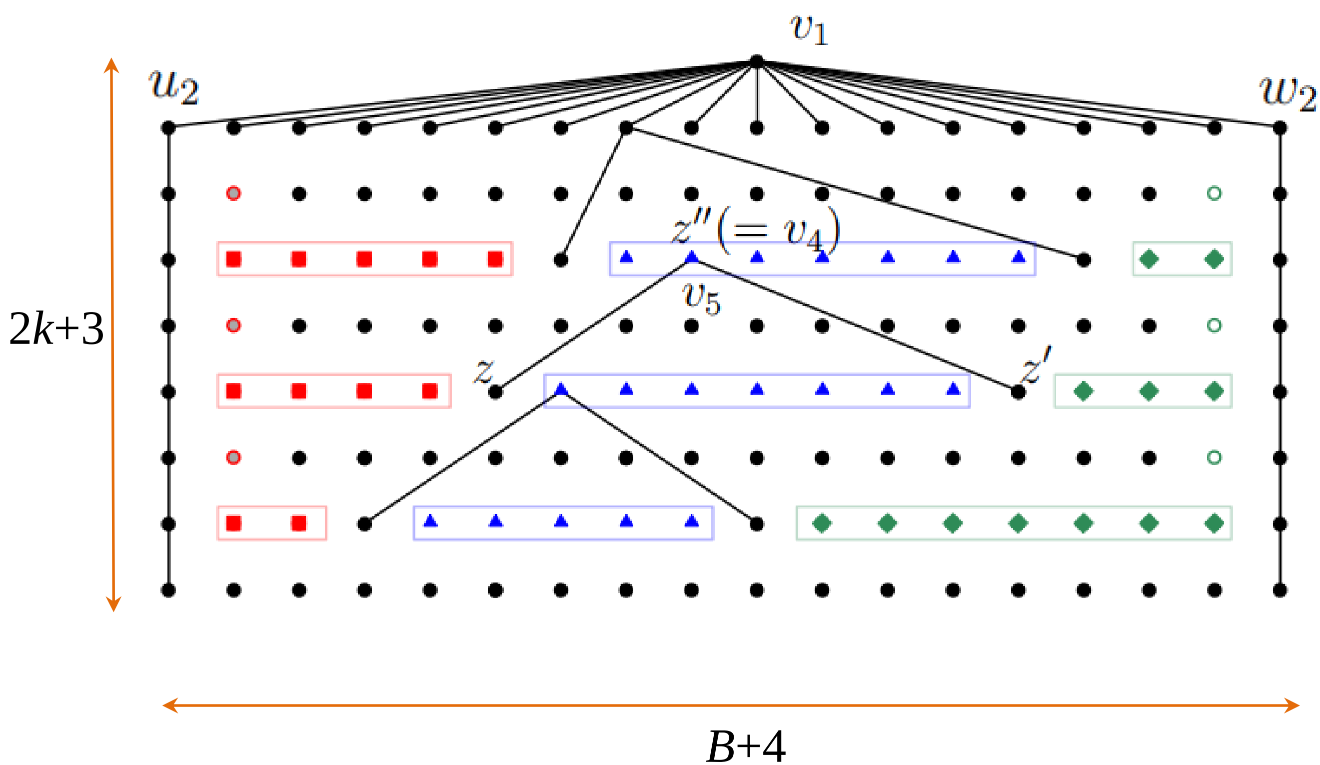

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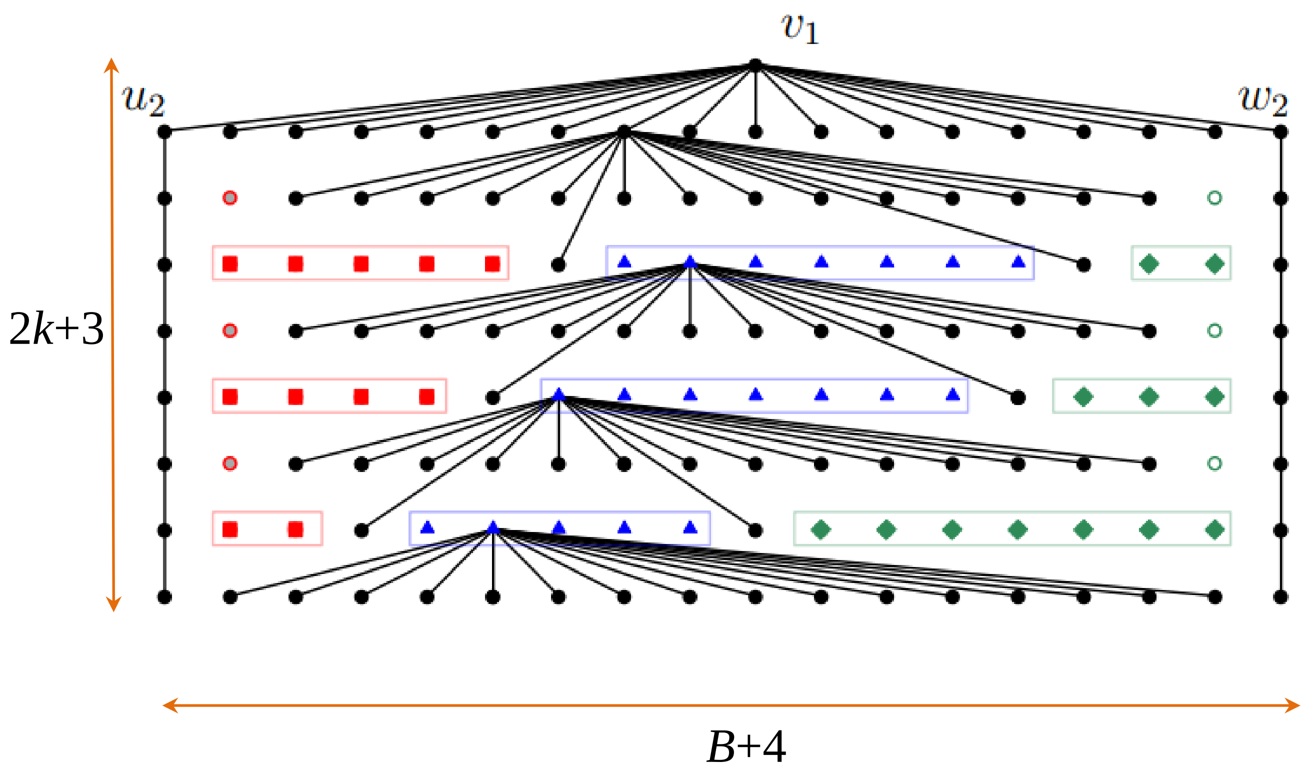

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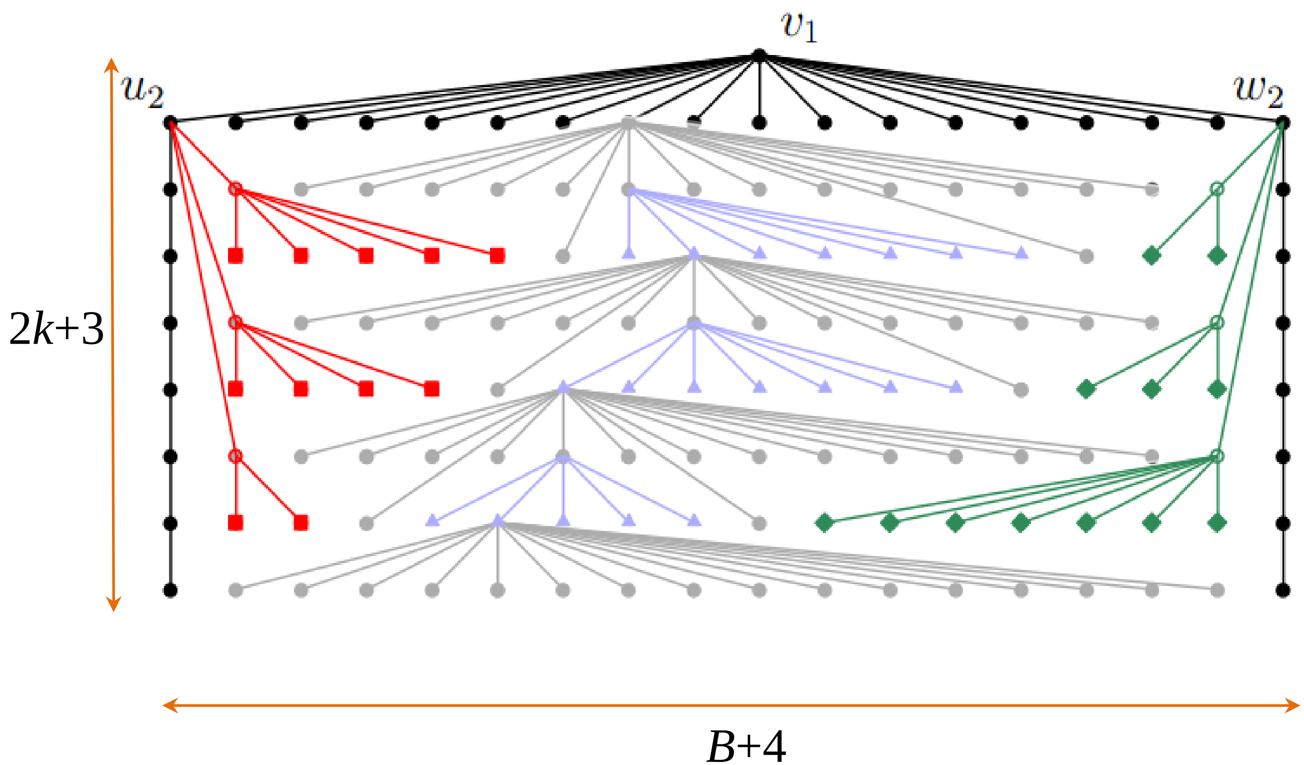
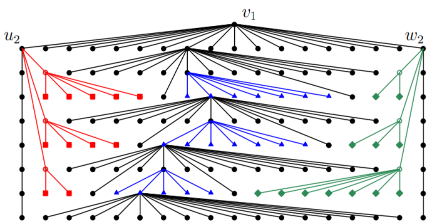


All the rows (except the topmost row) must be completely used up by the nodes of the tree.



From Tree Drawing to N3DM

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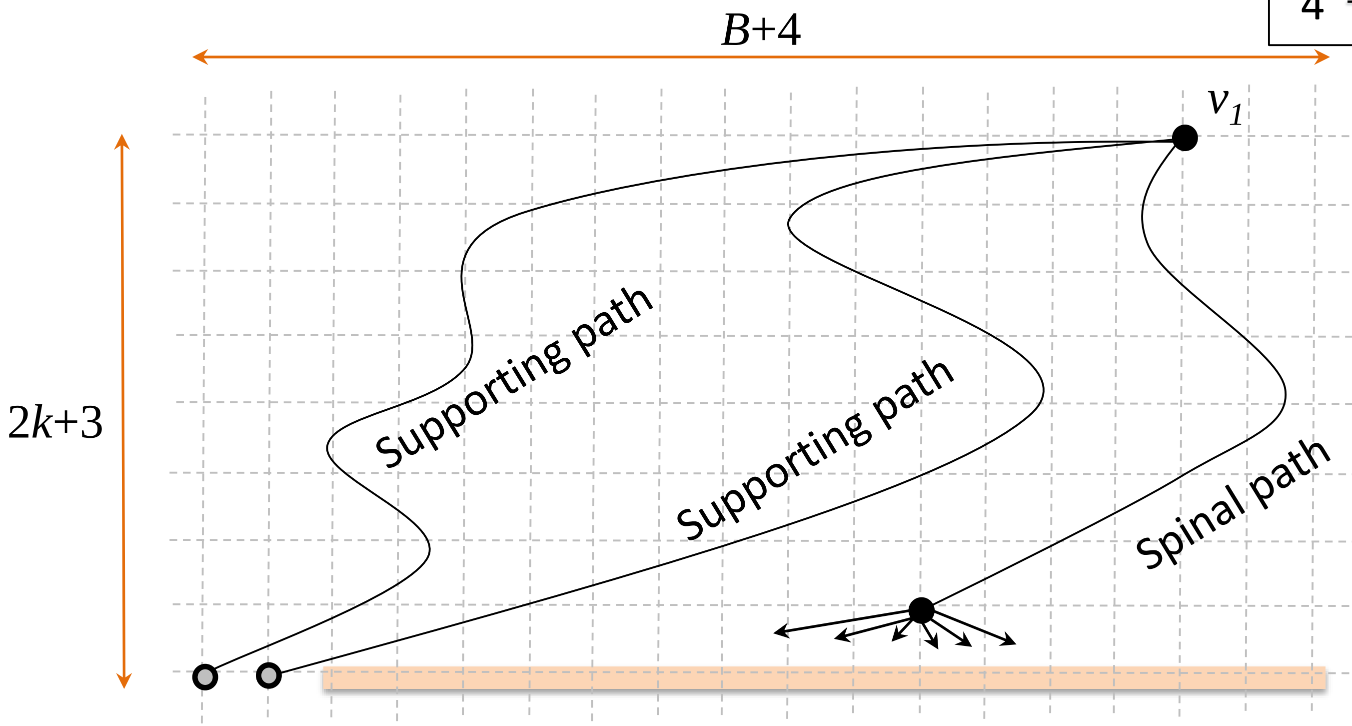
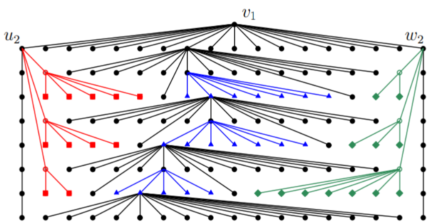


The bottommost layer contains

- two vertices from the supporting paths and
- Wall vertices of the bottommost wall parent.



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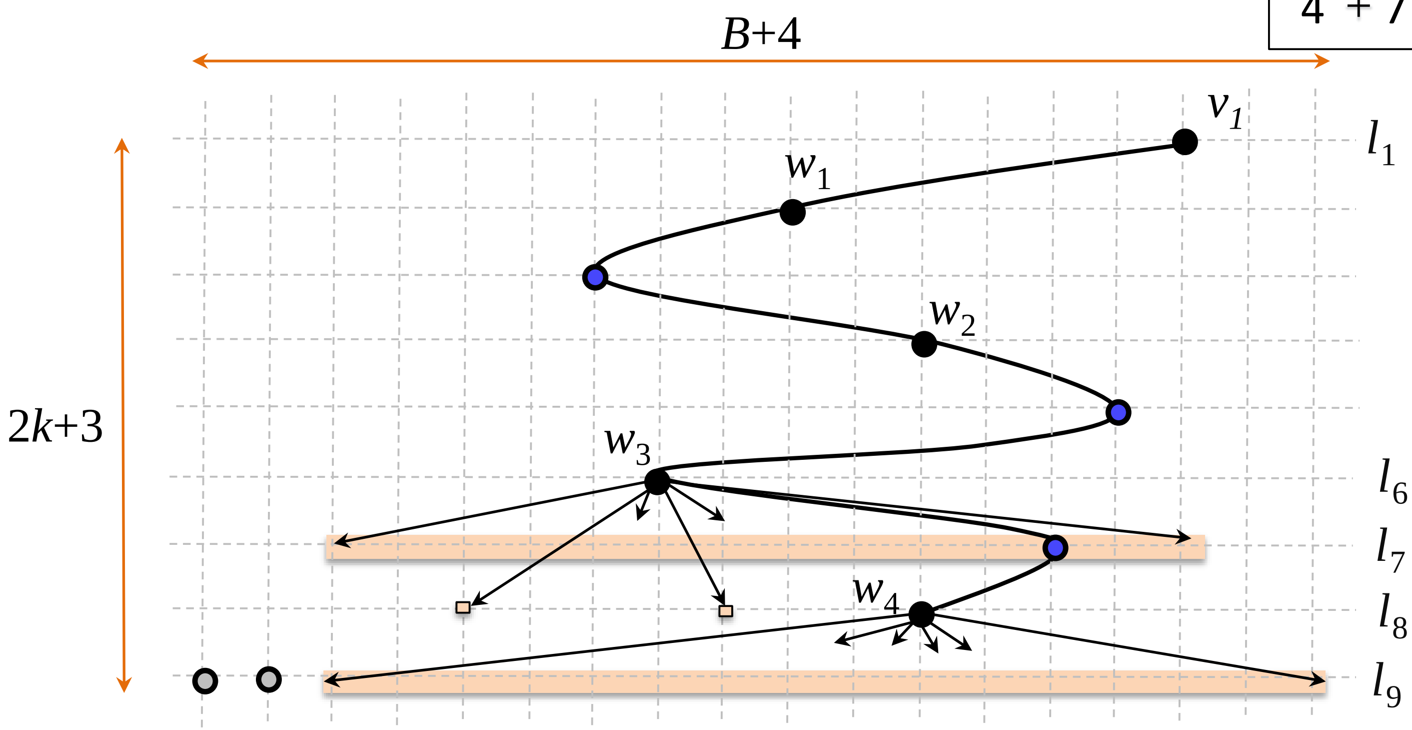
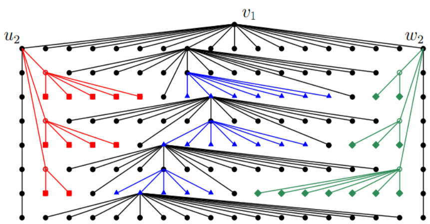


The next wall parent (w_3)

- can have at most two children on l_8 and
- all the remaining children lie consecutively on l_7



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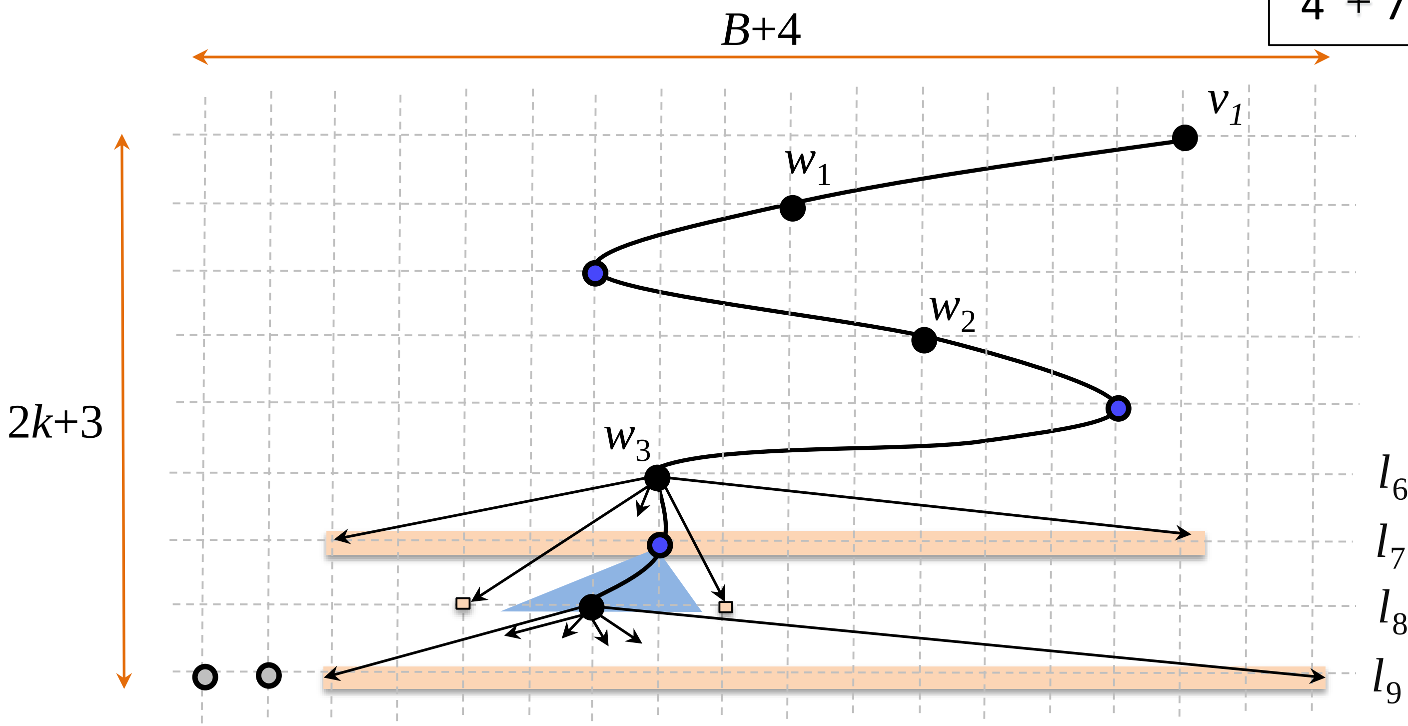

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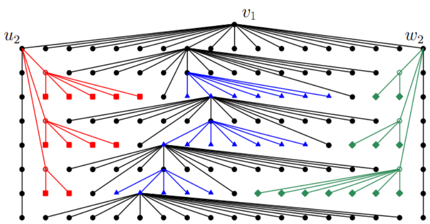

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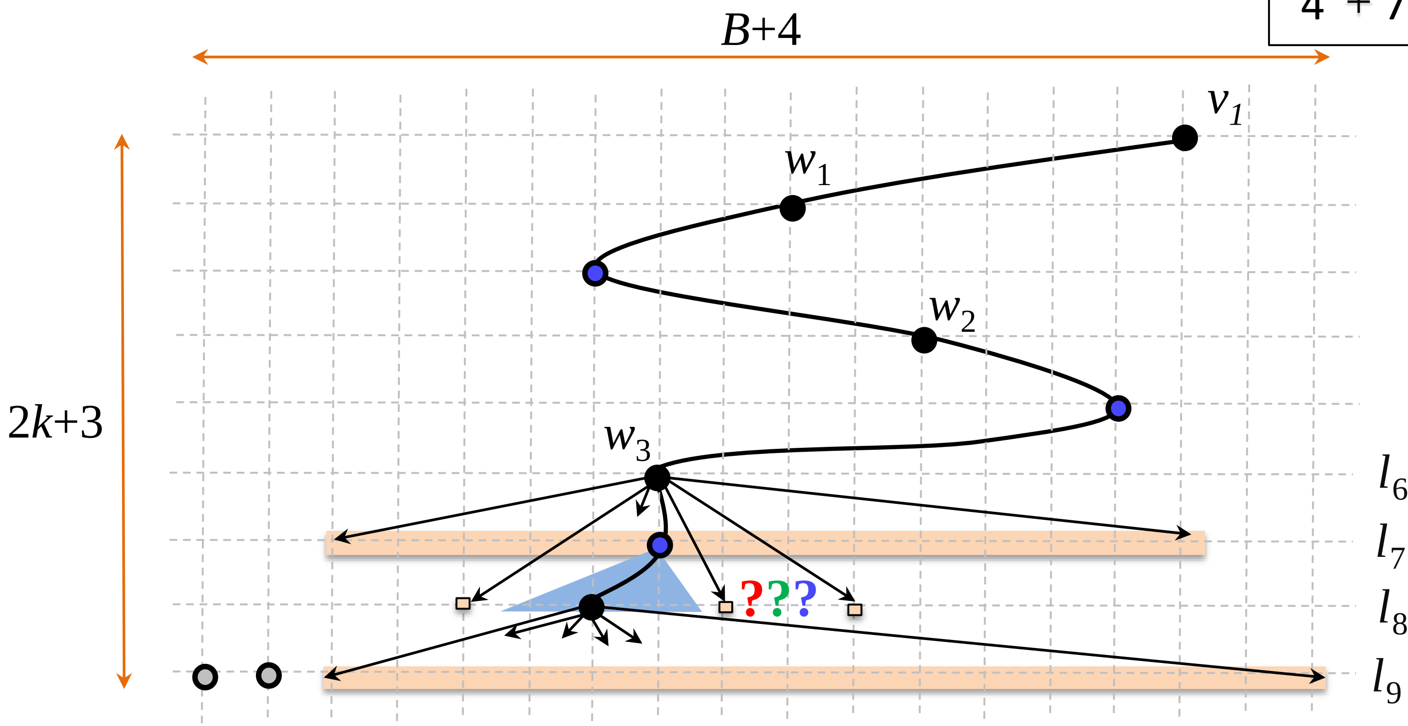

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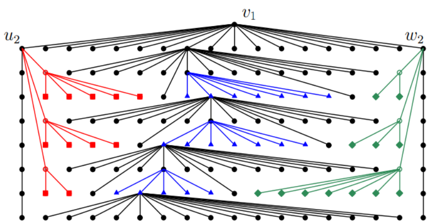

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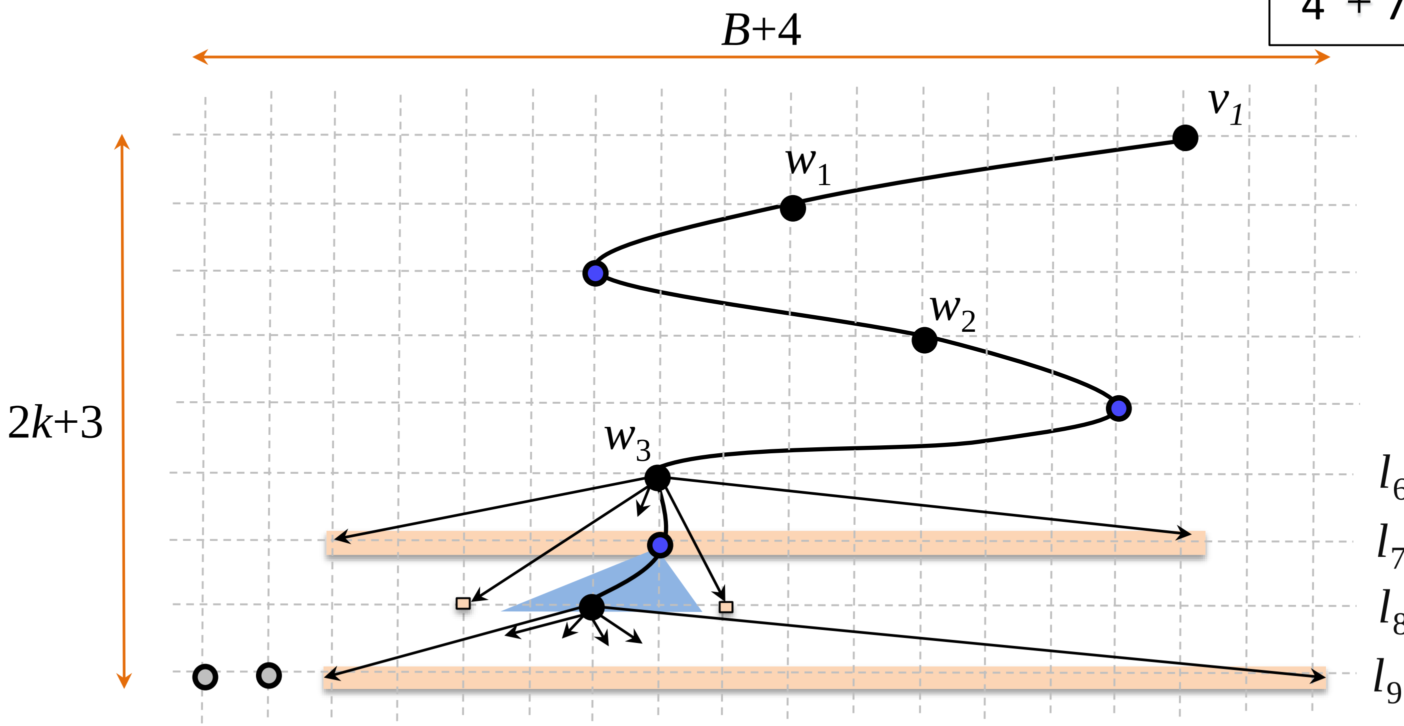

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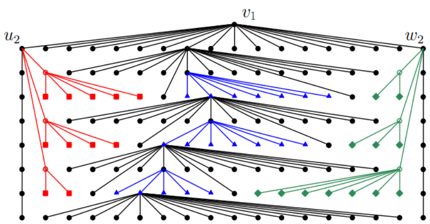

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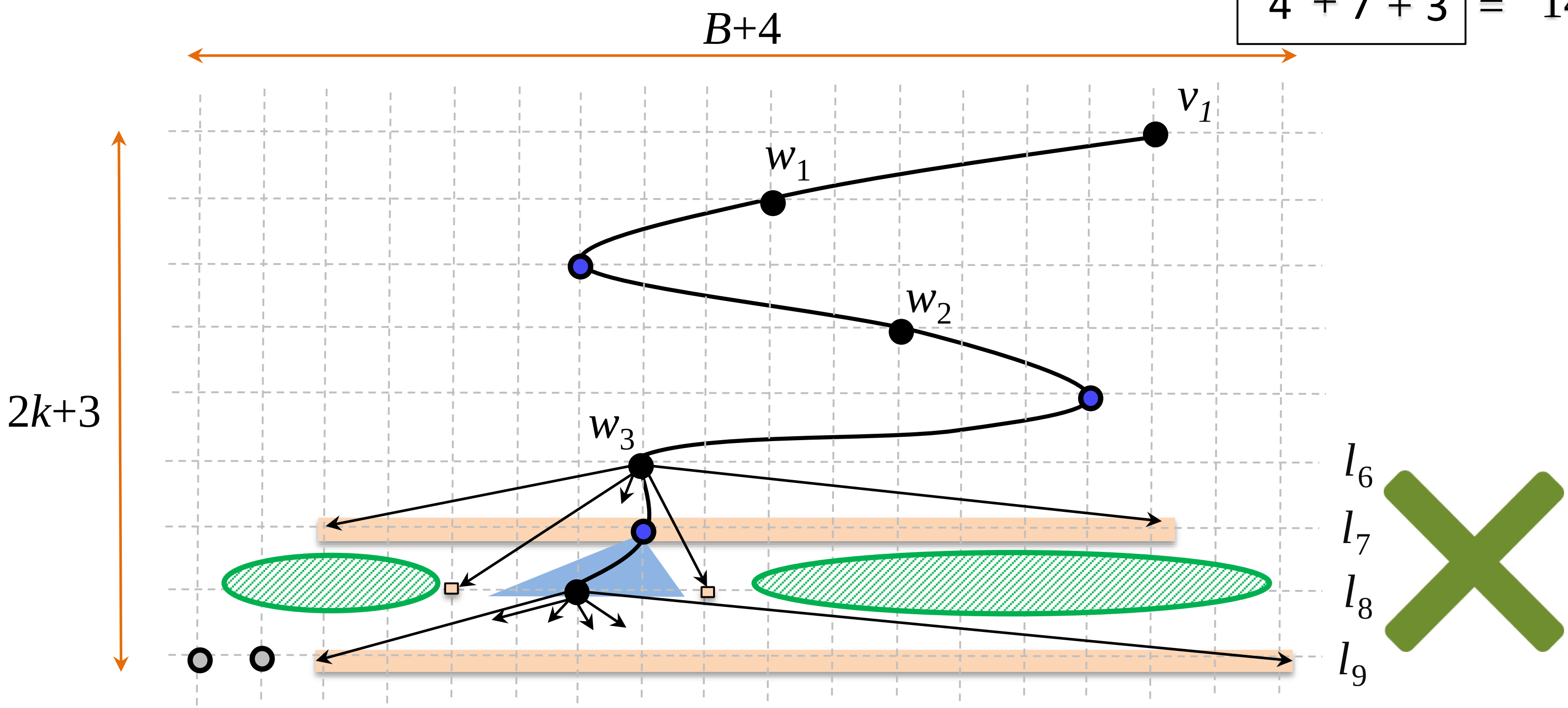
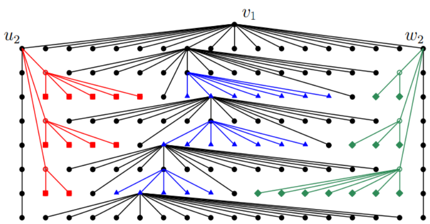


The remaining space is too large for two green stars.



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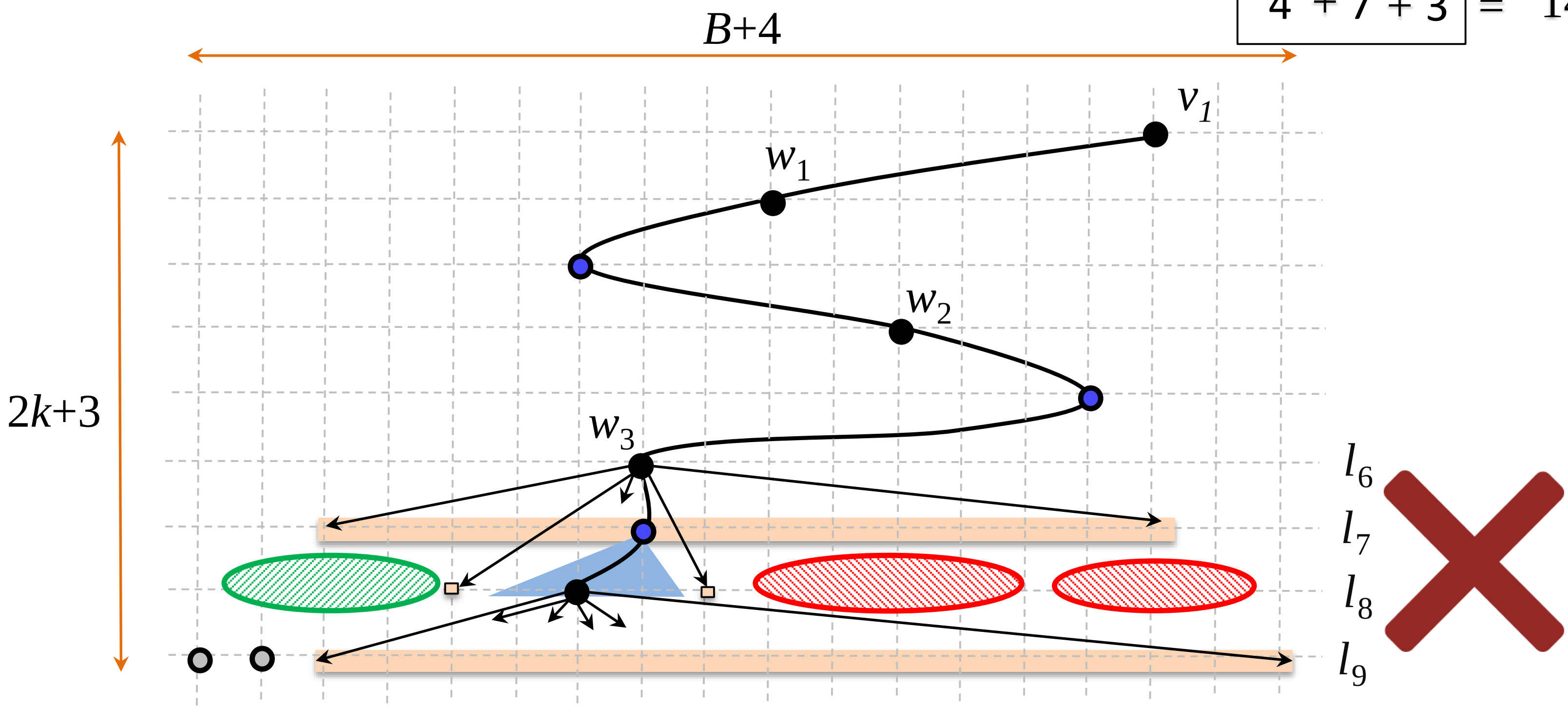
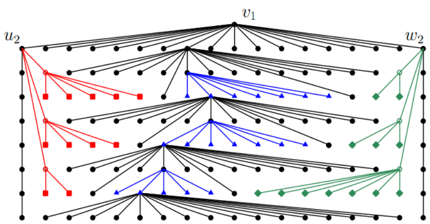


The remaining space is too large for two green stars, and placing two red stars would violate planarity.



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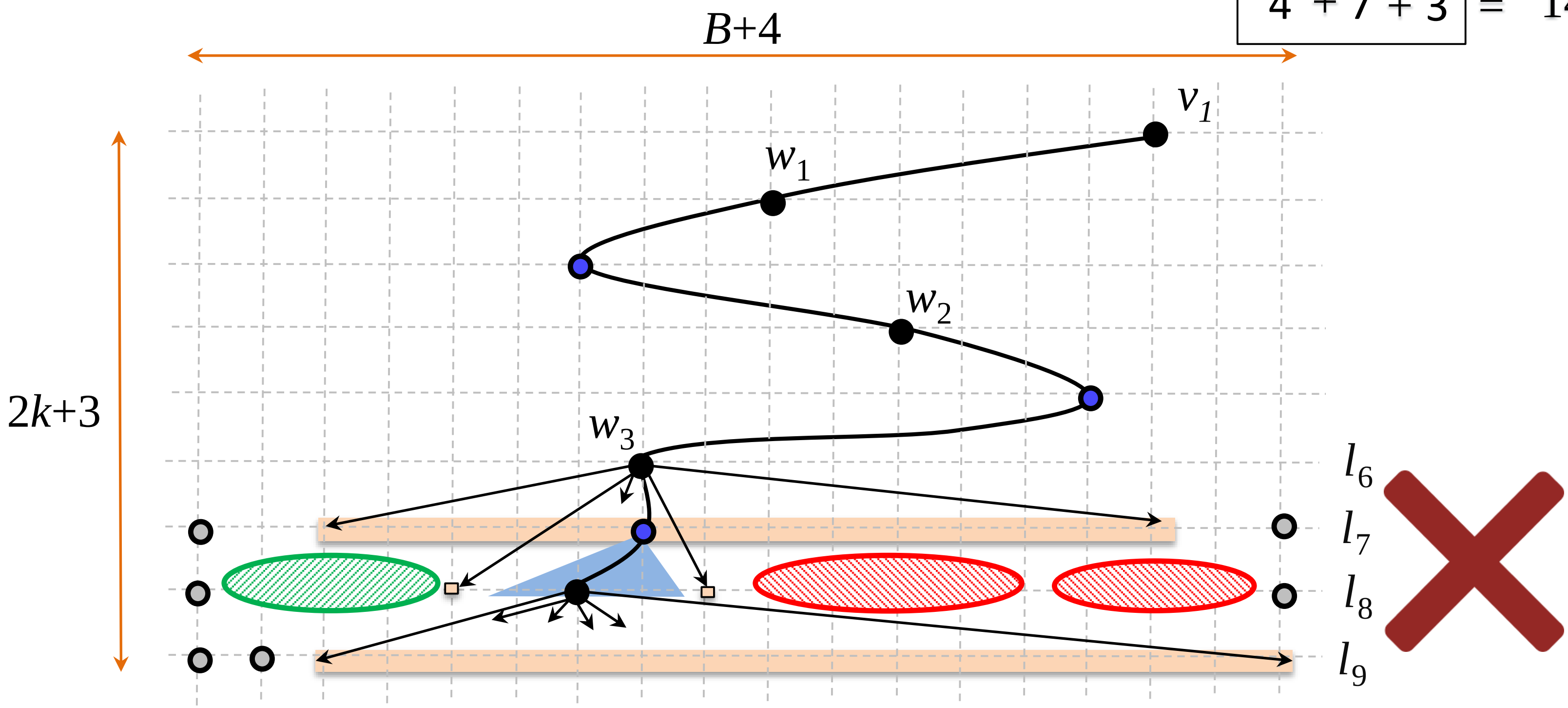

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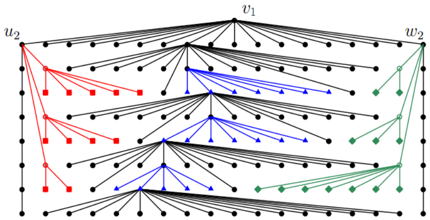

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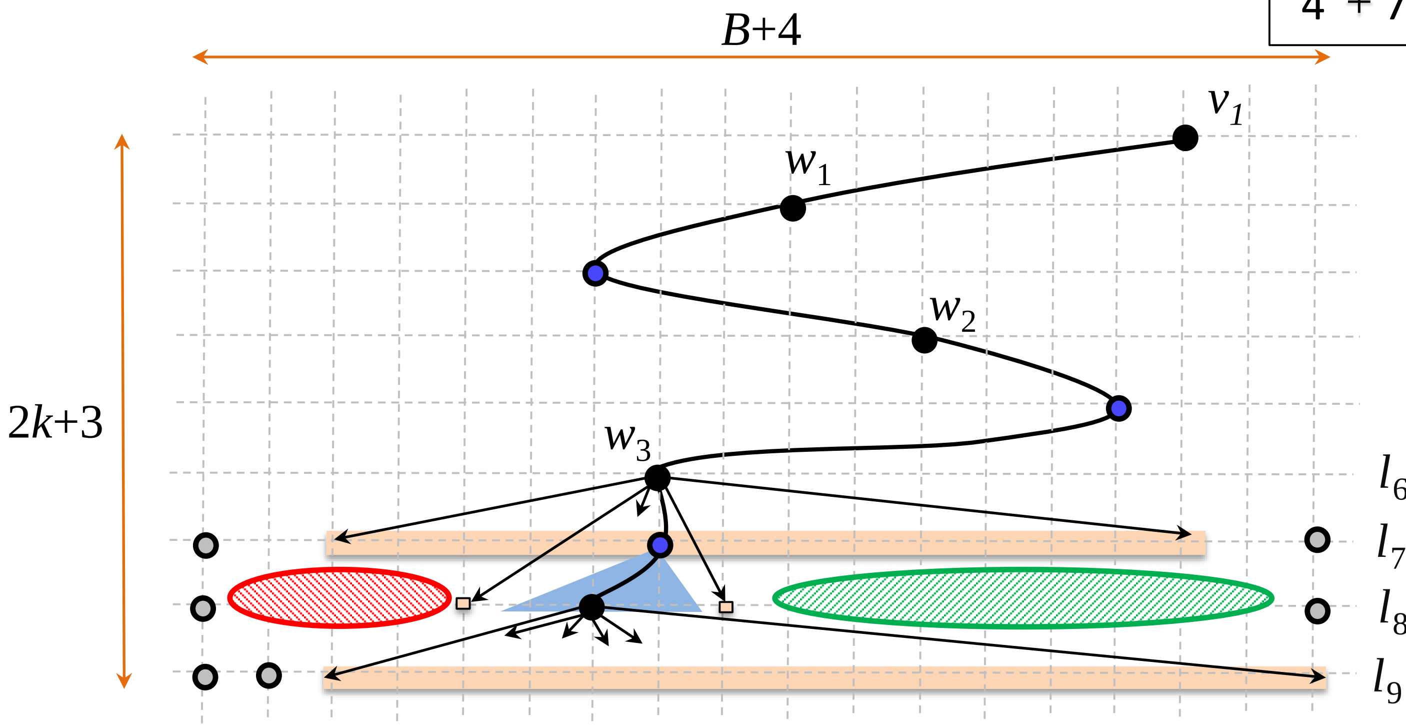

The remaining space is too large for two green stars, and placing two red stars would violate planarity.

Since all grid points must be used up, there can be exactly one green and one red star defining a triple that sum to B.

Future Research

Area Minimization for …

Straight-line drawings of Trees?

Upward **straight-line** drawings of **rooted** Trees?

Strictly **upward** **straight-line** drawings of **rooted** Trees?

Strictly **upward** **straight-line** drawings of **ordered** **rooted** Trees?

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r_i	b_i	g_i		B
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5 +	7 +	2	=	14
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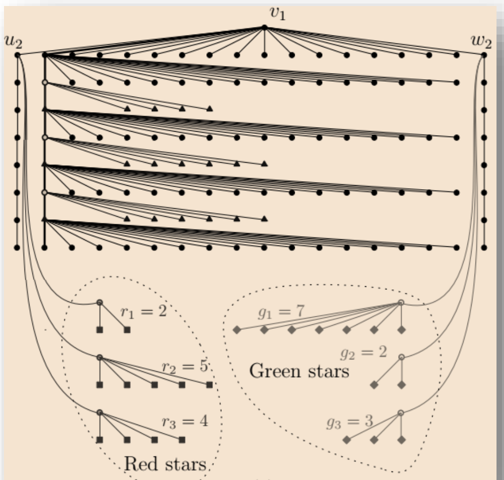
THANK YOU



(c)