gd17

14-27 September 2017
Boston, USA

Simple Compact Monotone Tree Drawings

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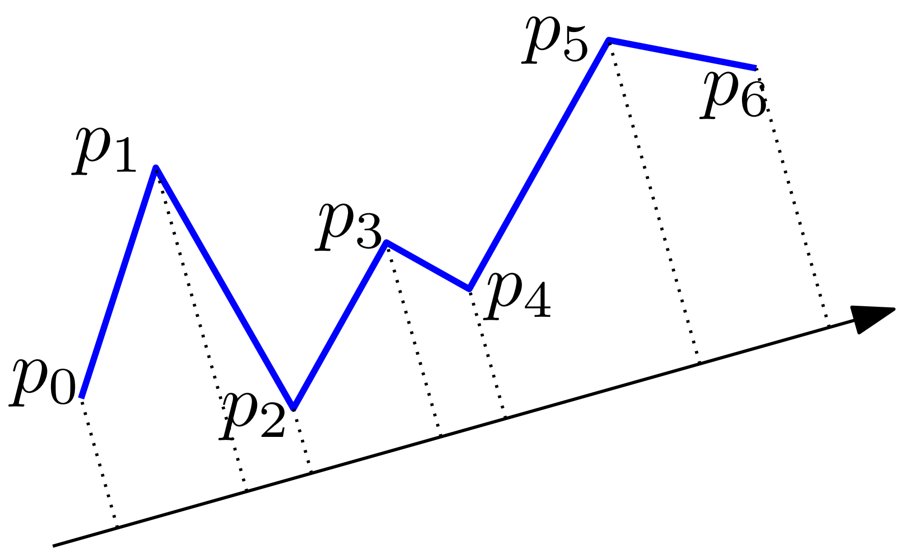
National Technical University of Athens, Greece

Monotone drawings

Monotone drawings

- A *path P is monotone* if there exists a line l such that the projections of the vertices of P on l appear on l in the same order as on P.



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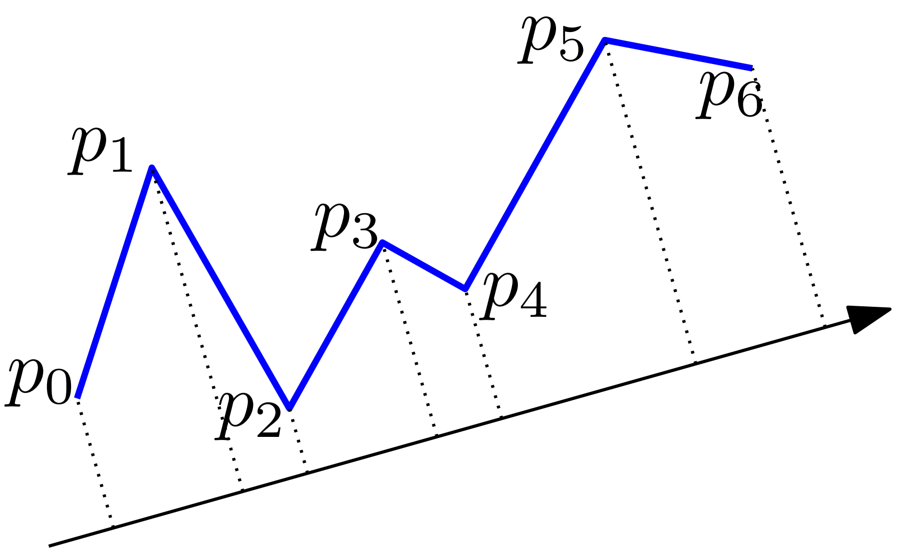
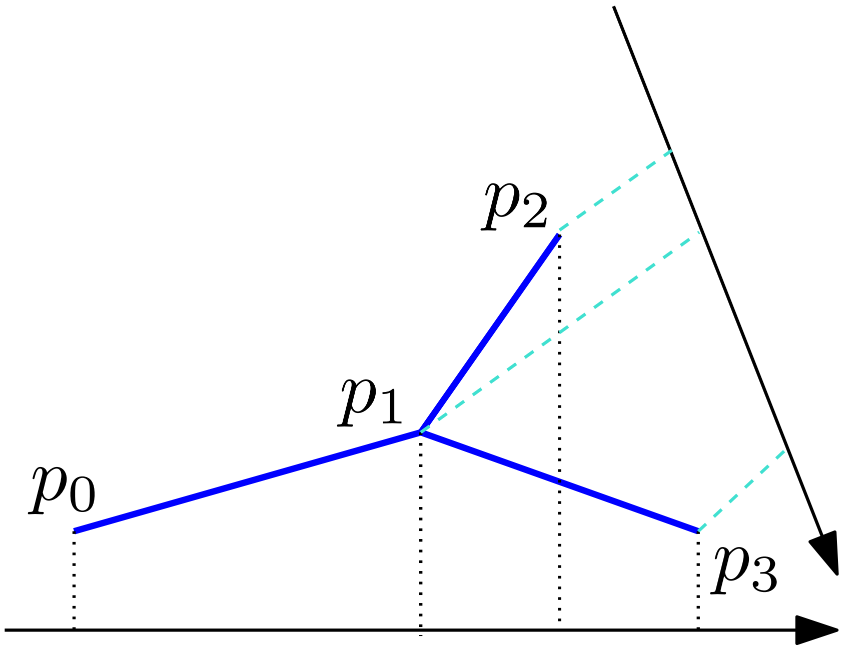


- A *straight-line drawing Γ of a graph G is monotone* if a *monotone* path connects every pair of vertices.



Monotone drawings of trees

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2010 Angelini, Colasante, Di Battista, Frati, Patrignani
[GD'10, JGAA'12]

- $O(n^{1.6}) \times O(n^{1.6})$ (BFS-based)
- $O(n) \times O(n^2)$ (DFS-based)
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Monotone drawings of trees

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2017 Oikonomou, Symvonis [GD'17]

- $n \times n$
- Simple weighting based on size of subtrees
- Some geometry

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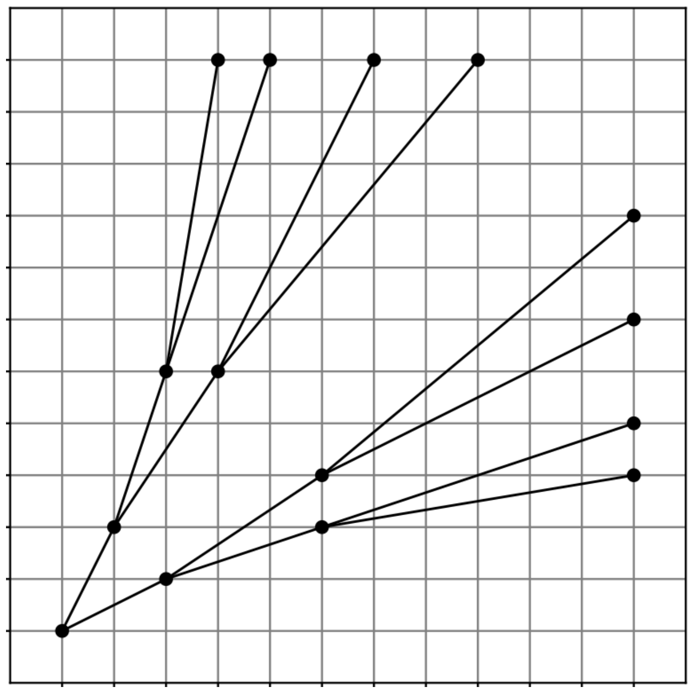
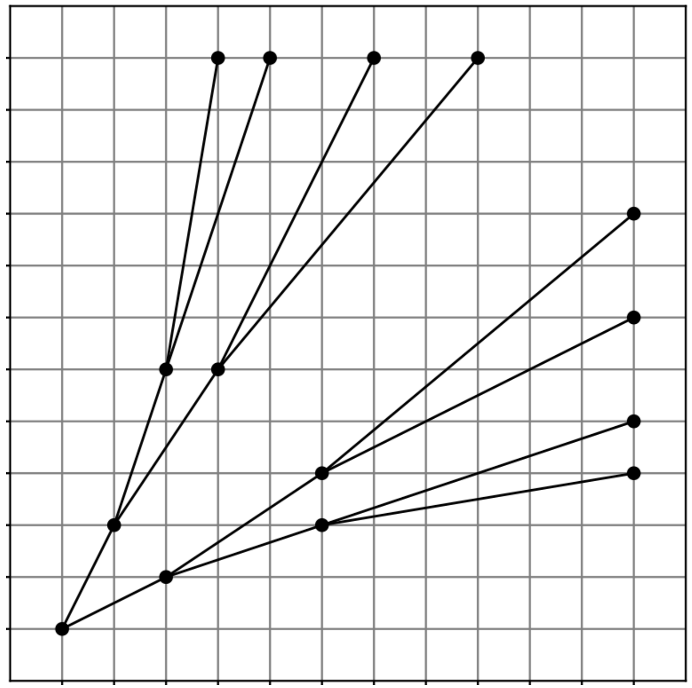


Complete binary (15 nodes)
[12 × 12] grid

Monotone drawings of trees

2017 Oikonomou, Symvonis [GD'17]



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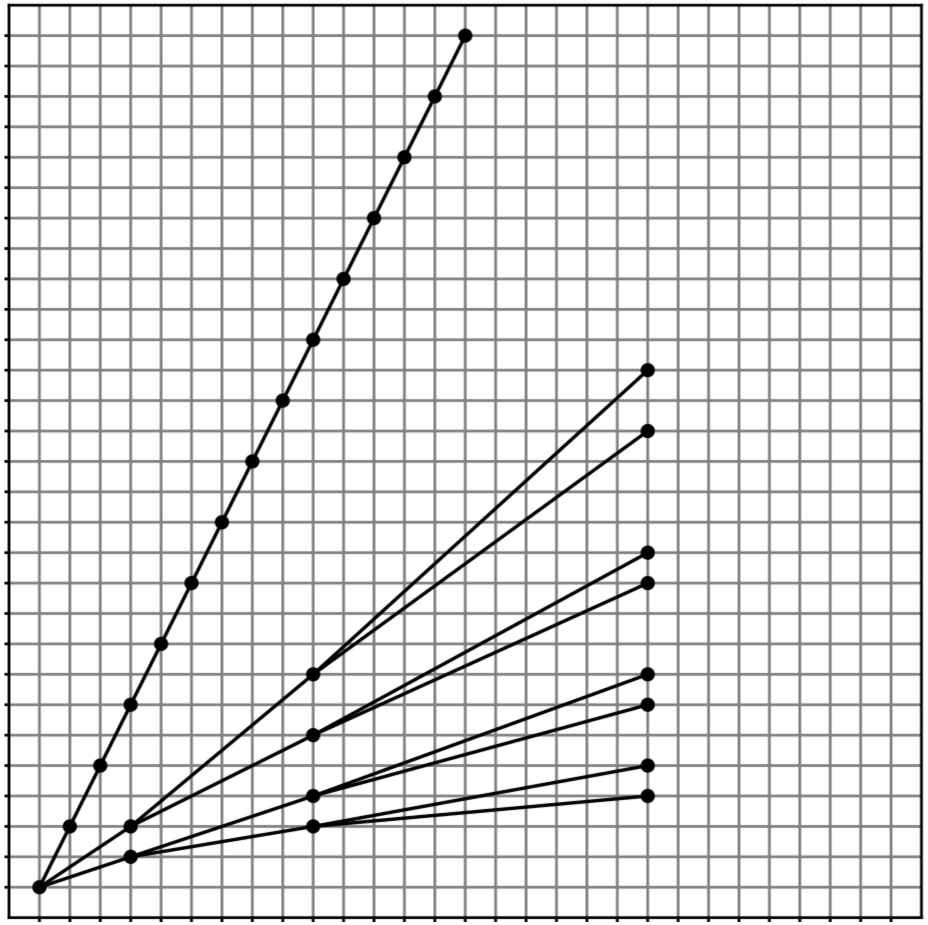

Complete binary + path (29 nodes)
[21 × 29] grid

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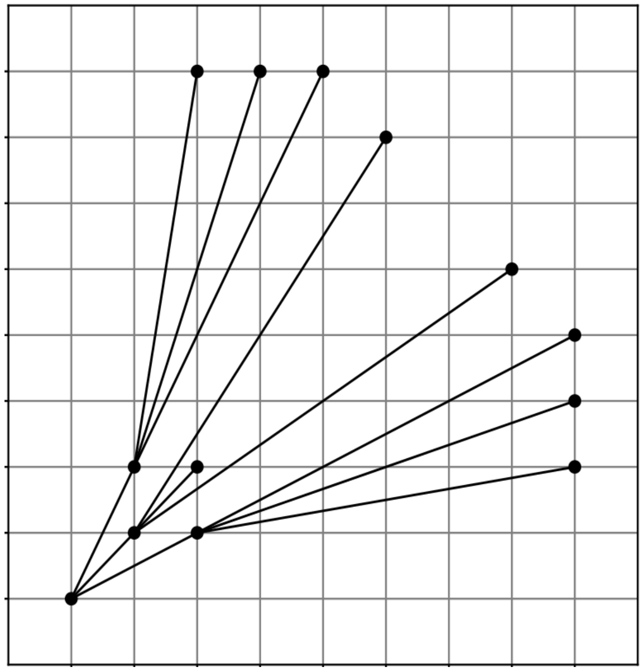

Complete ternary(13 nodes)
[9 × 9] grid

Monotone drawings of trees

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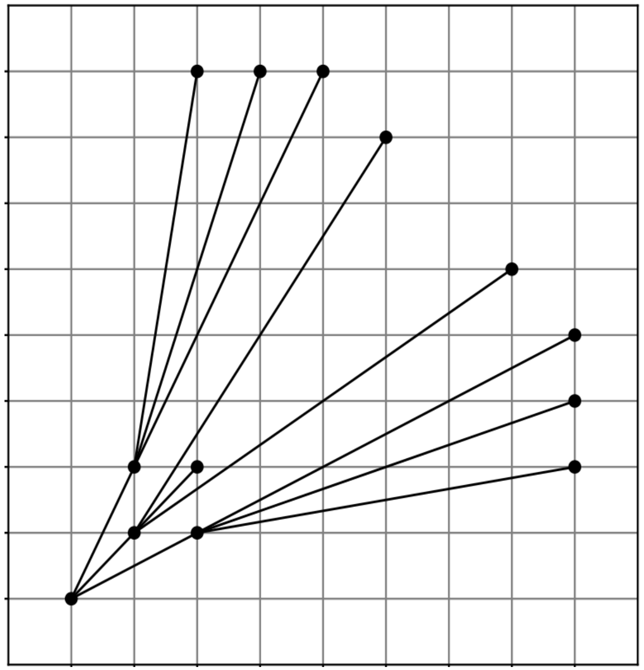

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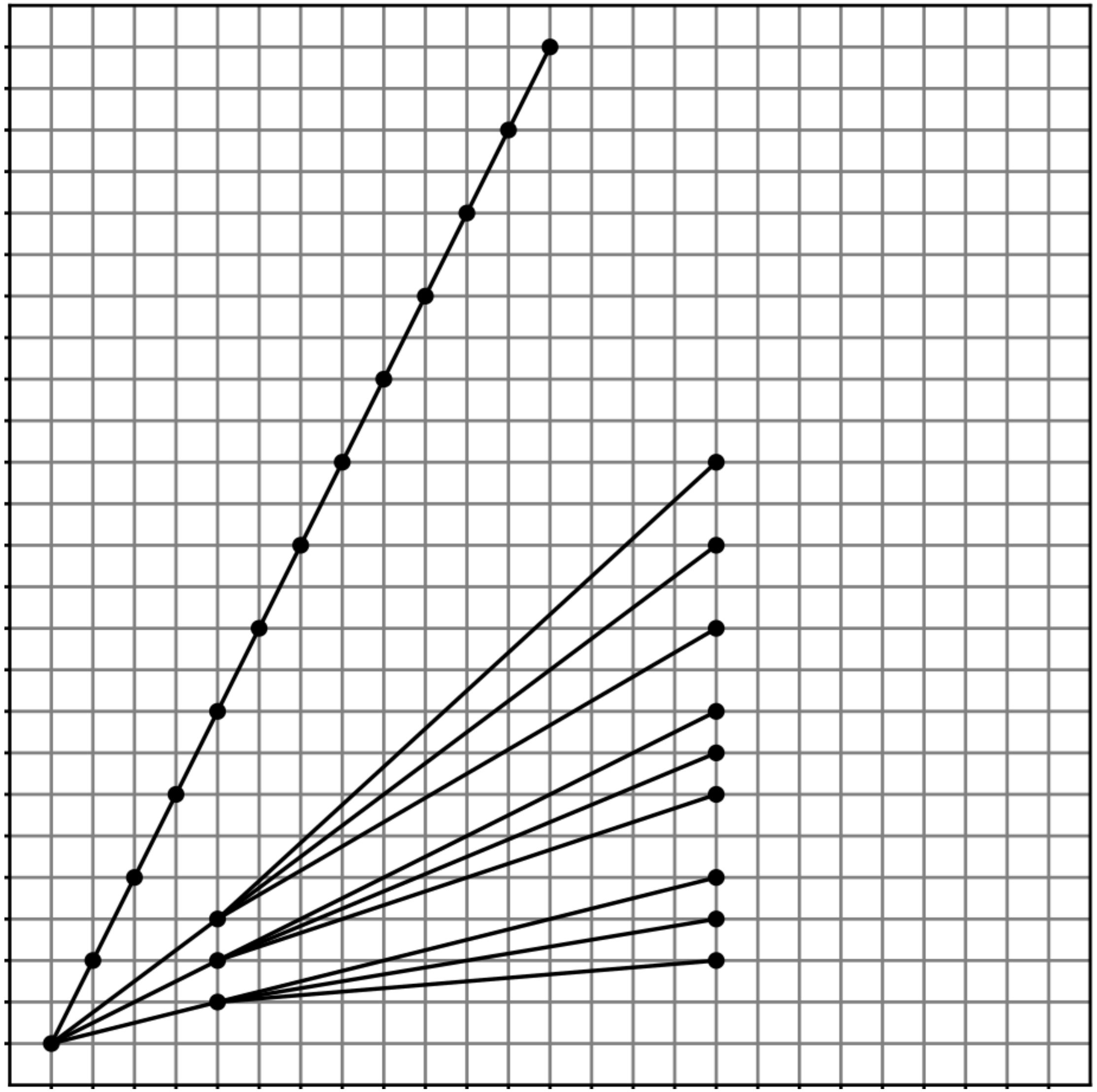

Complete ternary + path (25 nodes)
[17×25] grid

Slope disjoint tree drawings

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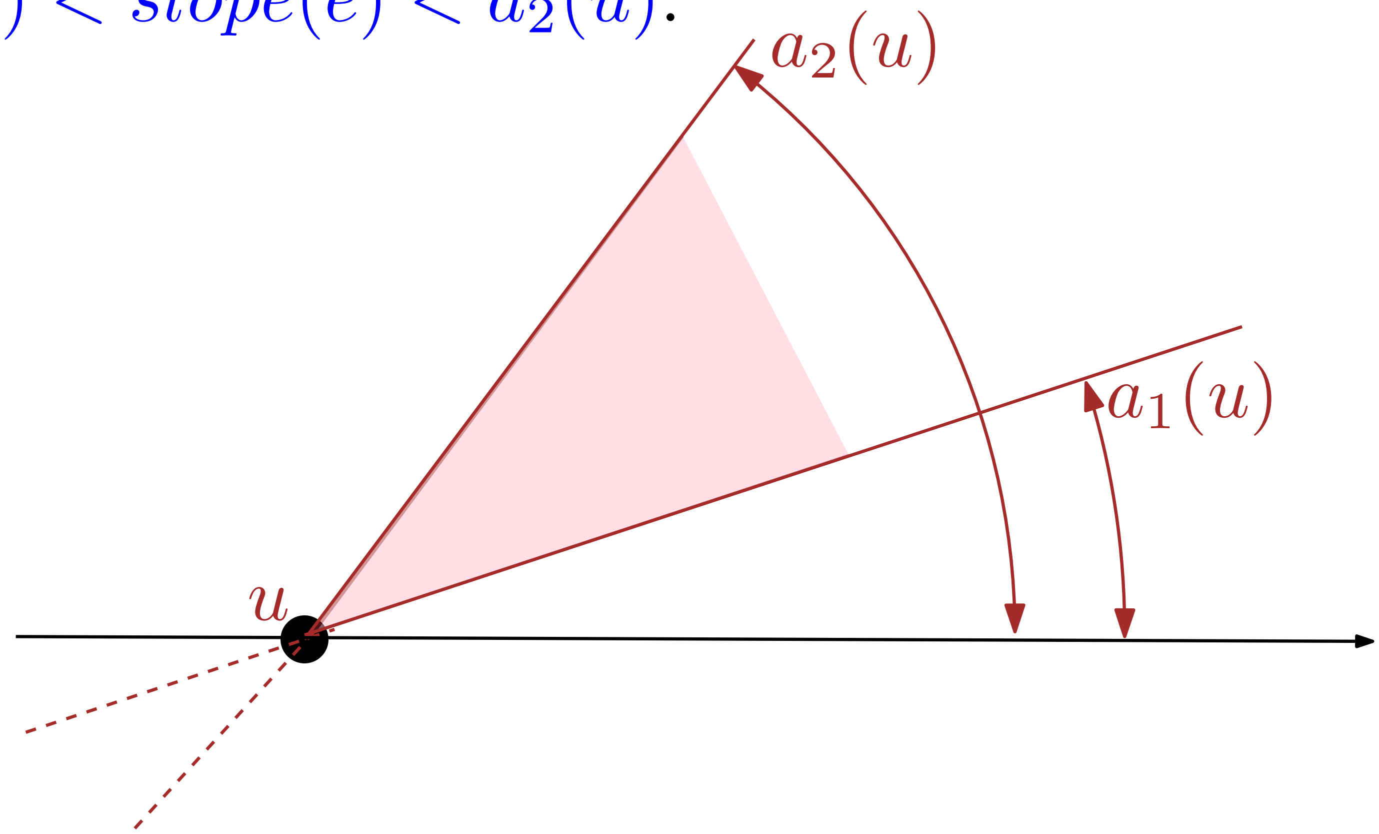

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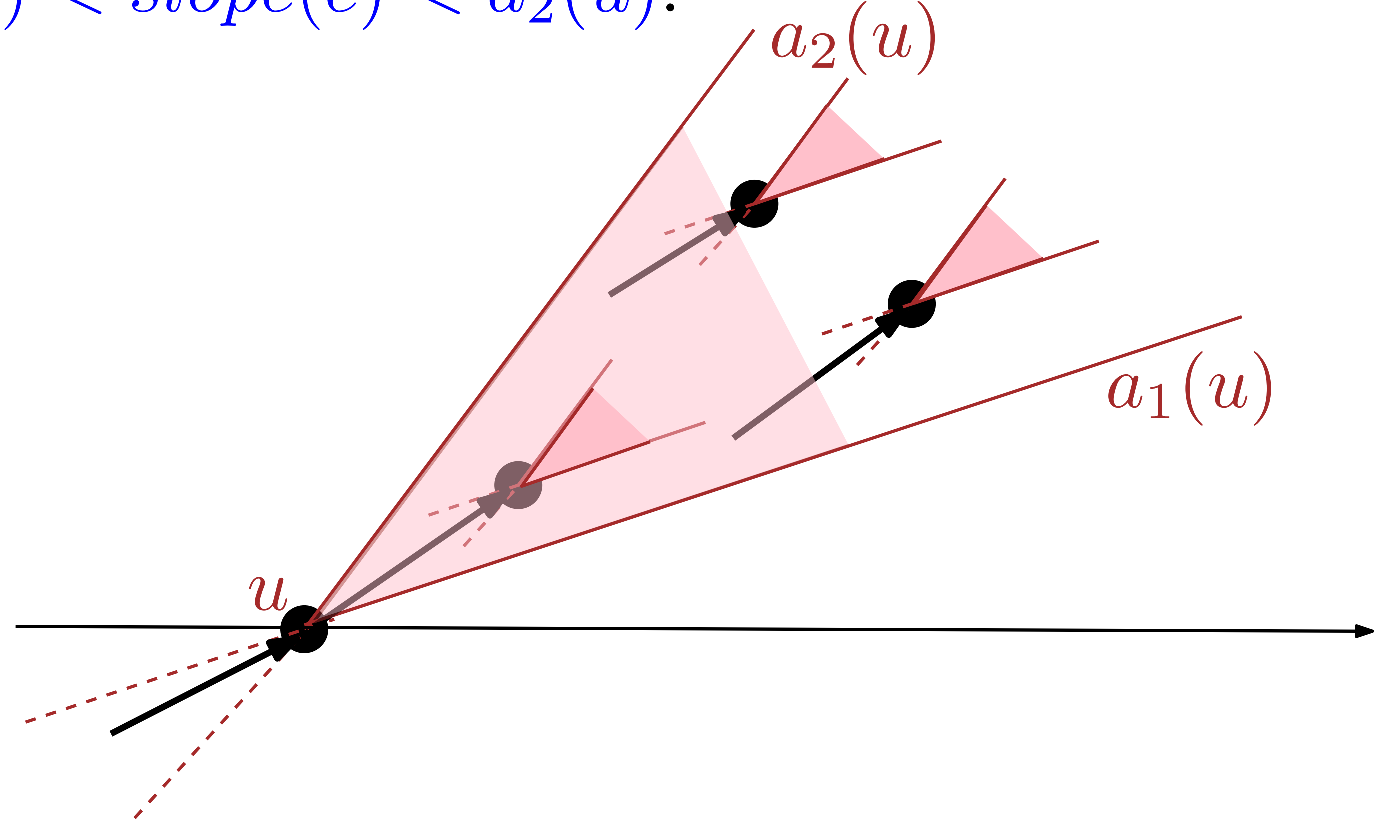

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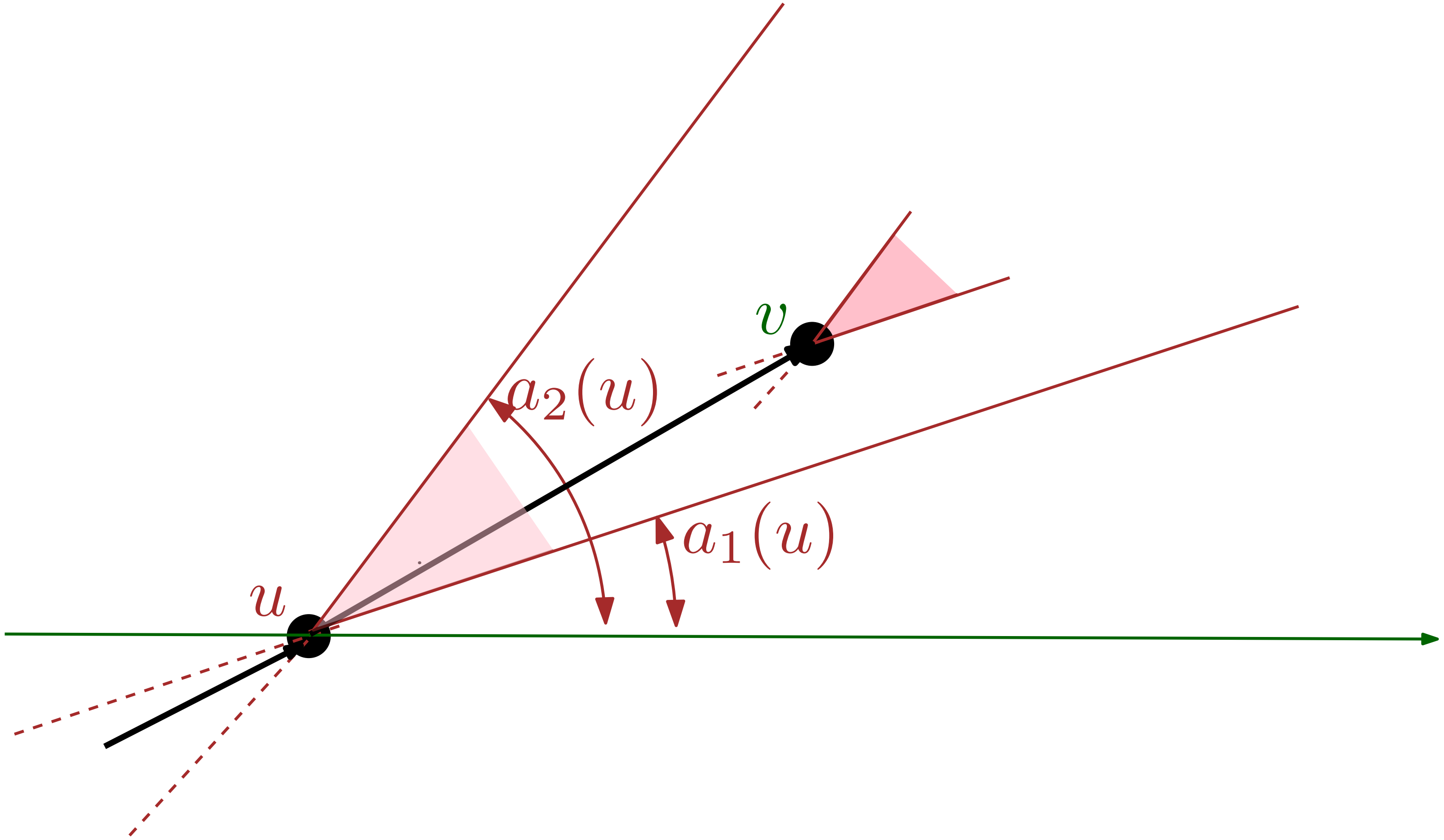

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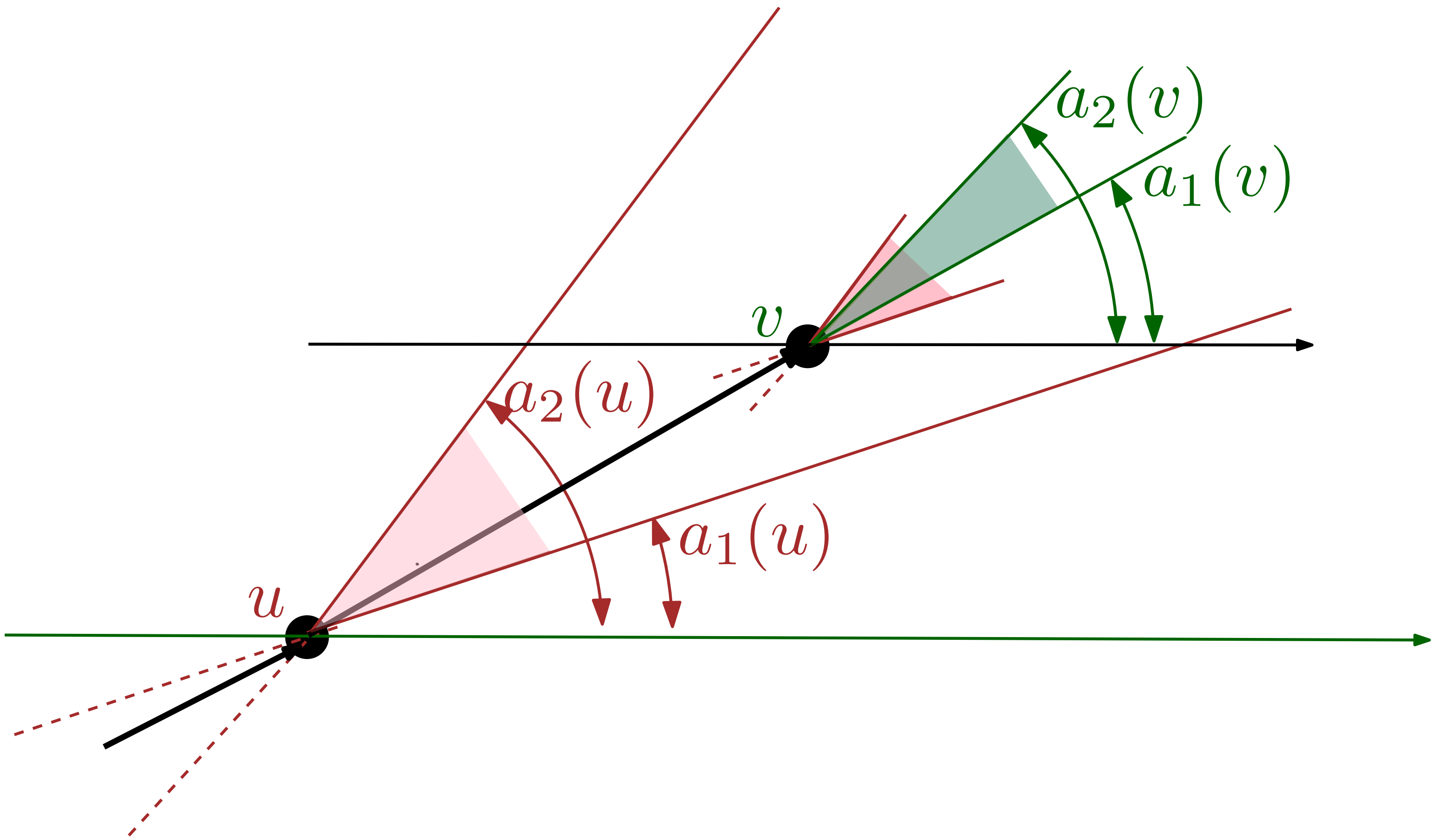

Slope disjoint tree drawings

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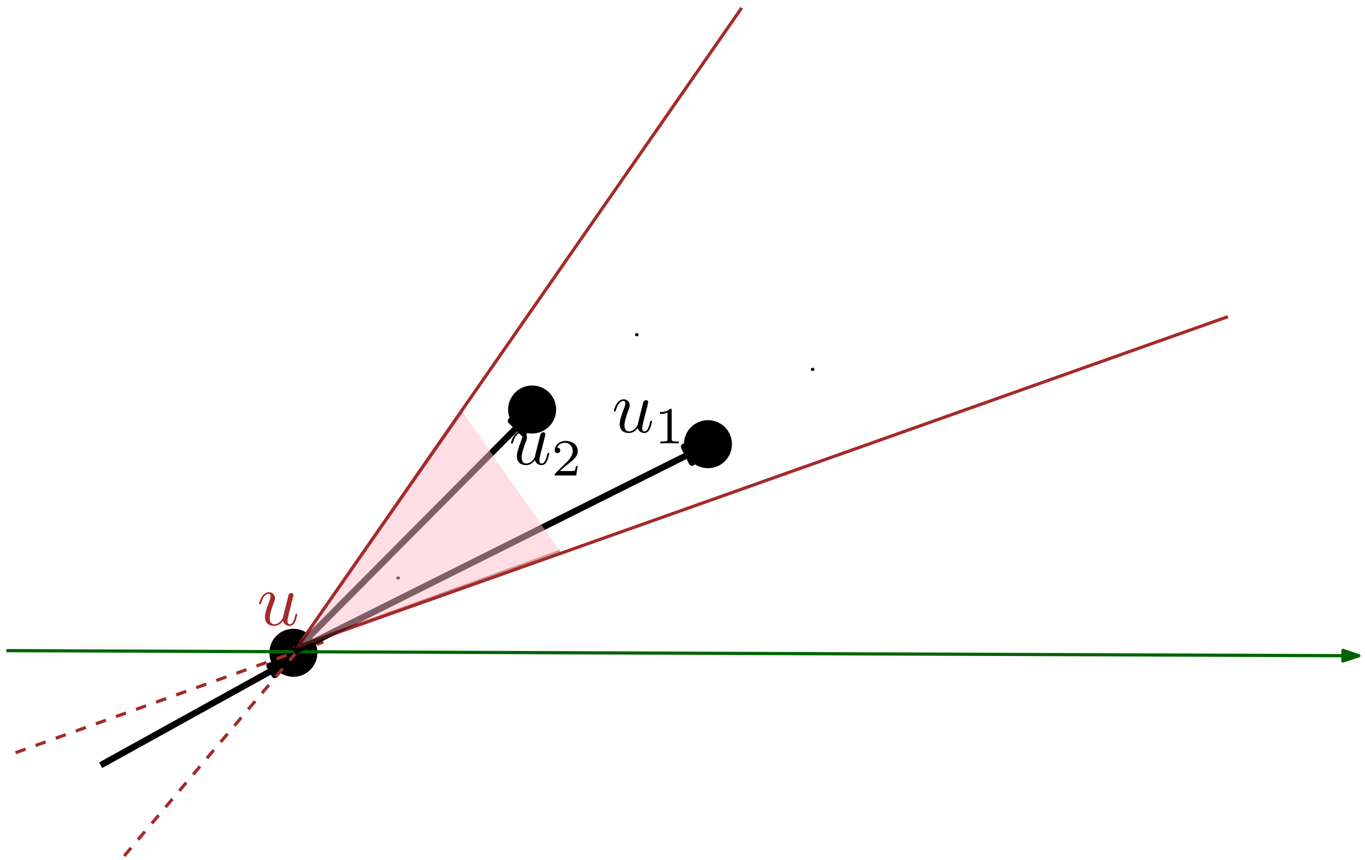

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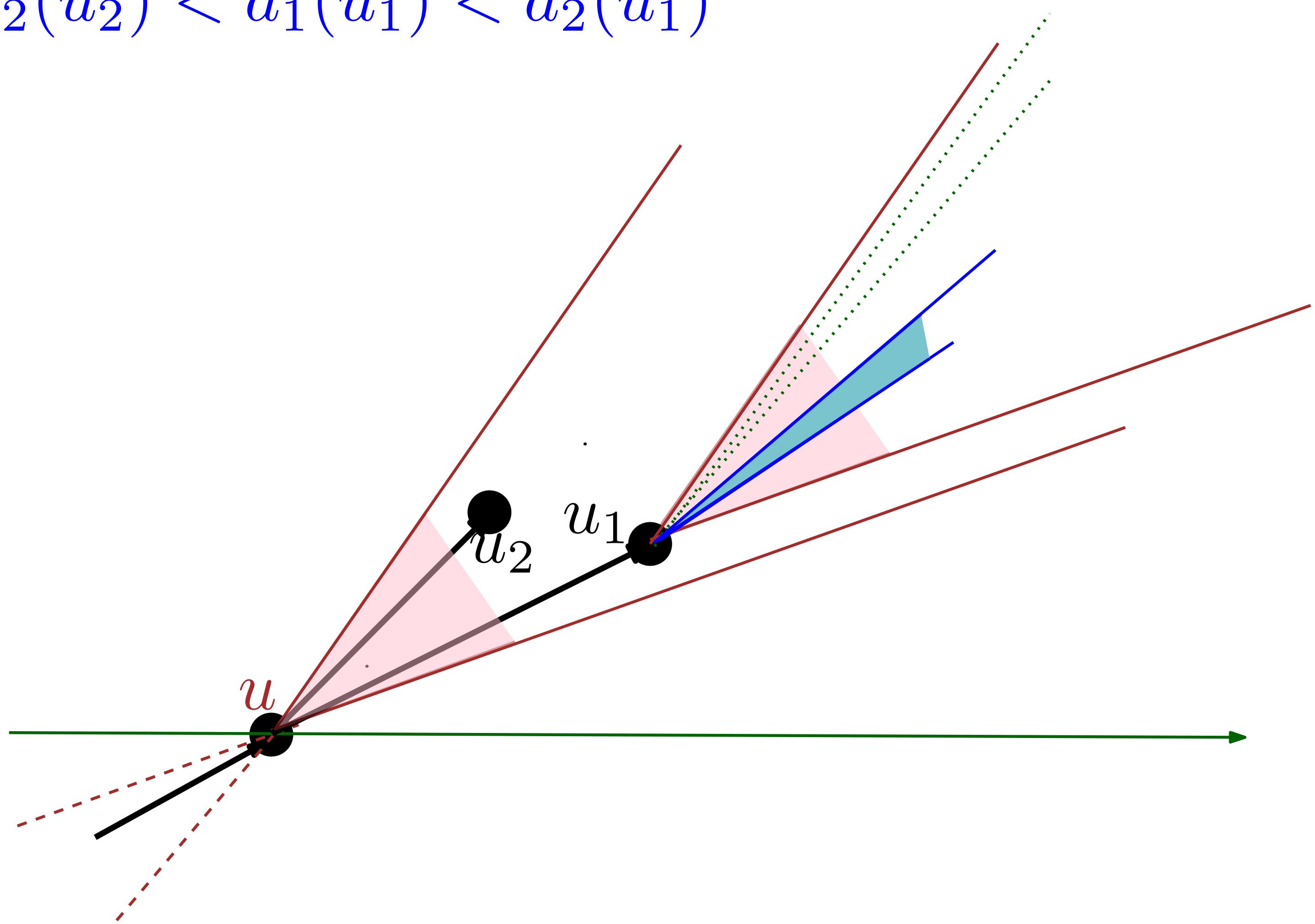

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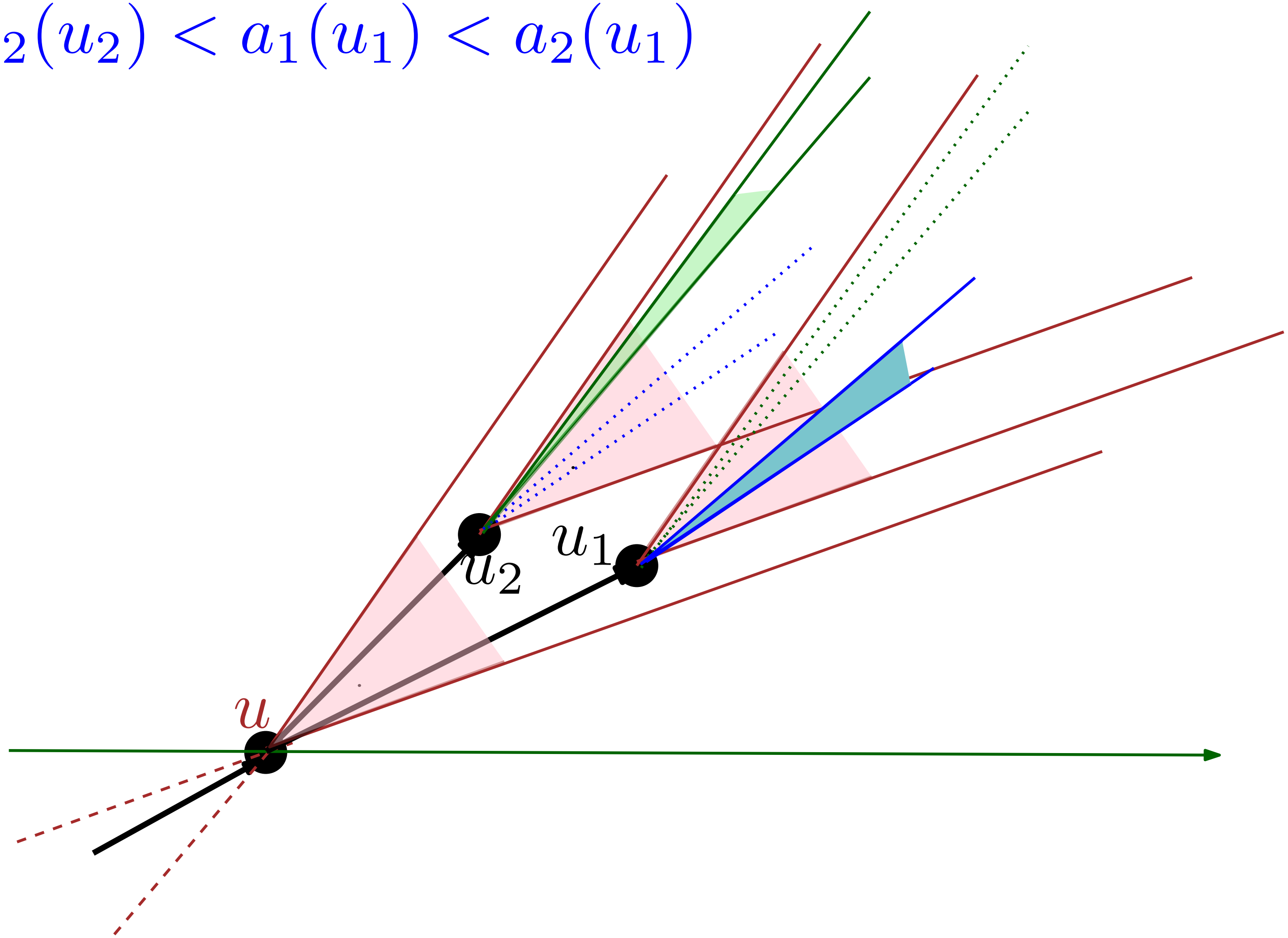

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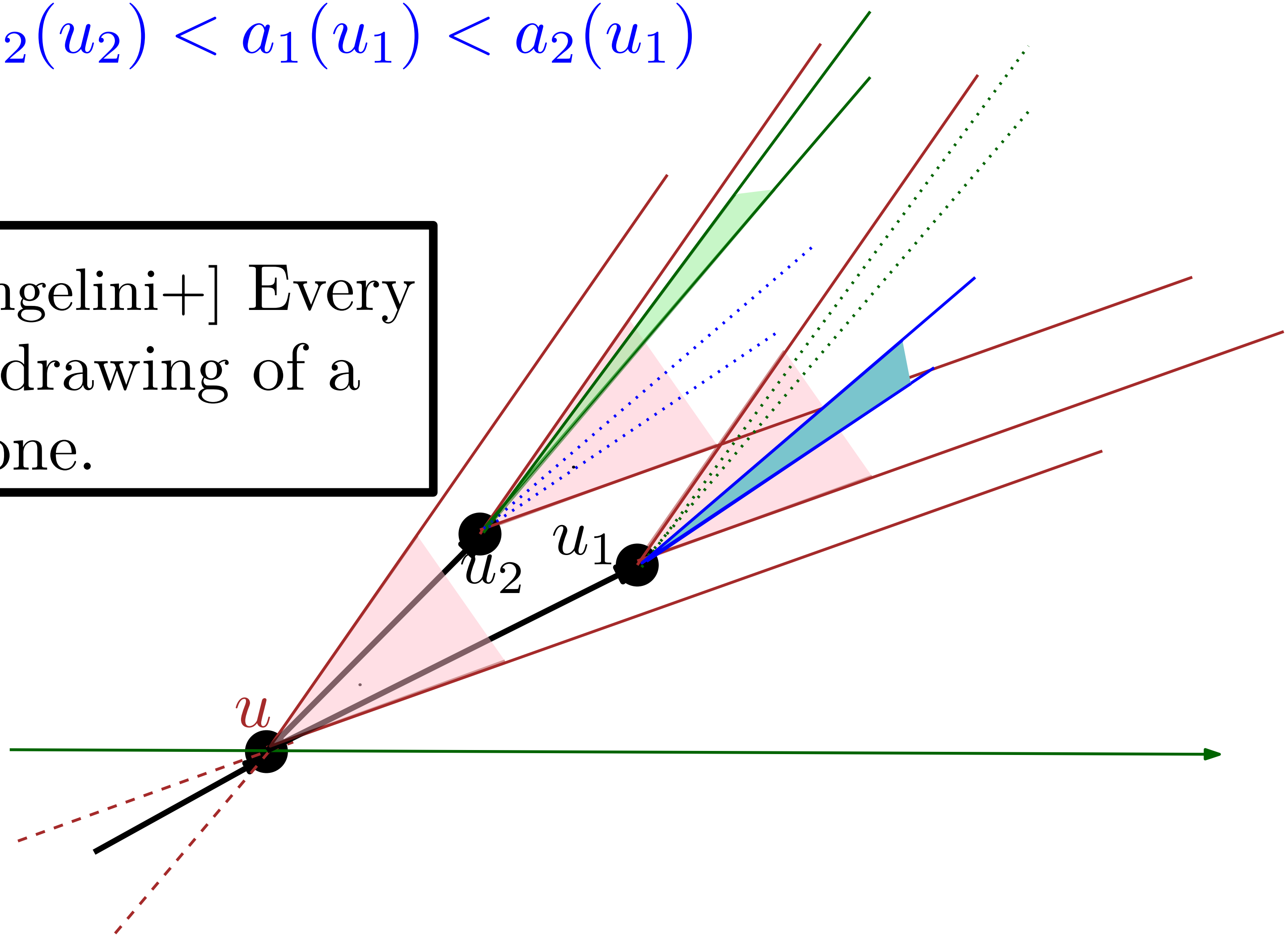
Theorem [Angelini+] Every slope-disjoint drawing of a tree is monotone.



Non-strictly slope disjoint tree drawings

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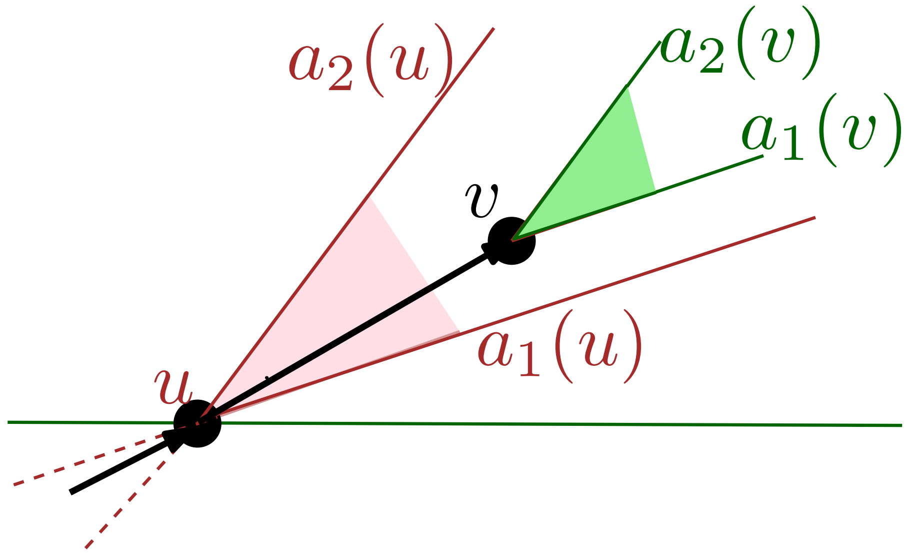

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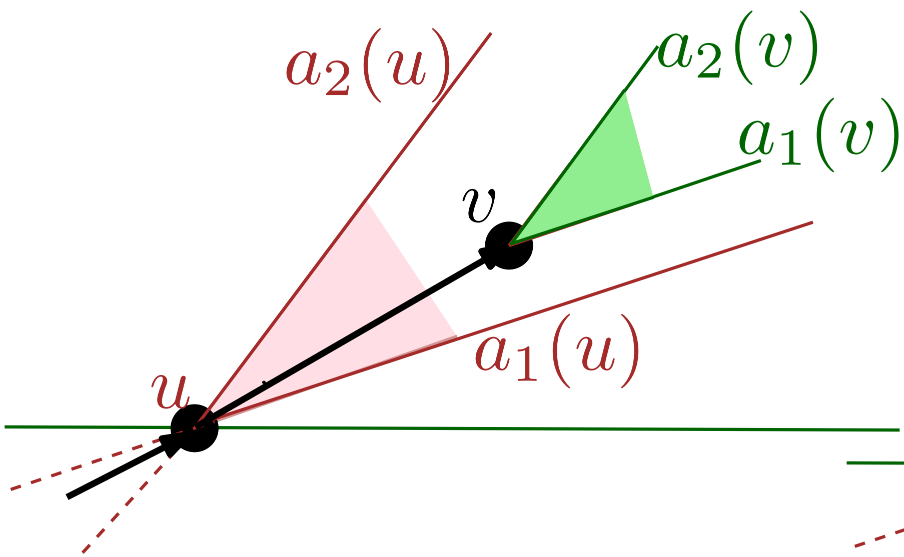

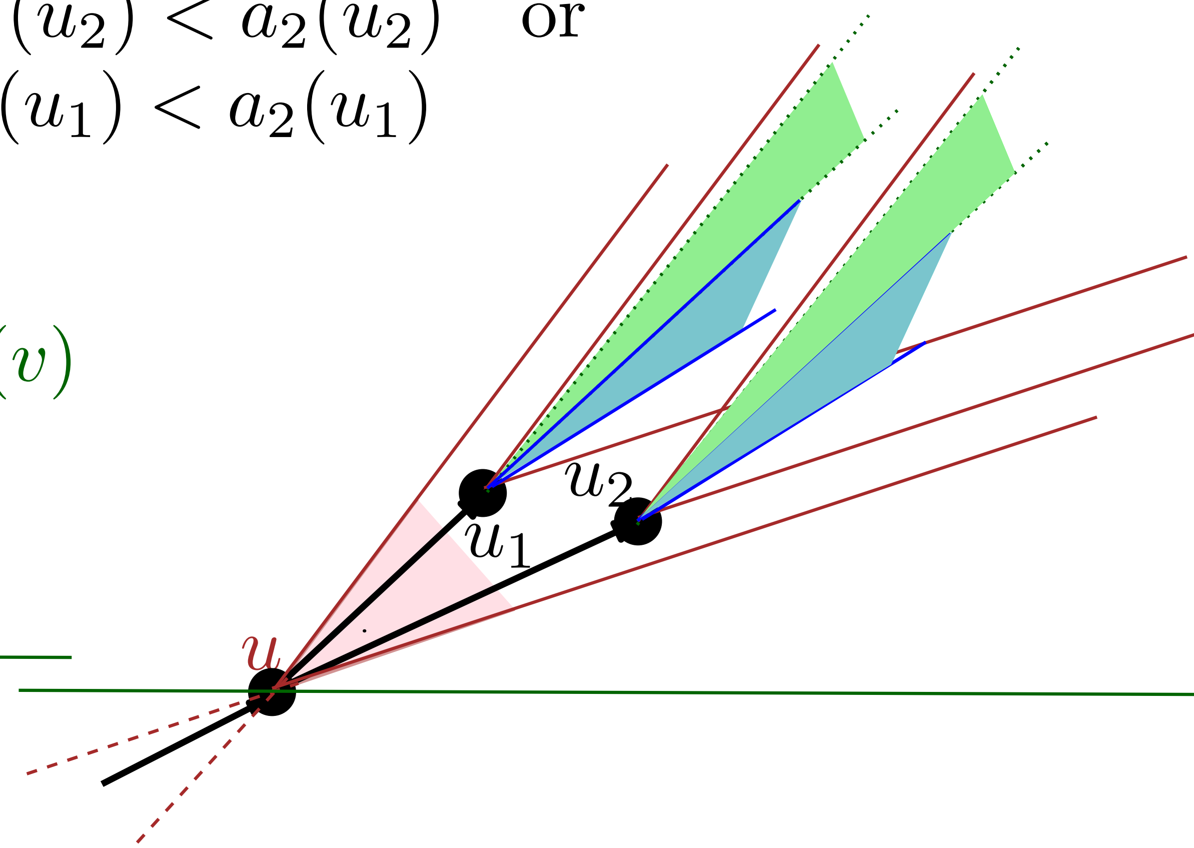

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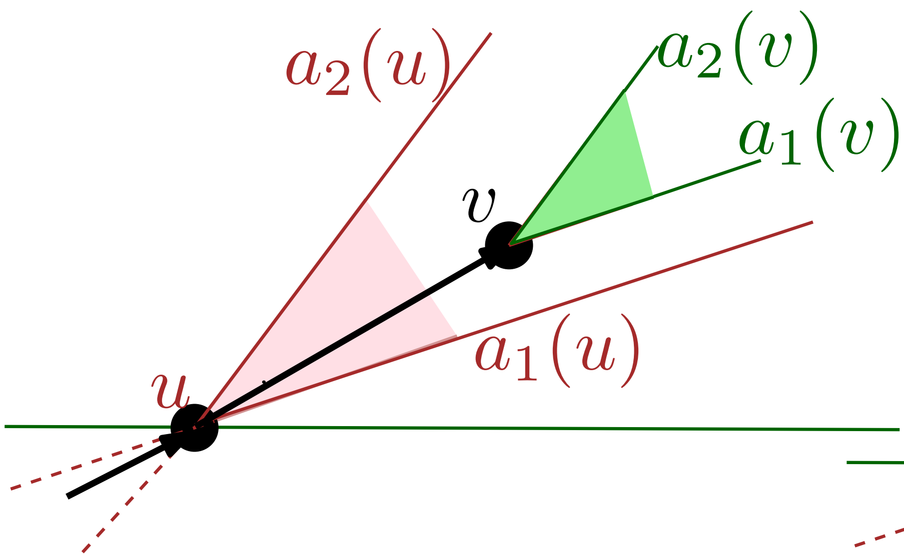

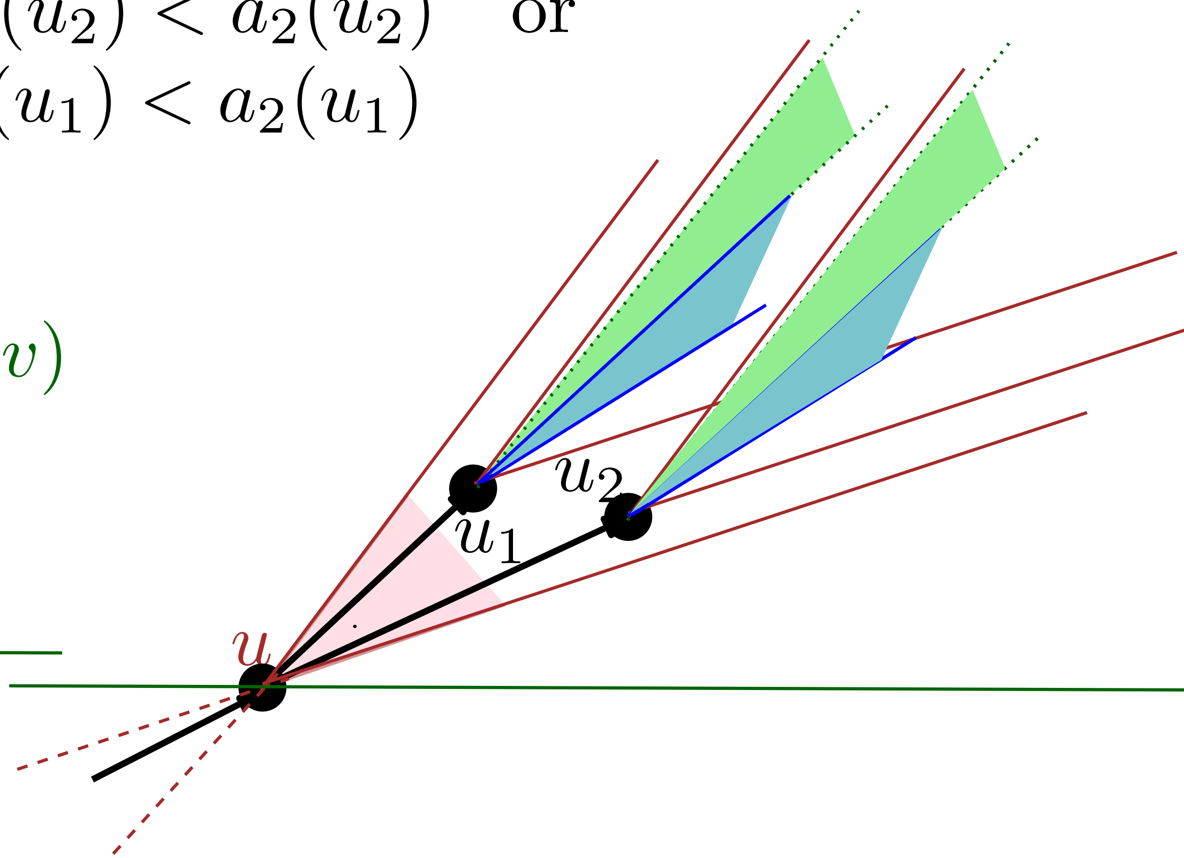

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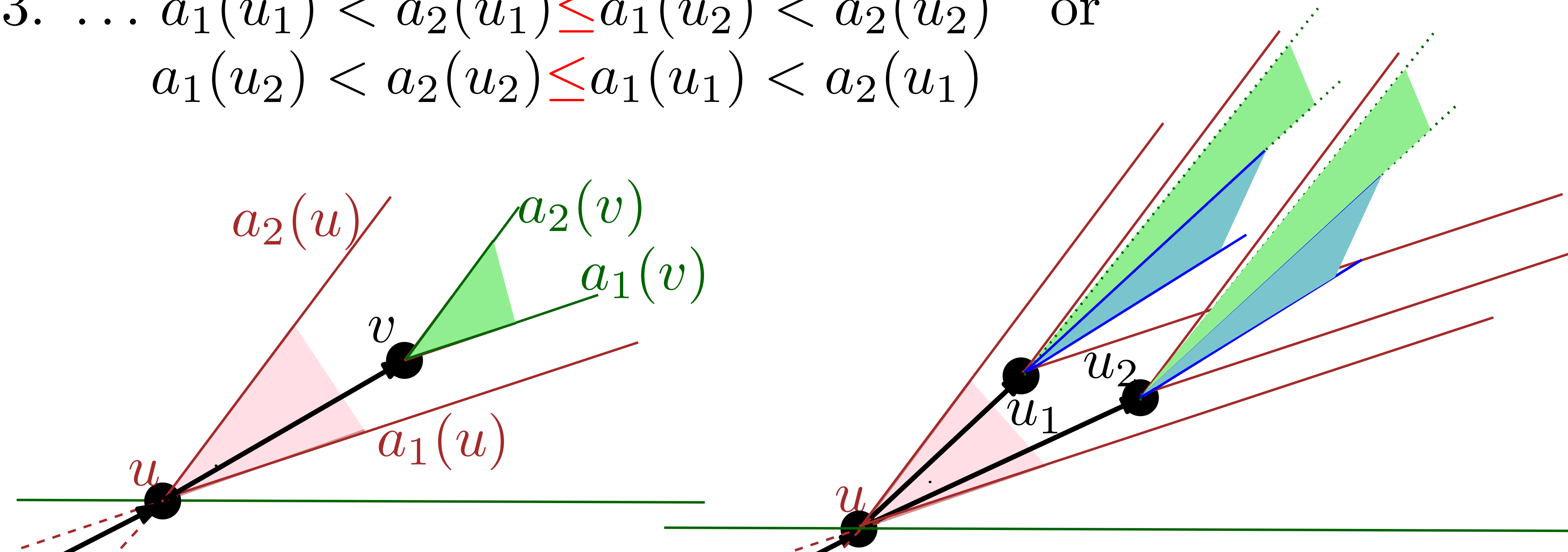
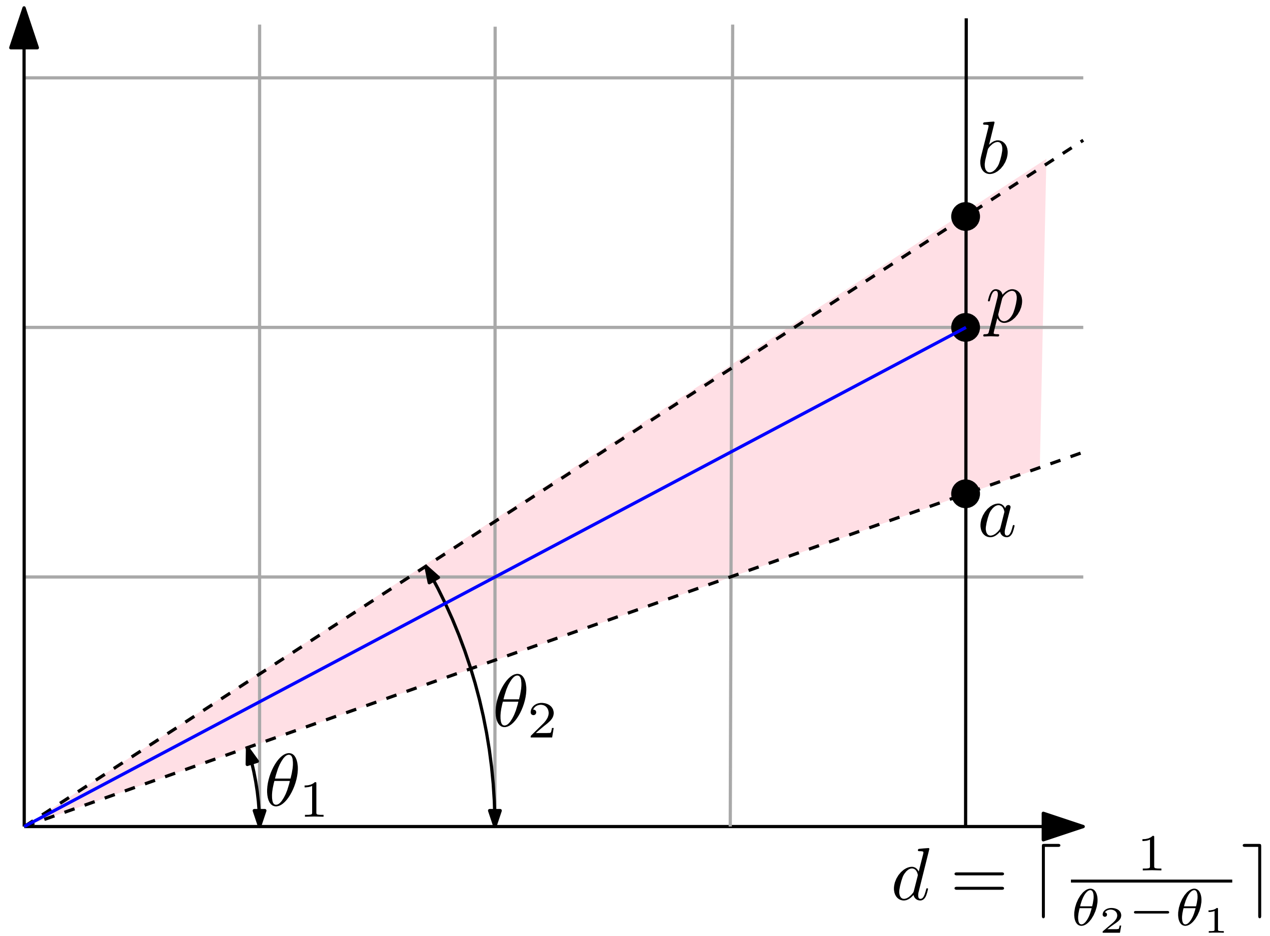


Theorem Every non-strictly slope disjoint drawing of a tree is monotone.

Locating points on the grid

Locating points on the grid

Lemma. Consider two angles θ_1, θ_2 with $0 \leq \theta_1 < \theta_2 \leq \frac{\pi}{4}$ and let $d = \lceil \frac{1}{\theta_2 - \theta_1} \rceil$. Then, edge e connecting the origin $(0, 0)$ to point $p = (d, \lfloor tan(\theta_1) \cdot d + 1 \rfloor)$ satisfies $\theta_1 < slope(e) < \theta_2$.



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Lemma-AssignPoint
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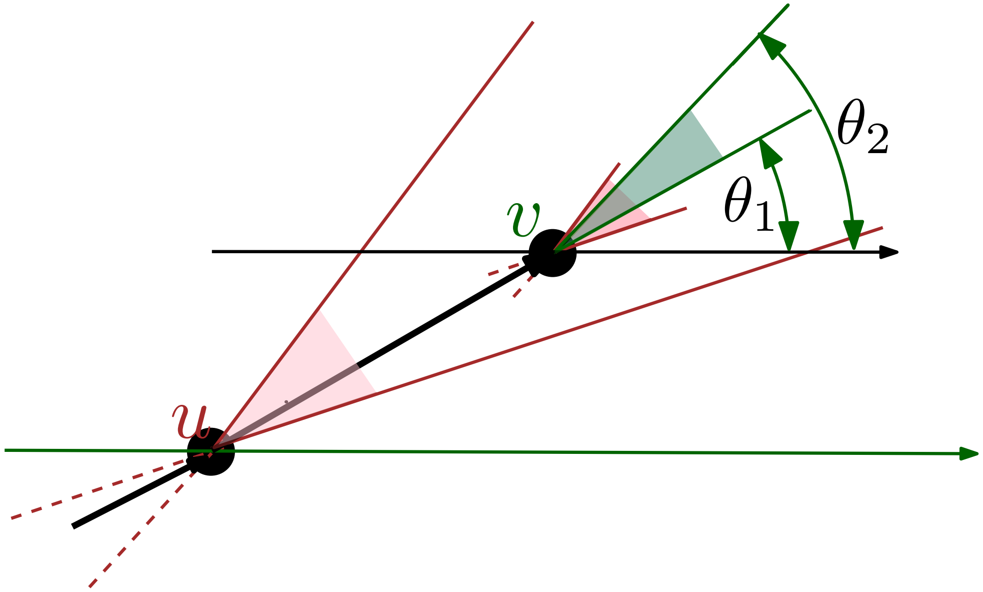

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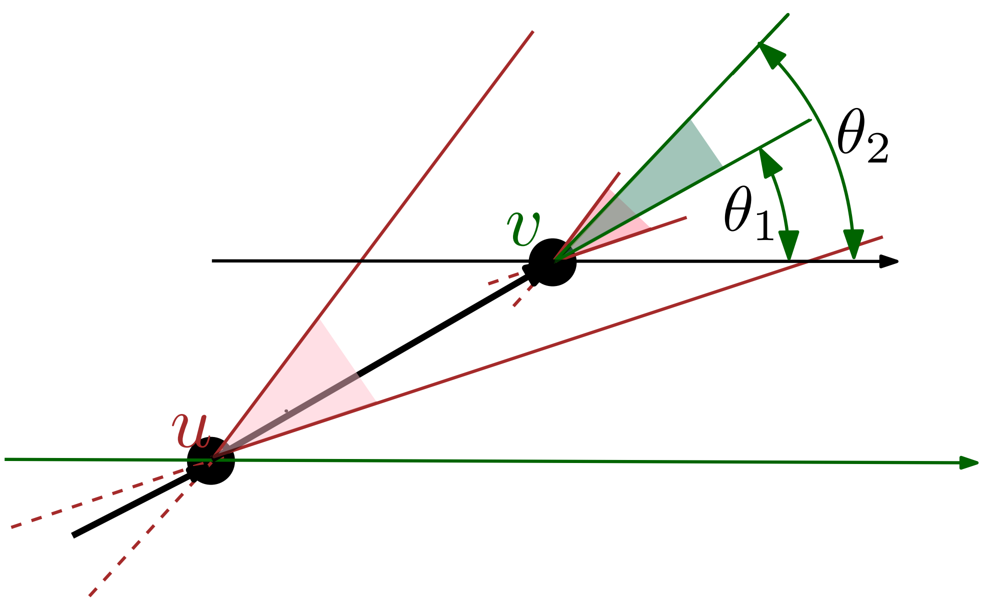

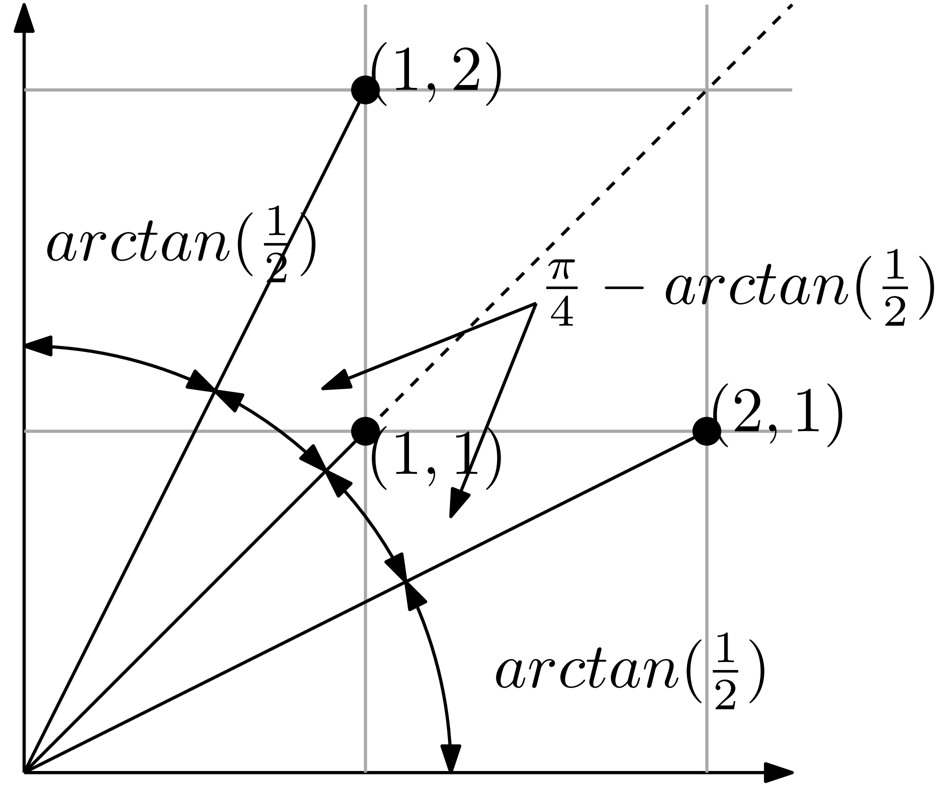

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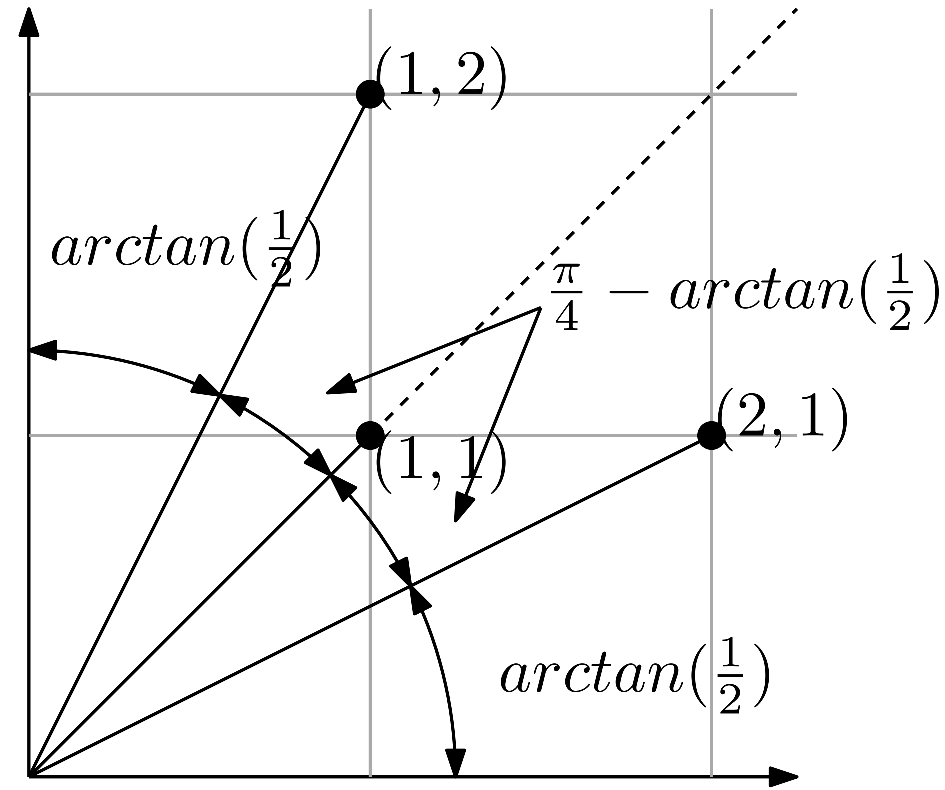

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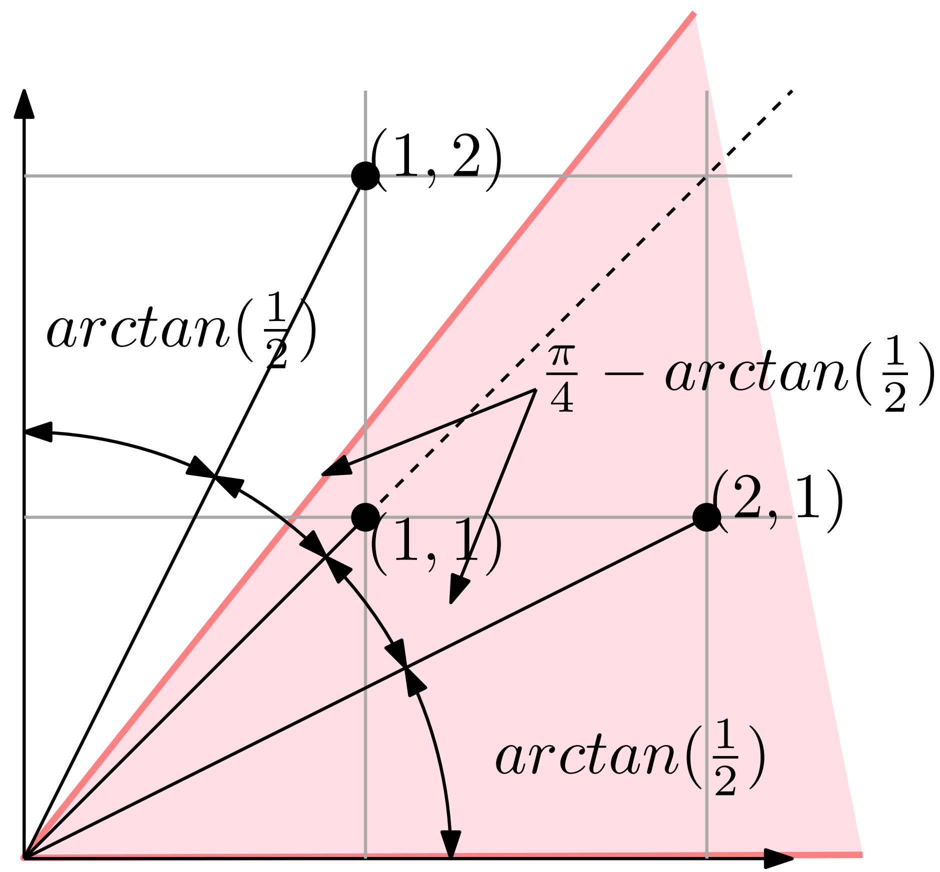
- $\theta_2 - \theta_1 > \frac{\pi}{4}$: $p = (1, 1)$



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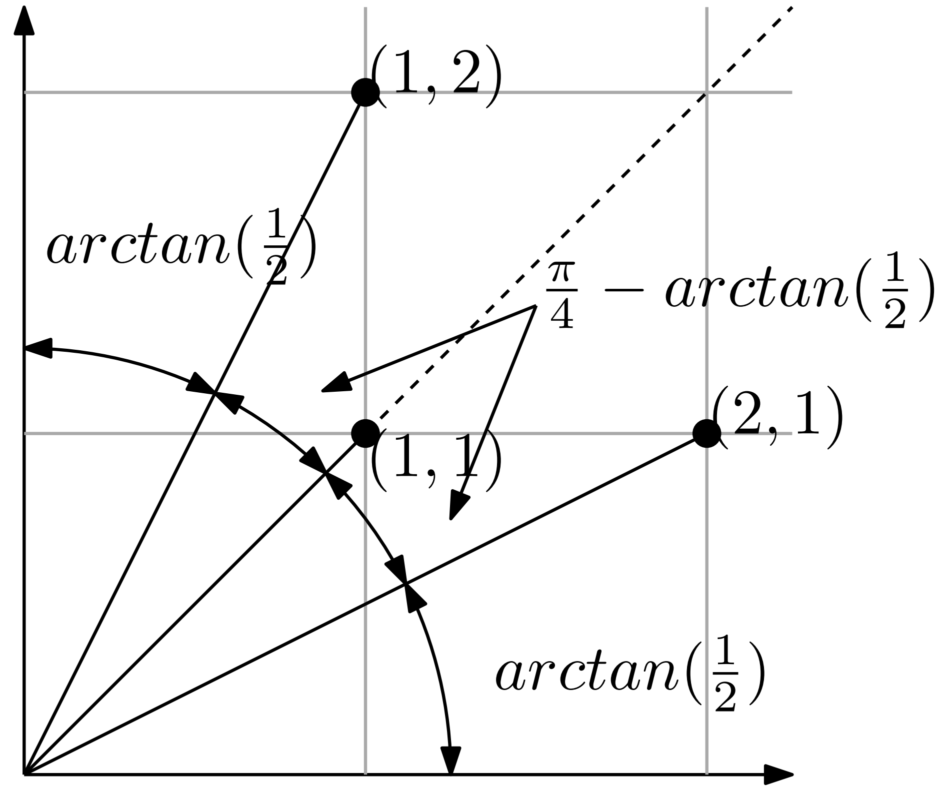

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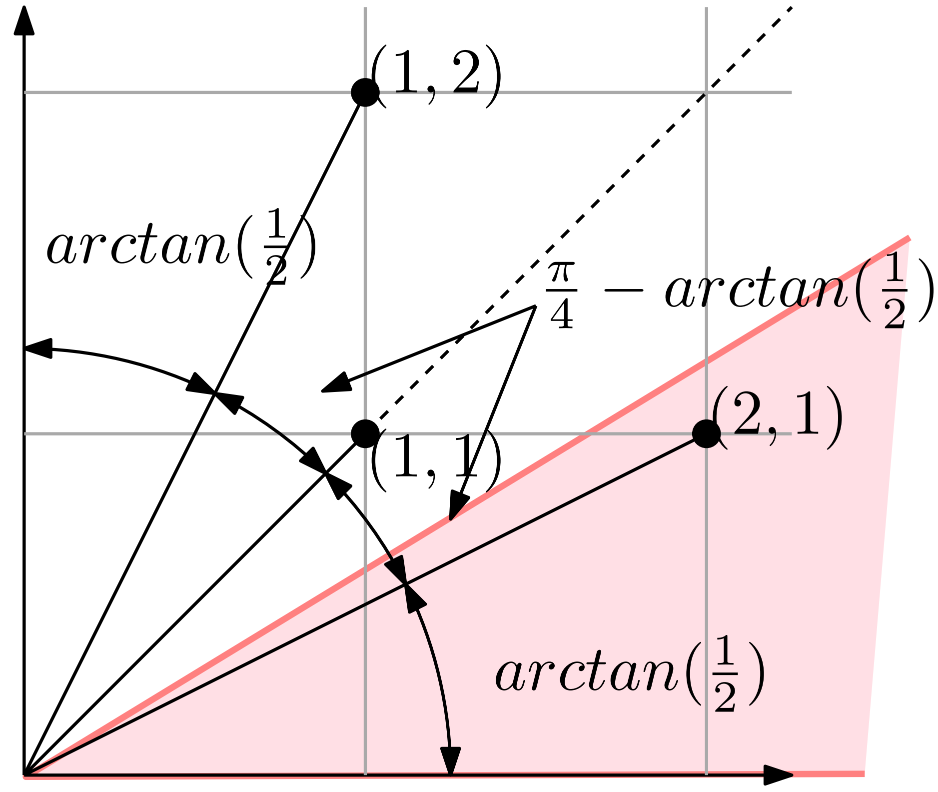

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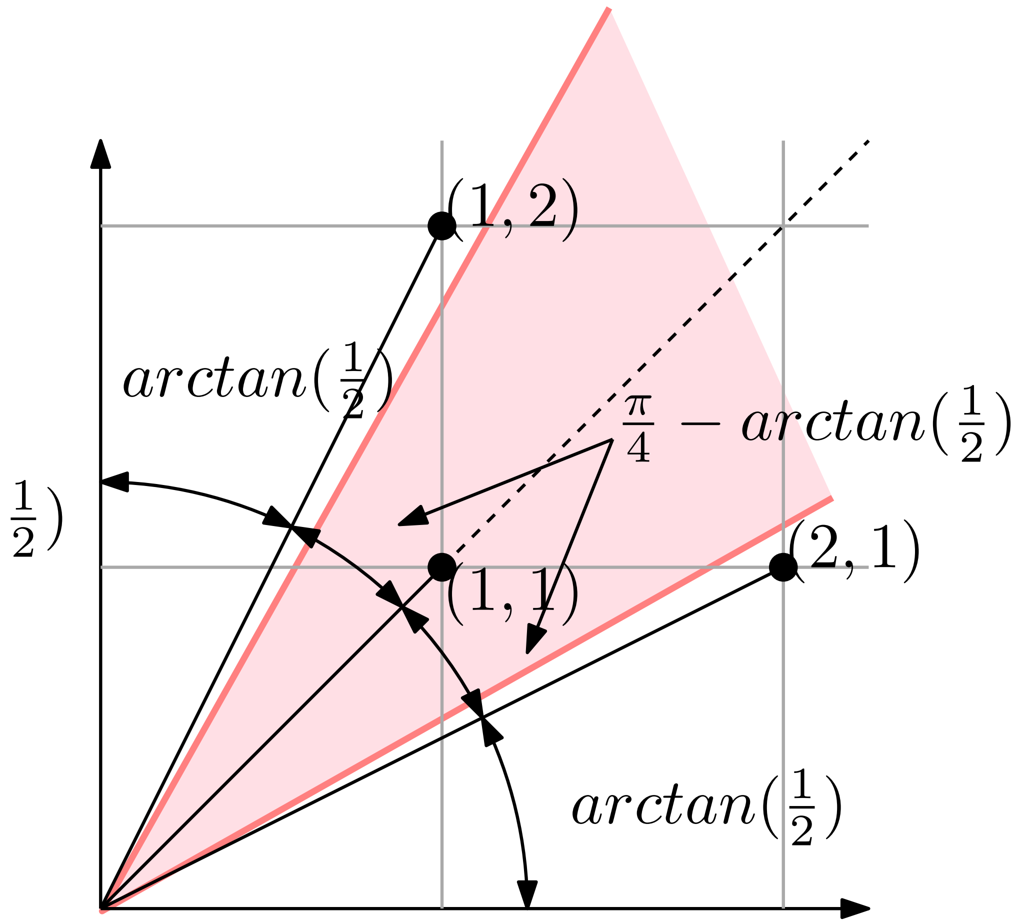

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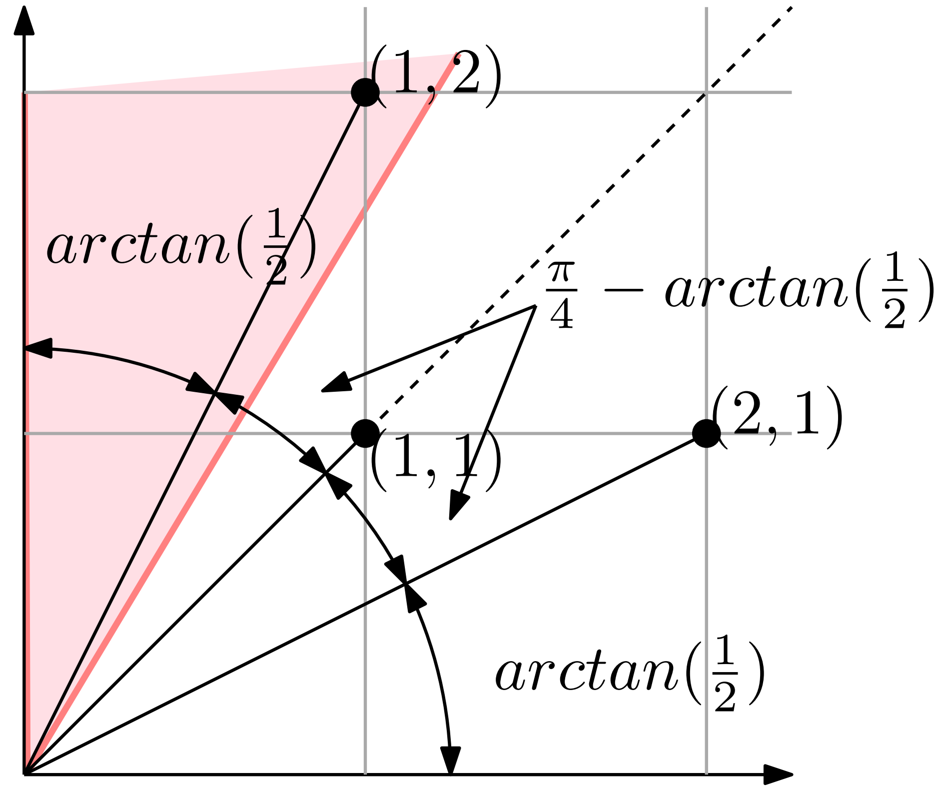

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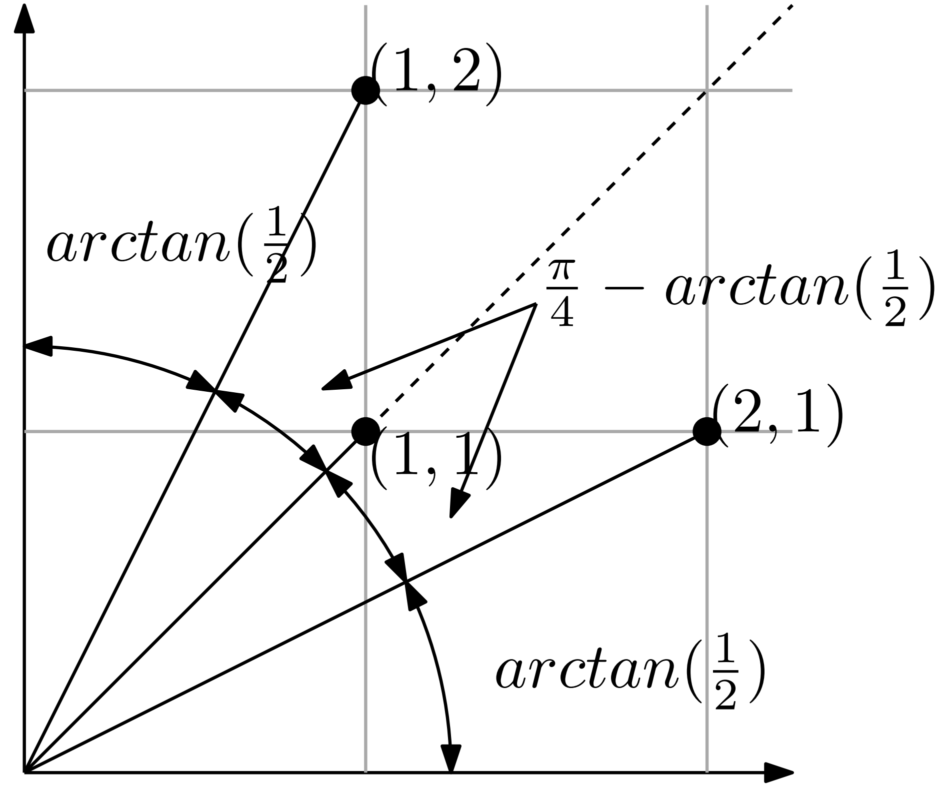

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Lemma-AssignPoint
Consider angles θ_1, θ_2 with $0 \leq \theta_1 < \theta_2 \leq \frac{\pi}{2}$ and let $d = \lceil \frac{1}{\theta_2 - \theta_1} \rceil$. Then, a grid point p such that the edge e that connects the origin $(0,0)$ to p satisfies $\theta_1 < slope(e) < \theta_2$ can be identified as follows:

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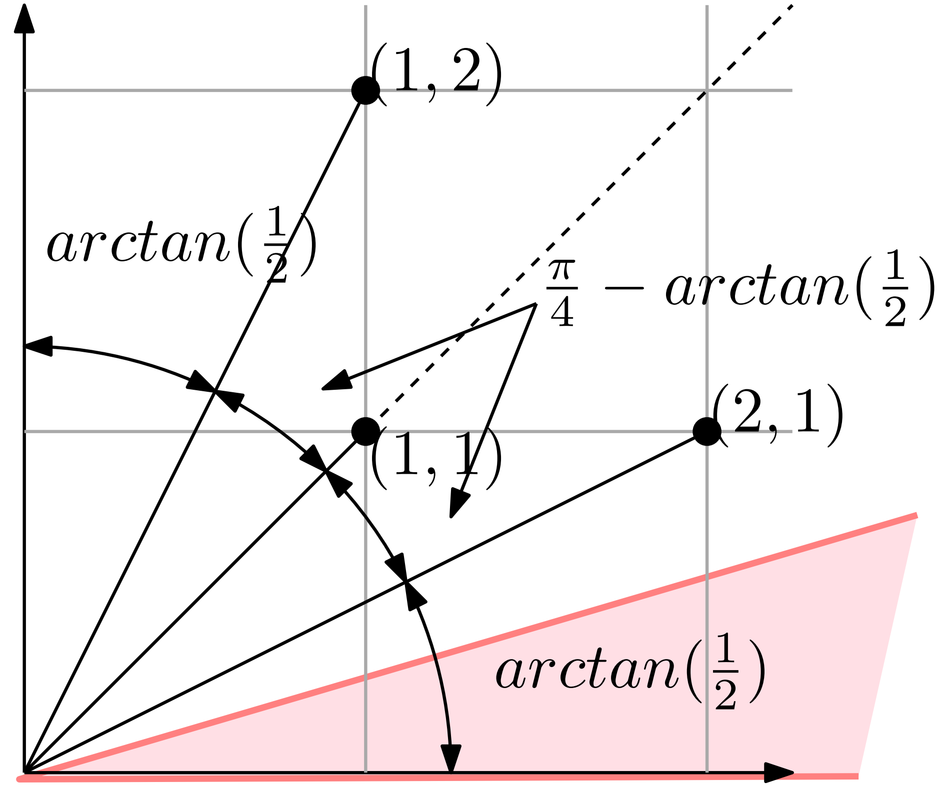

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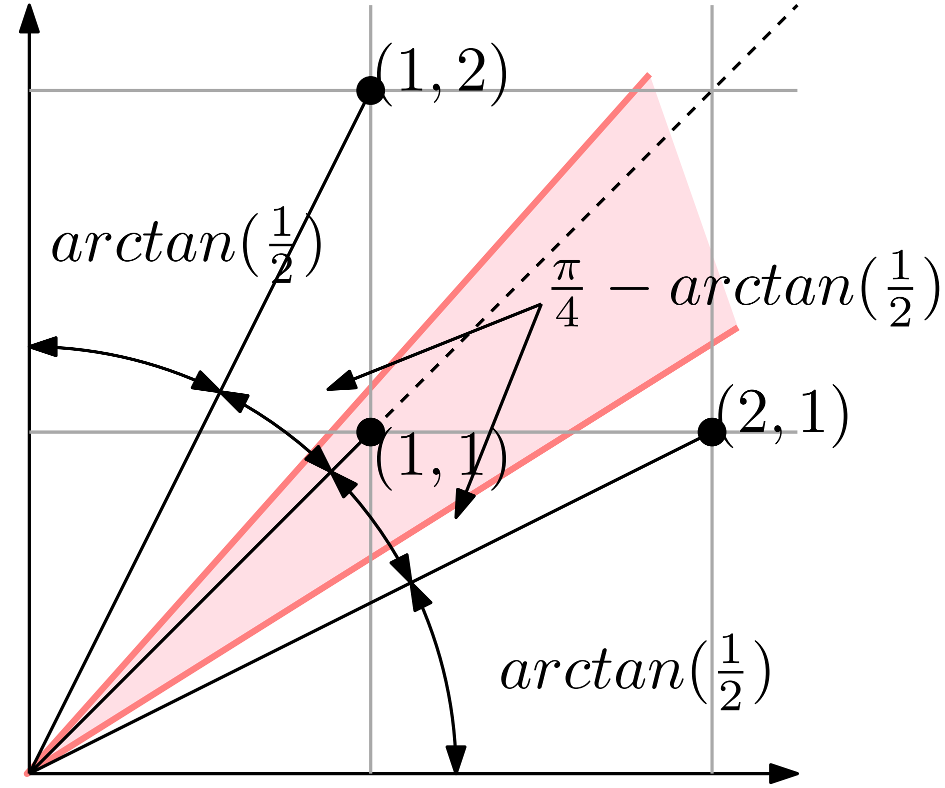

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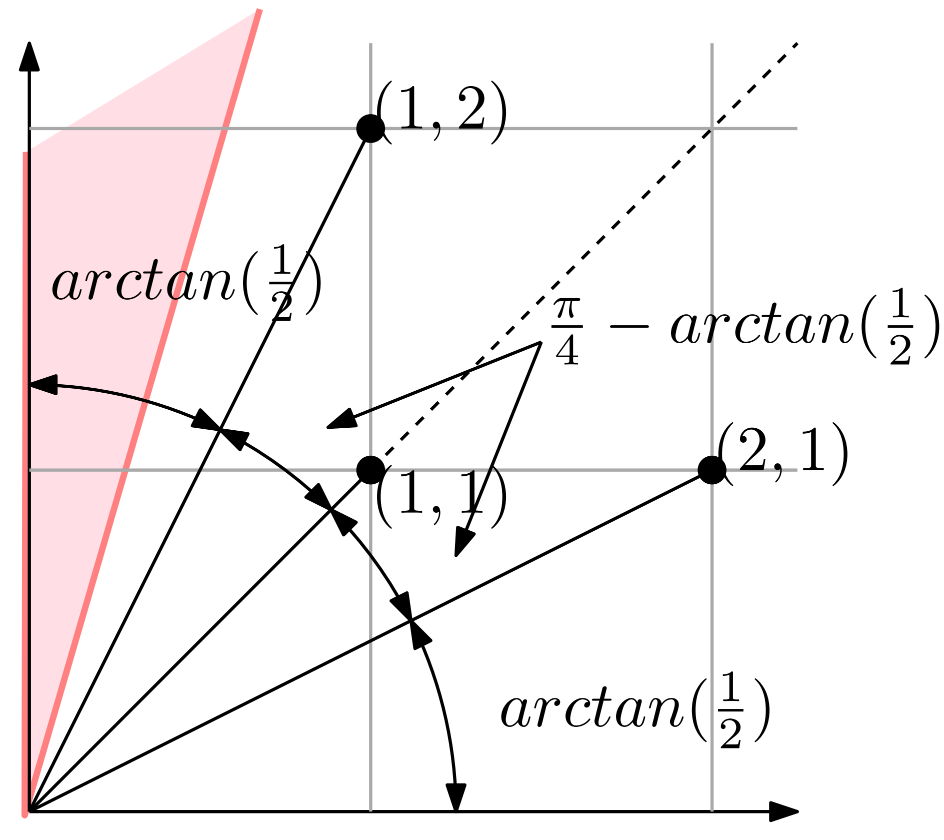

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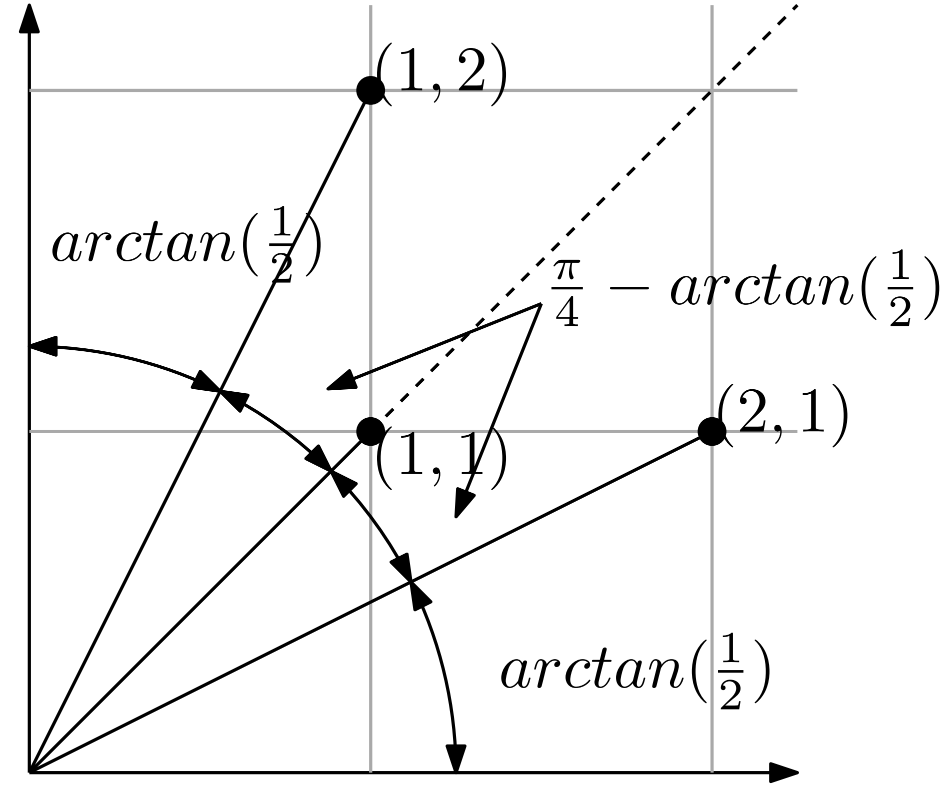

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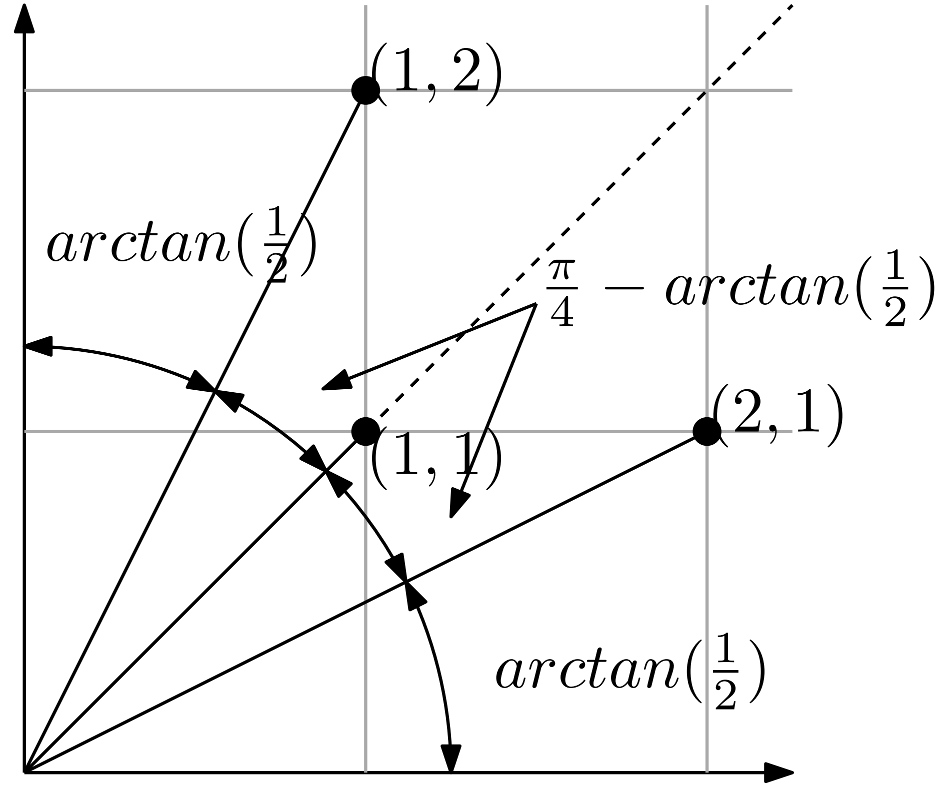
- If $p = (x, y)$ is the identified point, it also holds that:

$$\max(x, y) \leq \frac{\pi}{2} \cdot \frac{1}{\theta_2 - \theta_1}$$



Balanced angle-range assignment for tree nodes

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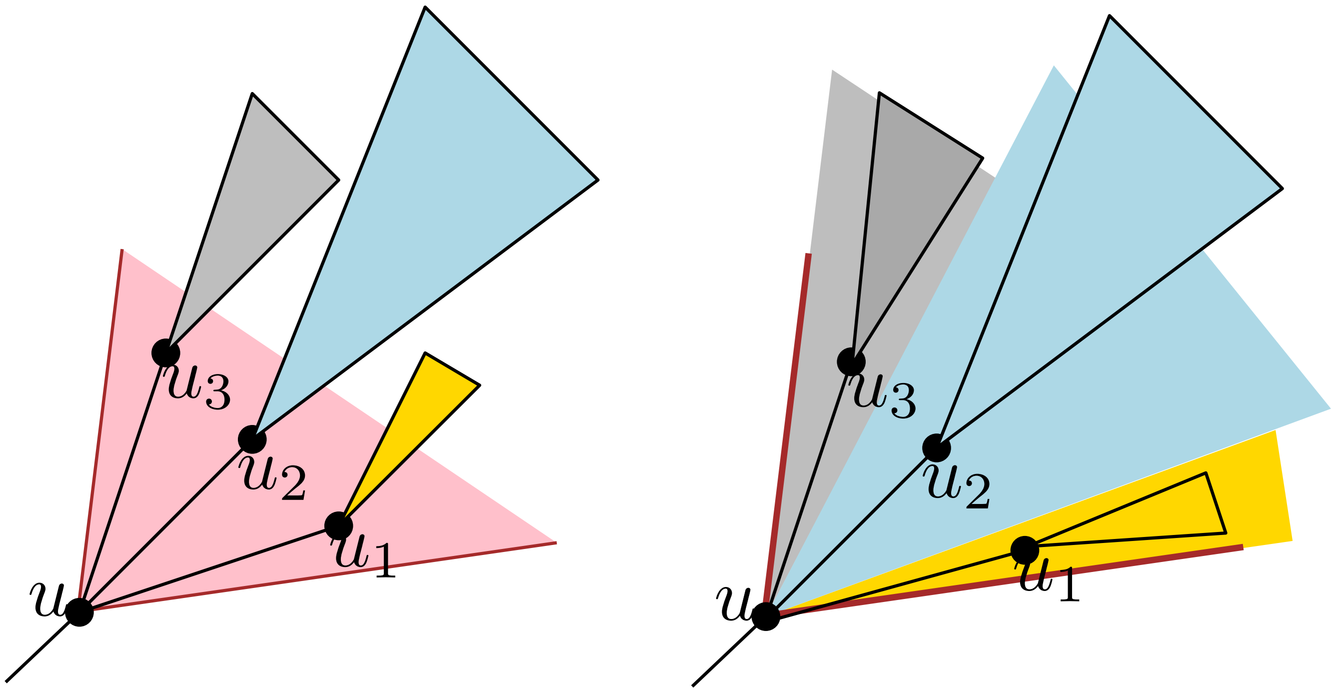
- **Strategy: Balanced assignment**
  - Spil the angle range of a node u to its childen in proportion to the size of the subtree rooted at each child.



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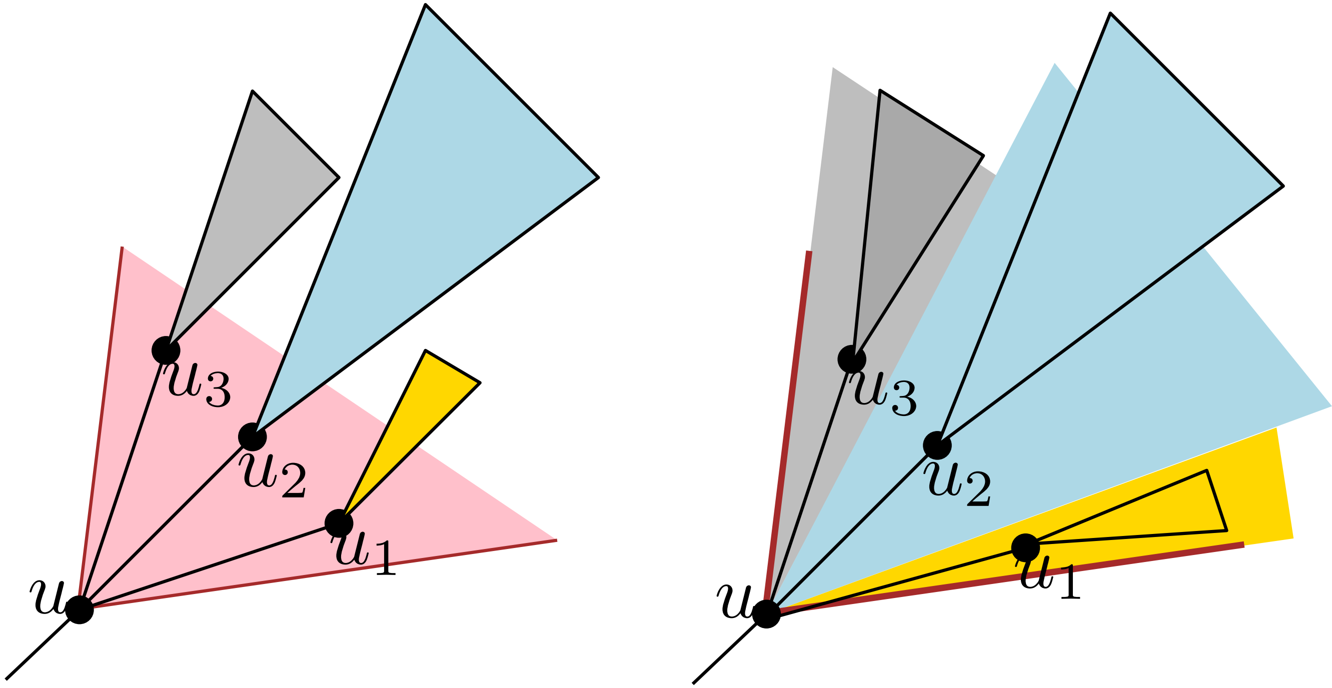

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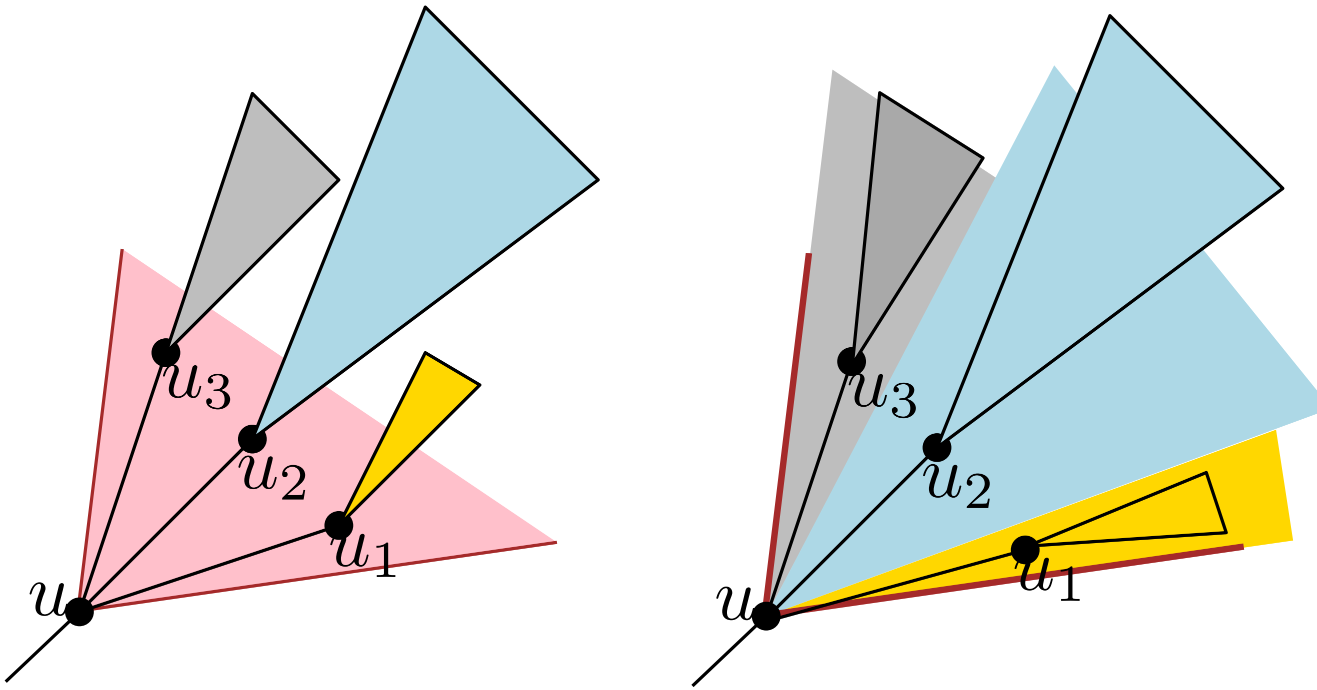

Lemma

"Balanced assignment" leads to a non-strictly slope disjoint drawing.

The tree drawing algorithm

The tree drawing algorithm

Algorithm-1 Balanced Monotone Tree Drawing
Input: An n-vertex tree T rooted at vertex r.
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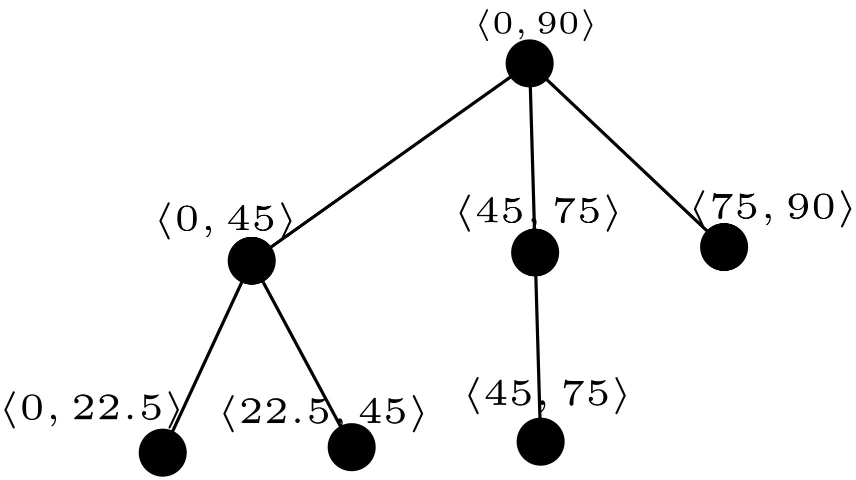

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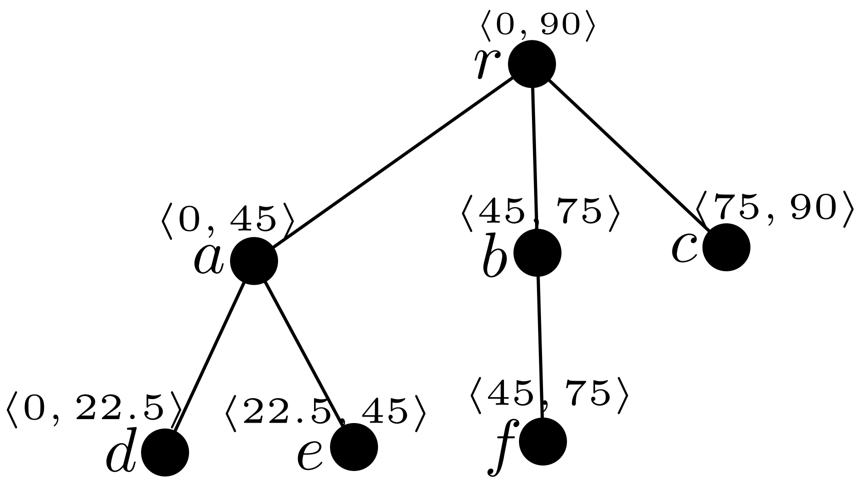

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3. Draw the root r at $(0, 0)$

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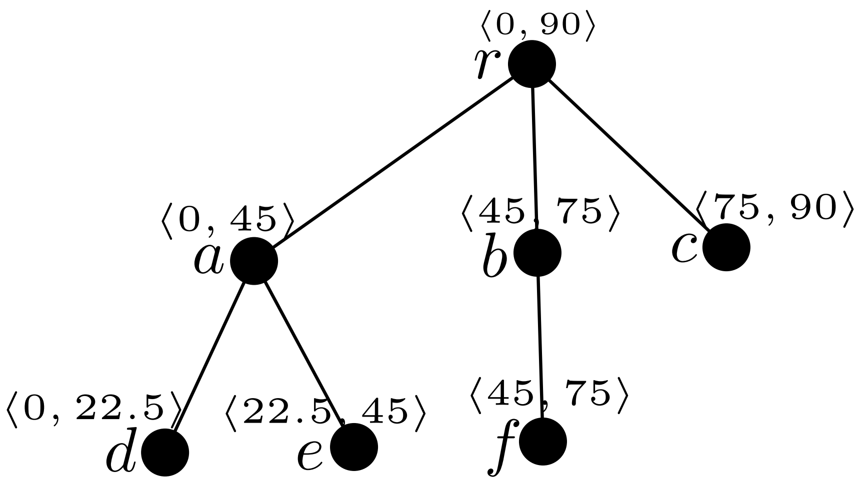

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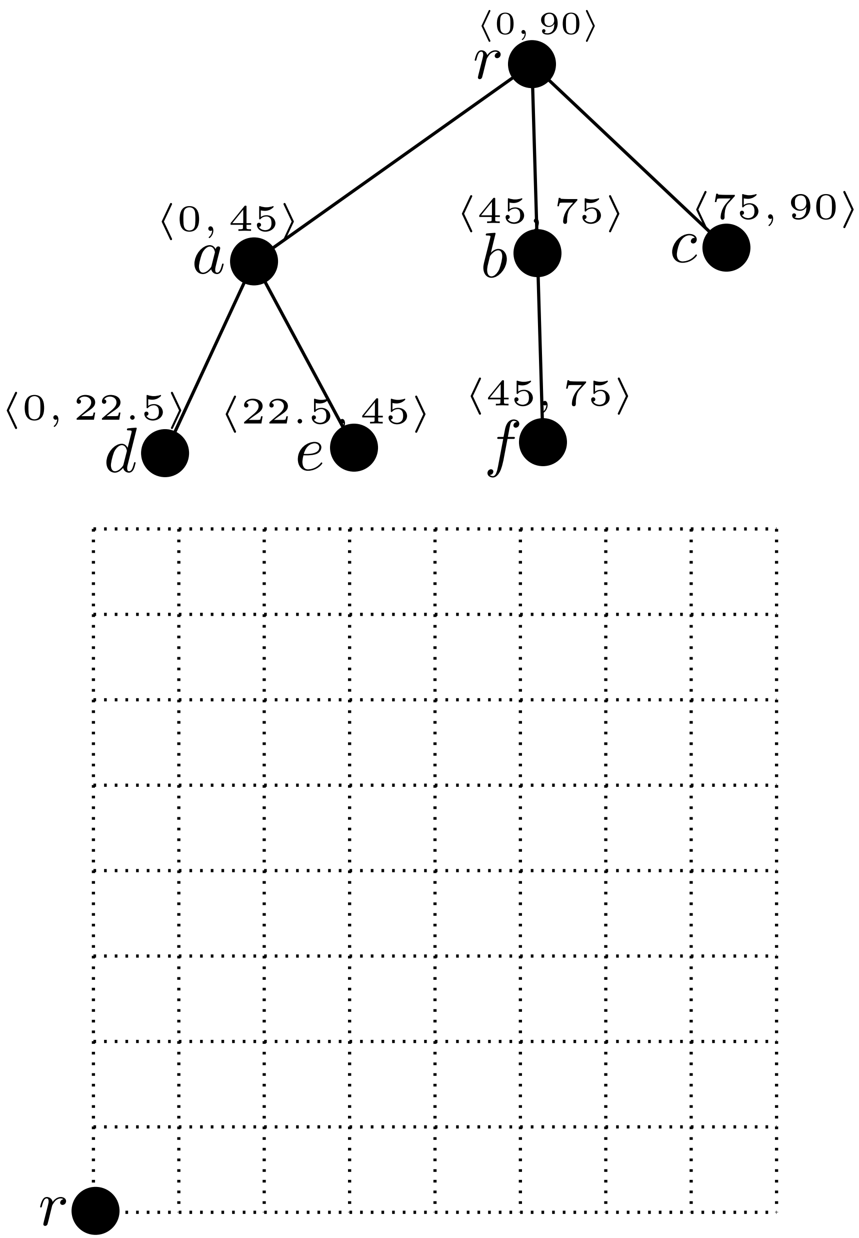

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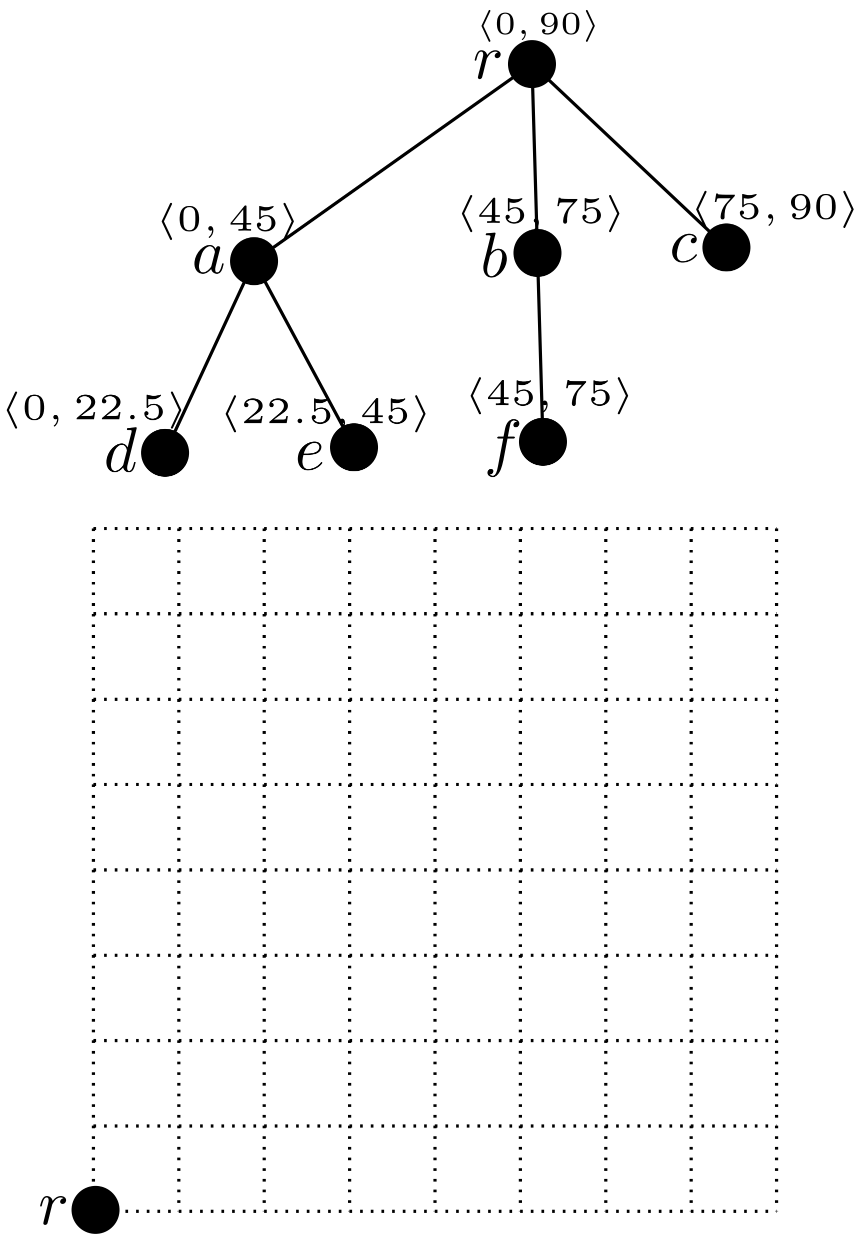
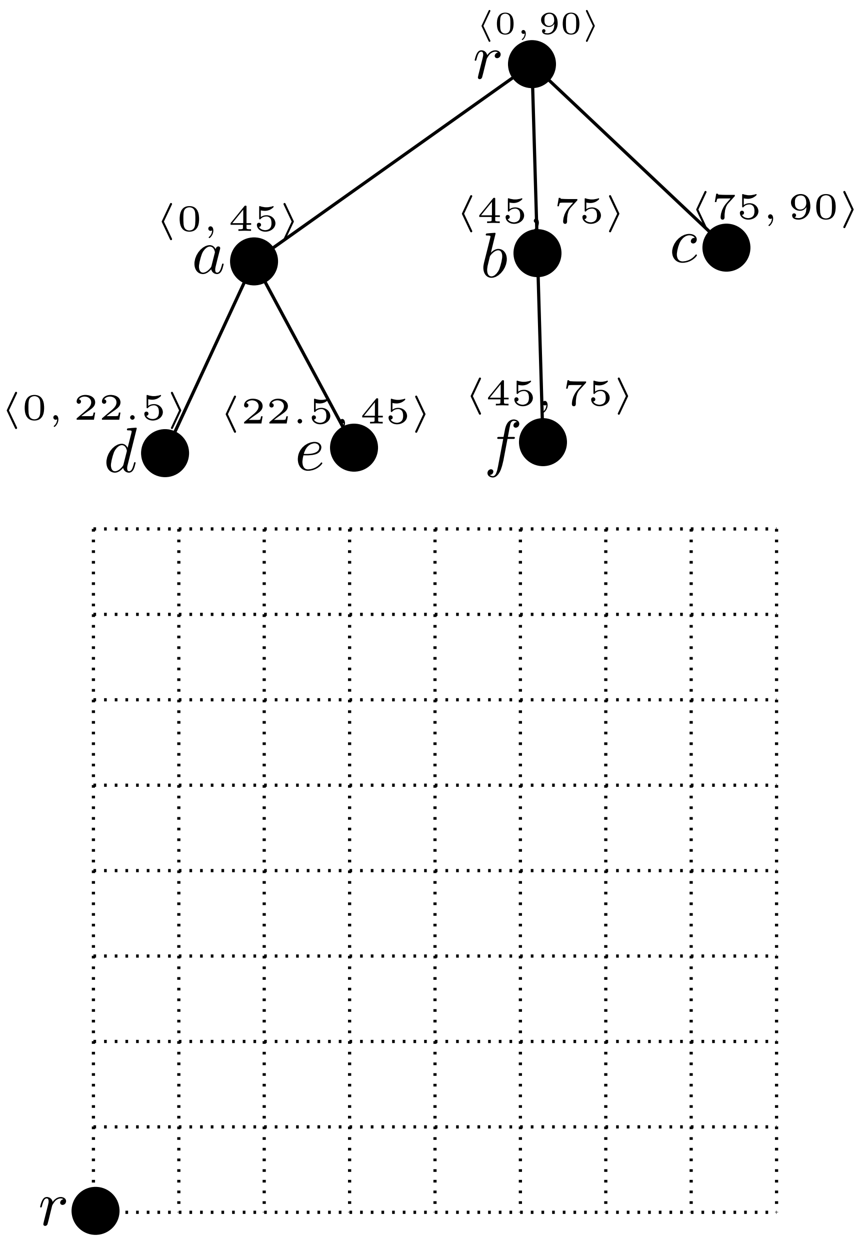


The tree drawing algorithm

$a: \langle 0, 45 \rangle$



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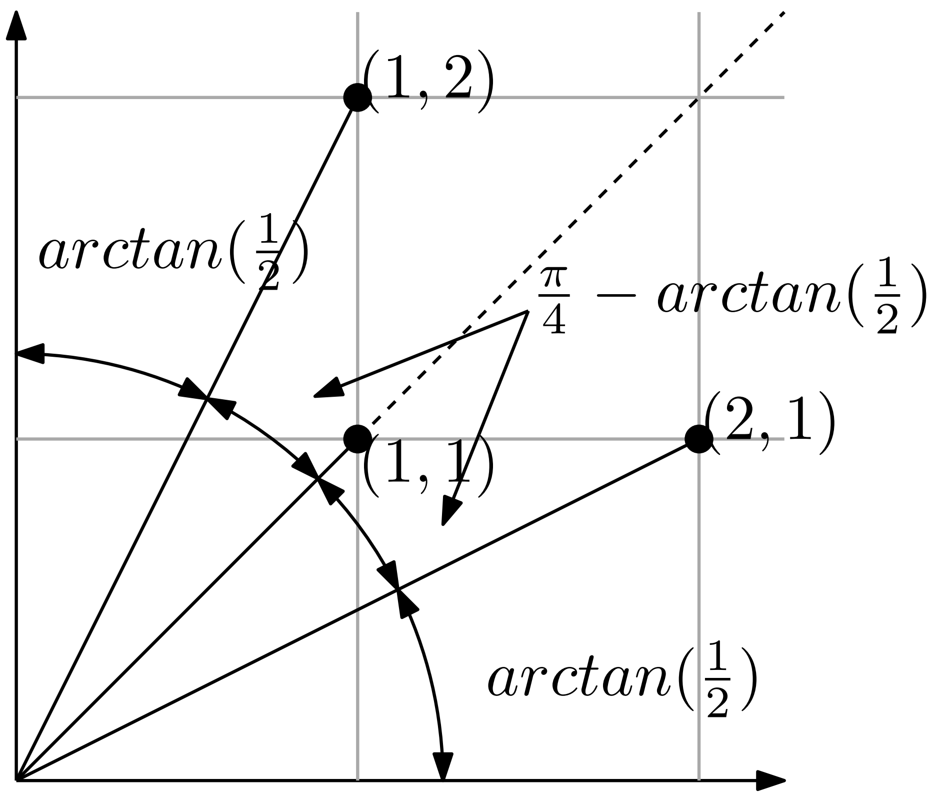

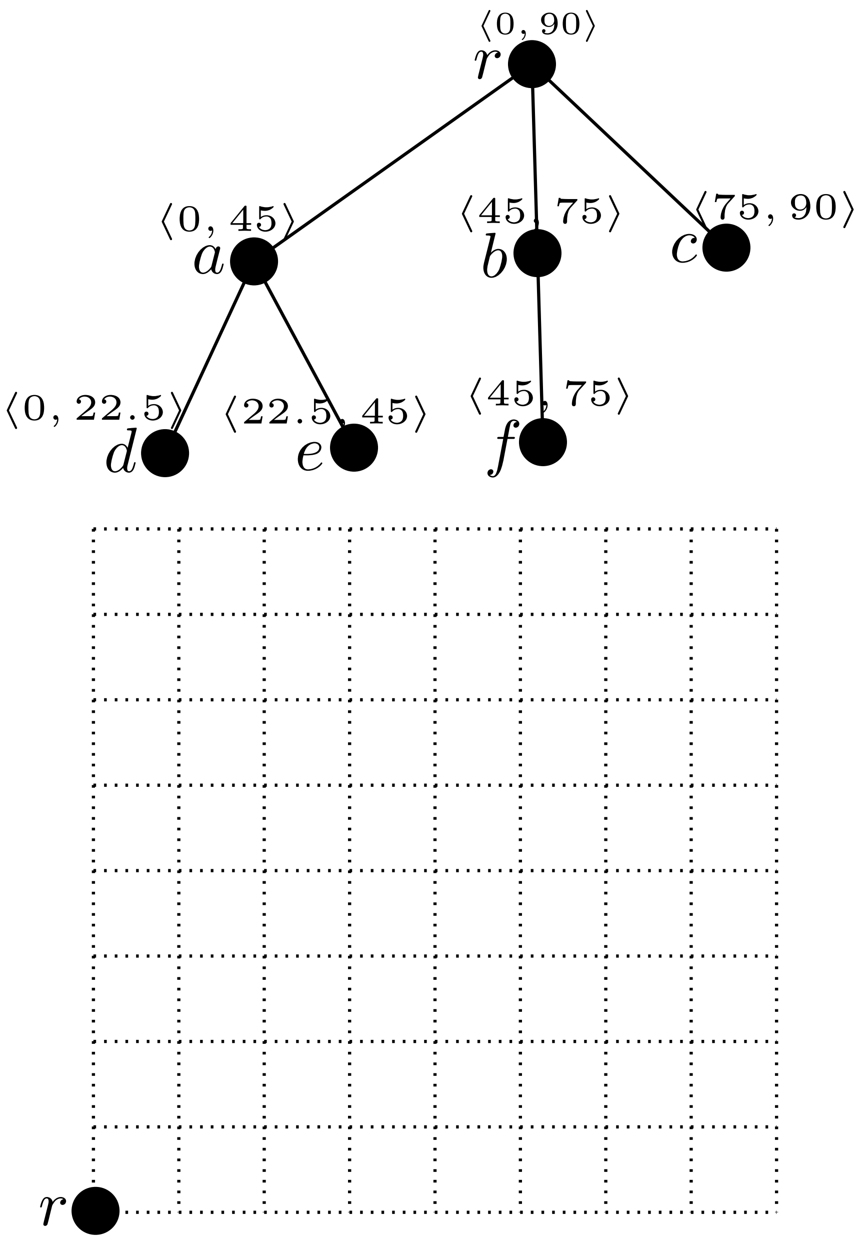

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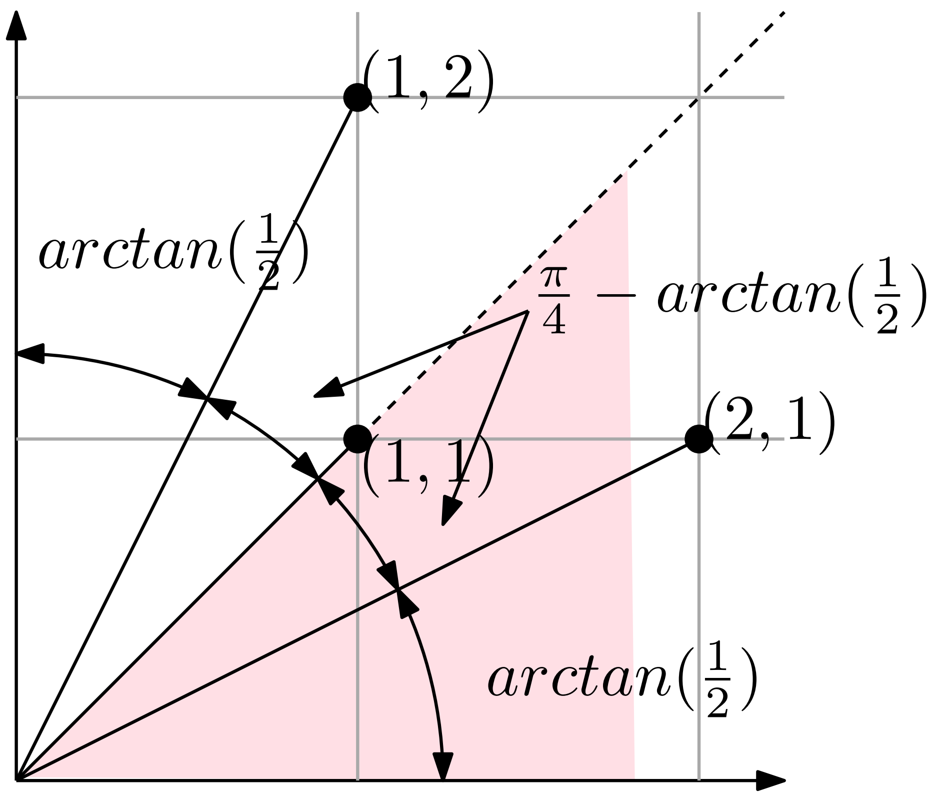

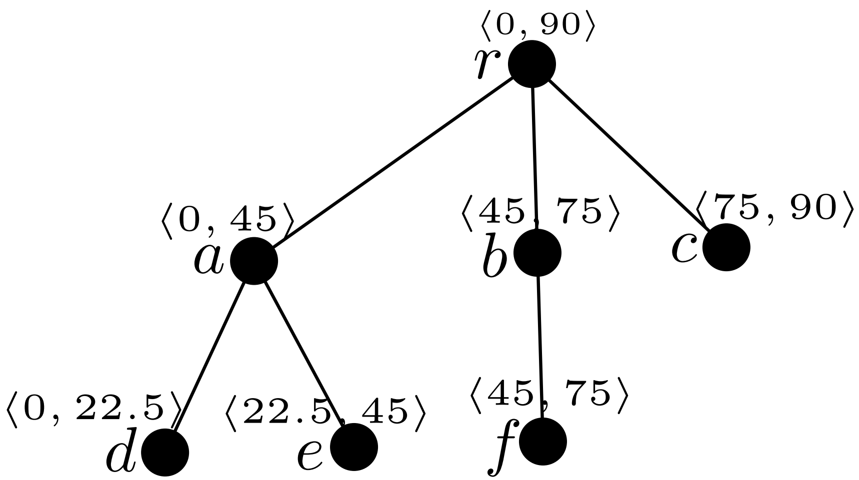

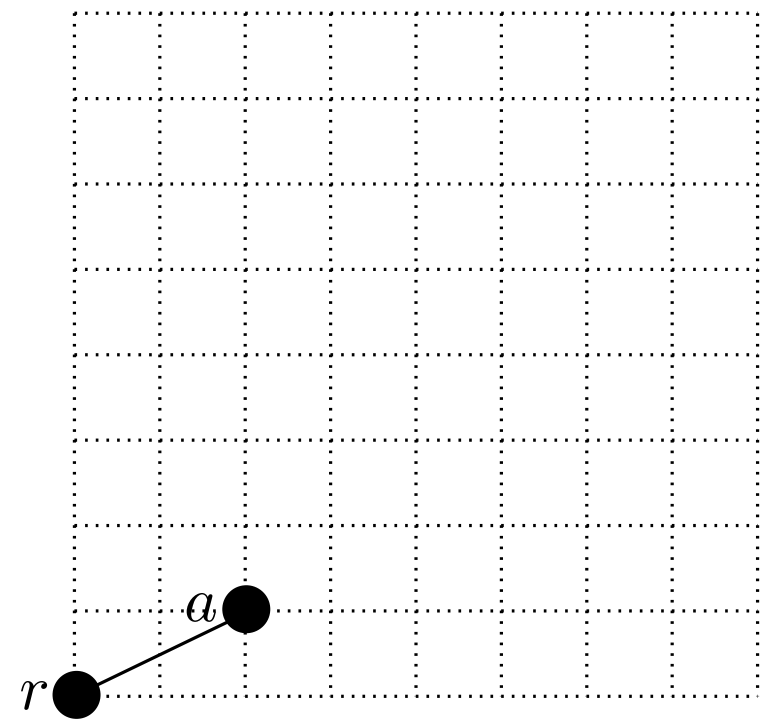

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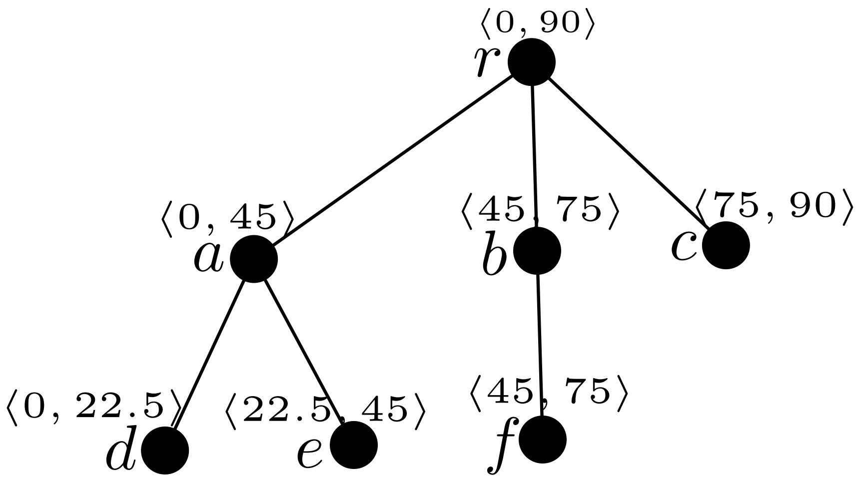

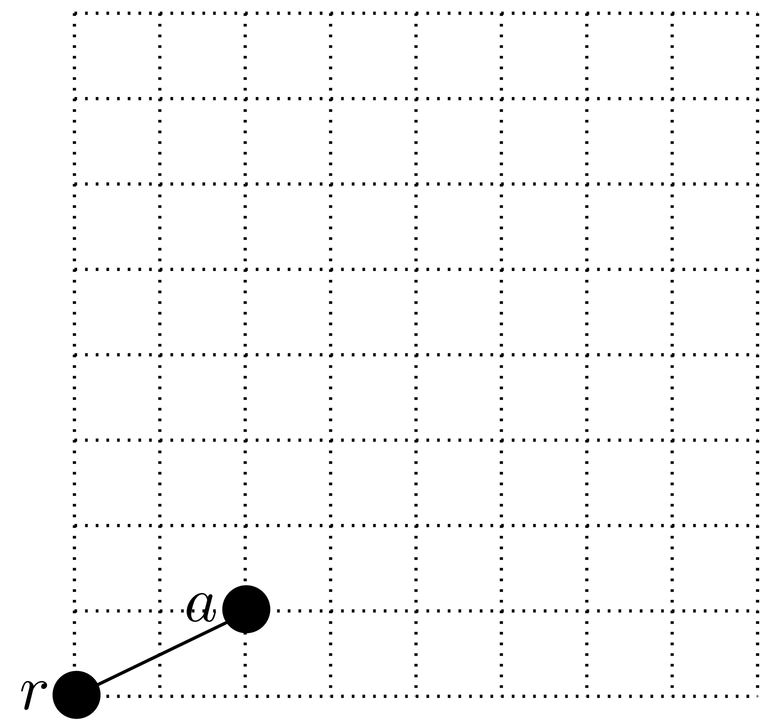

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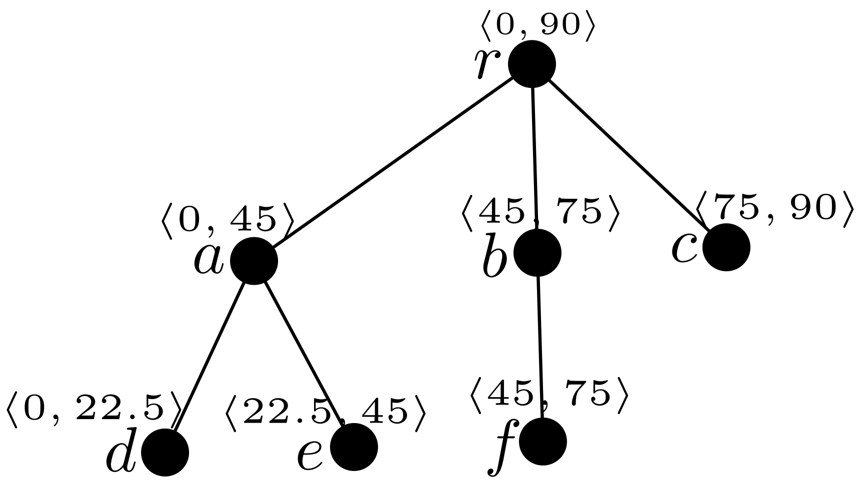
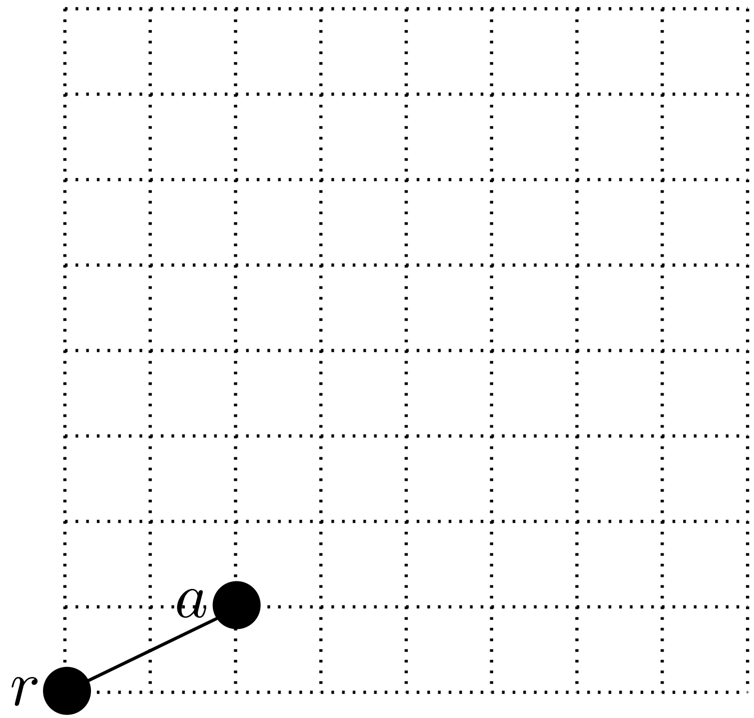
$b: \langle 45, 75 \rangle$ $f: \langle 45, 75 \rangle$





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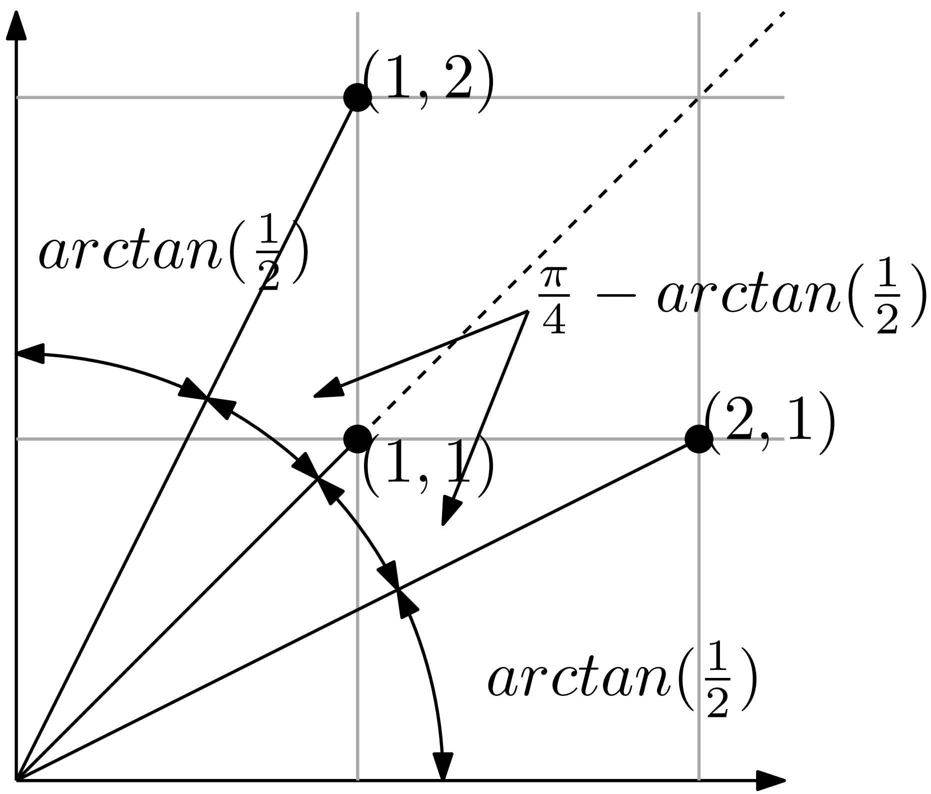

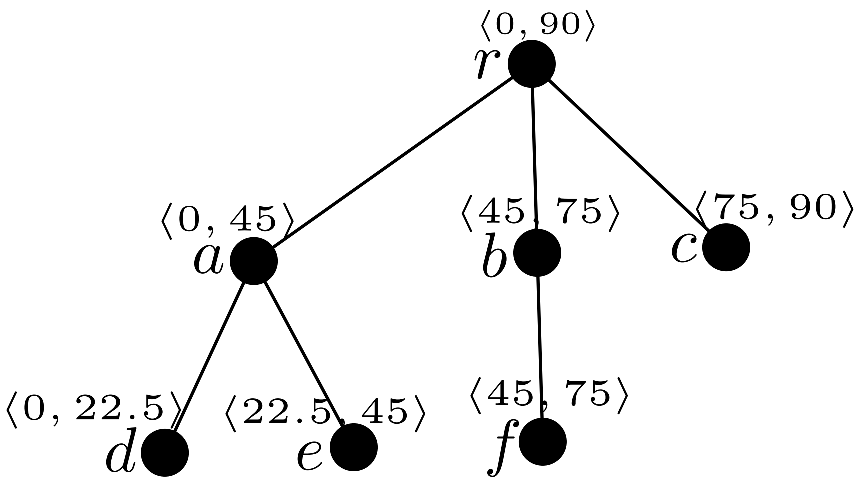

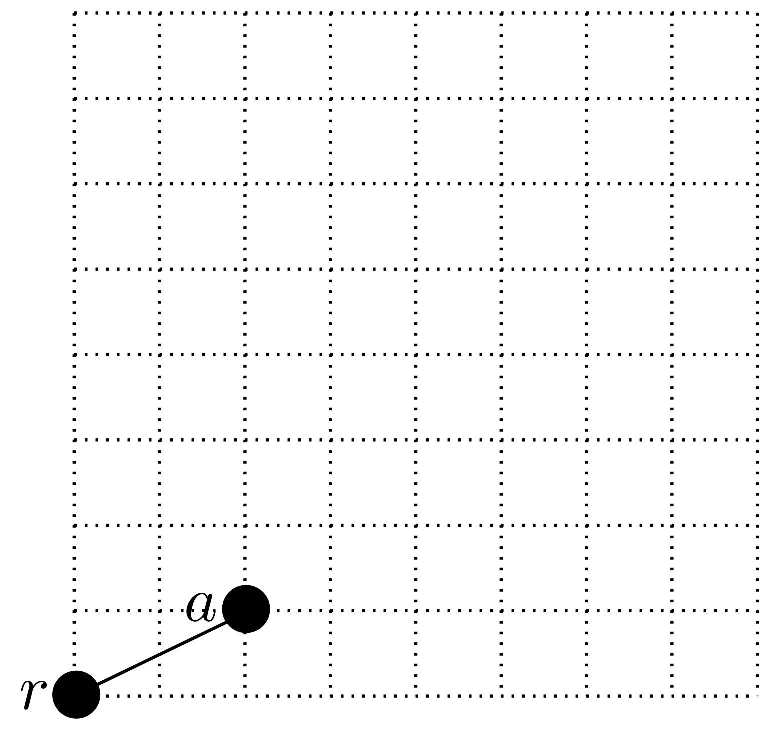

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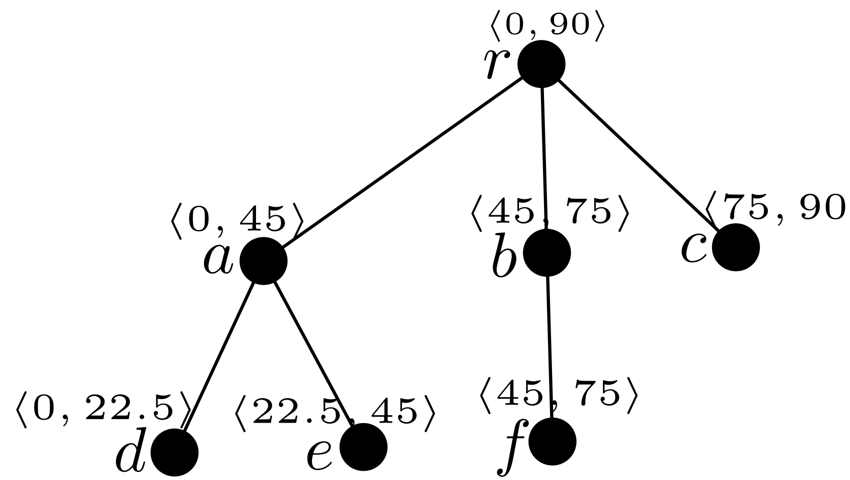

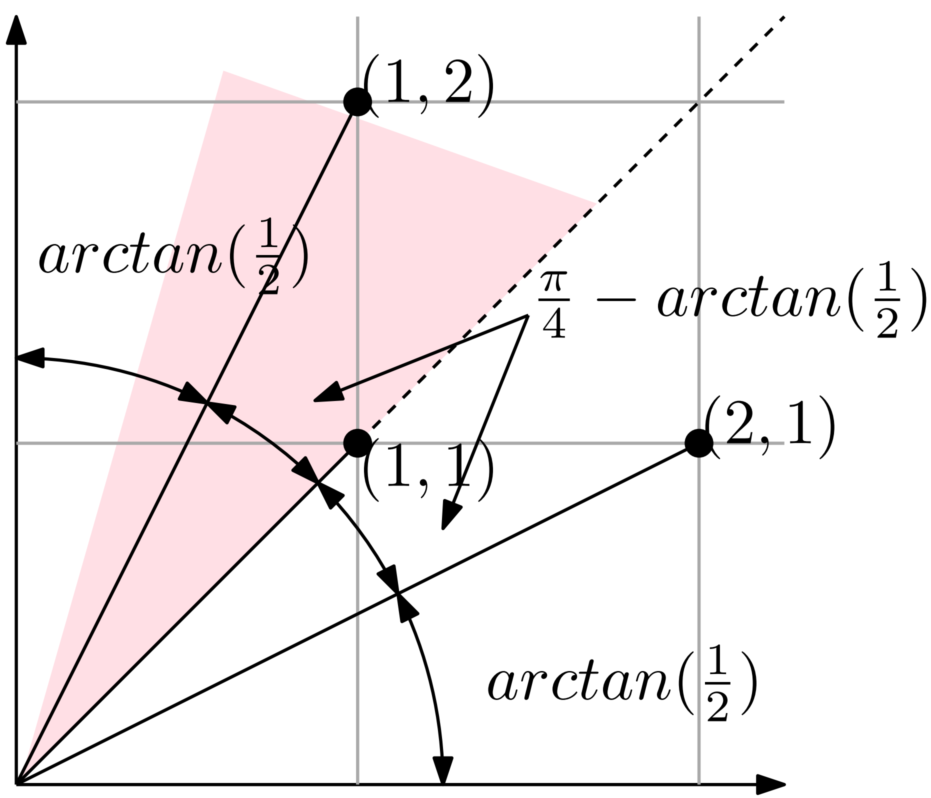

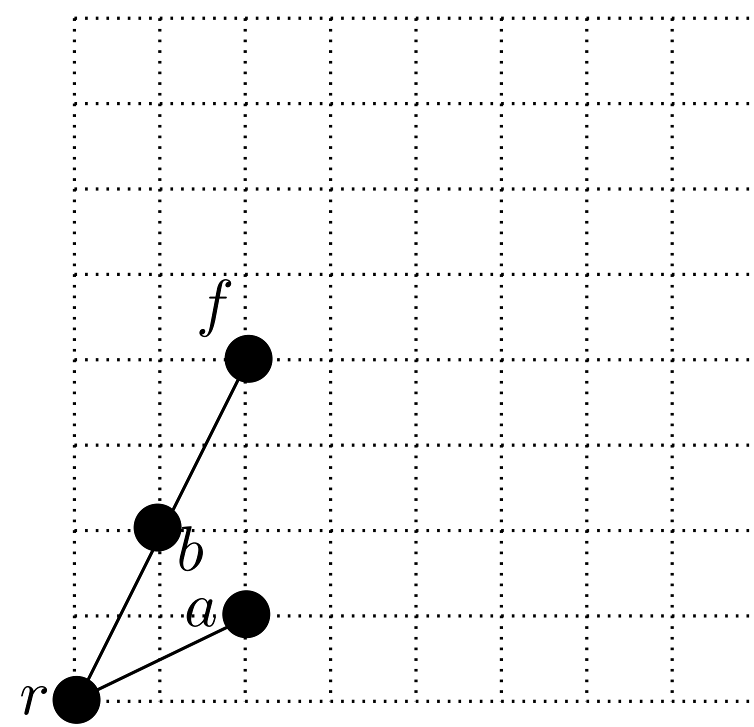

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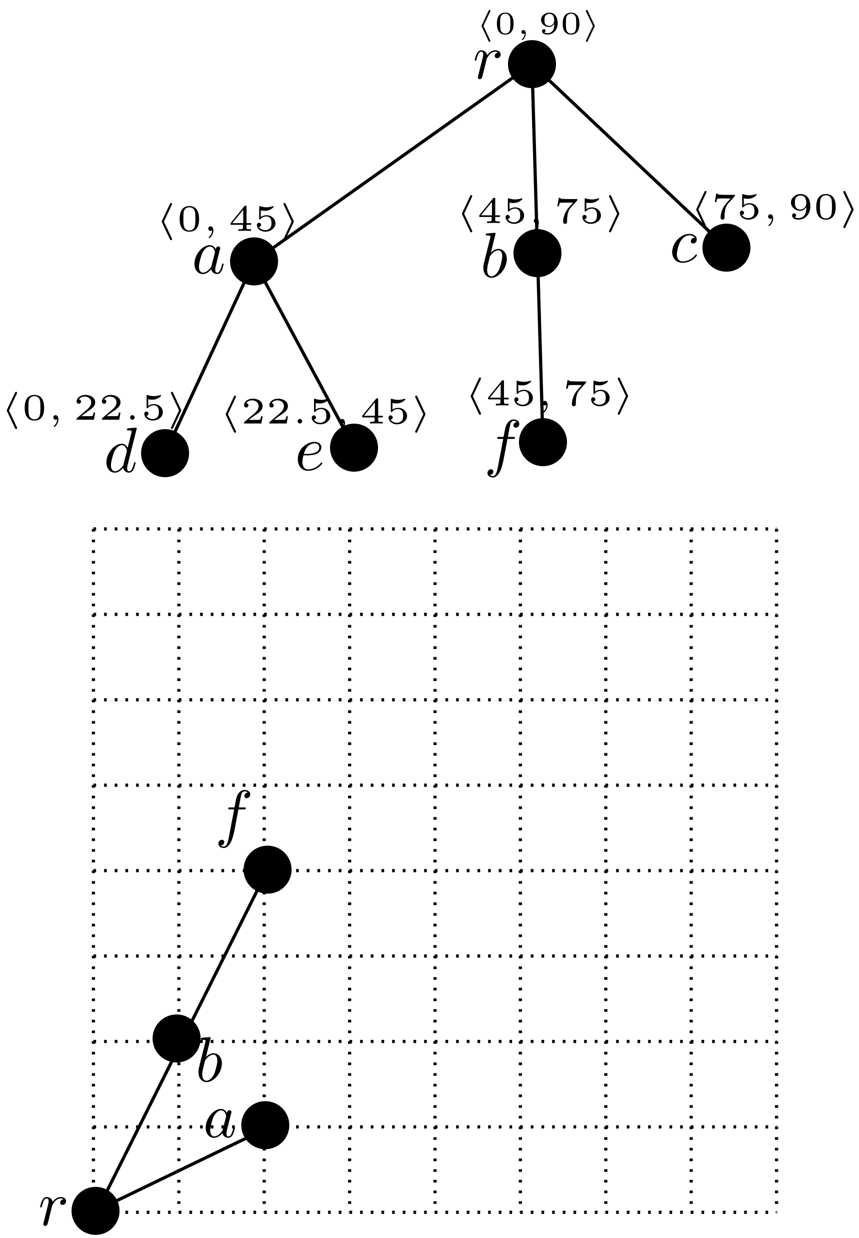

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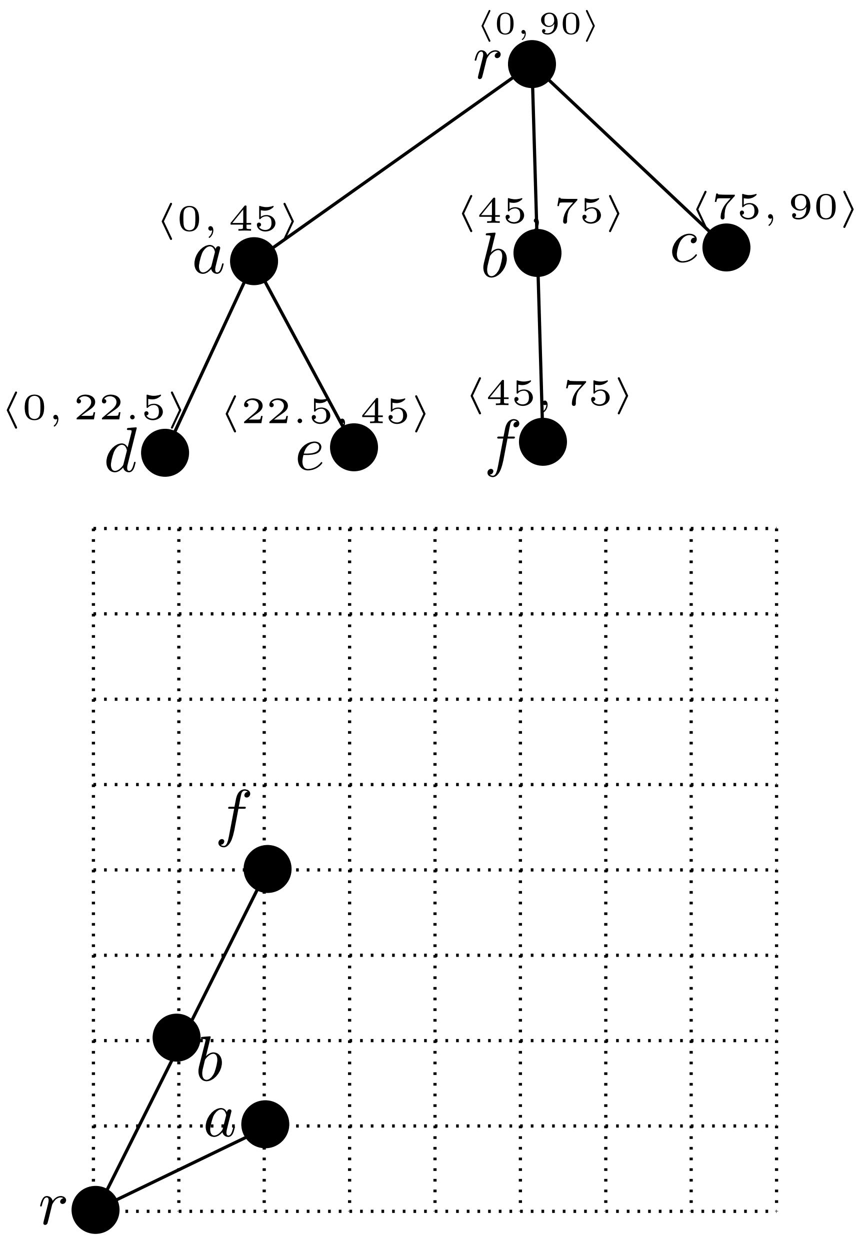

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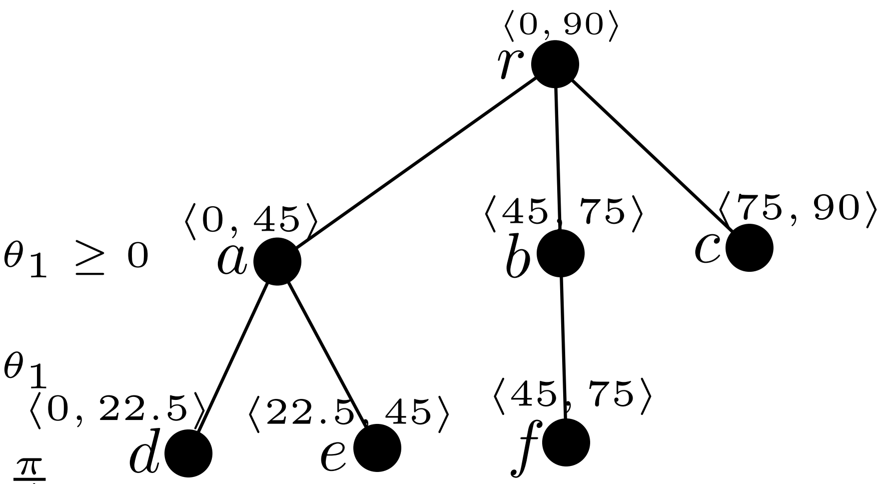

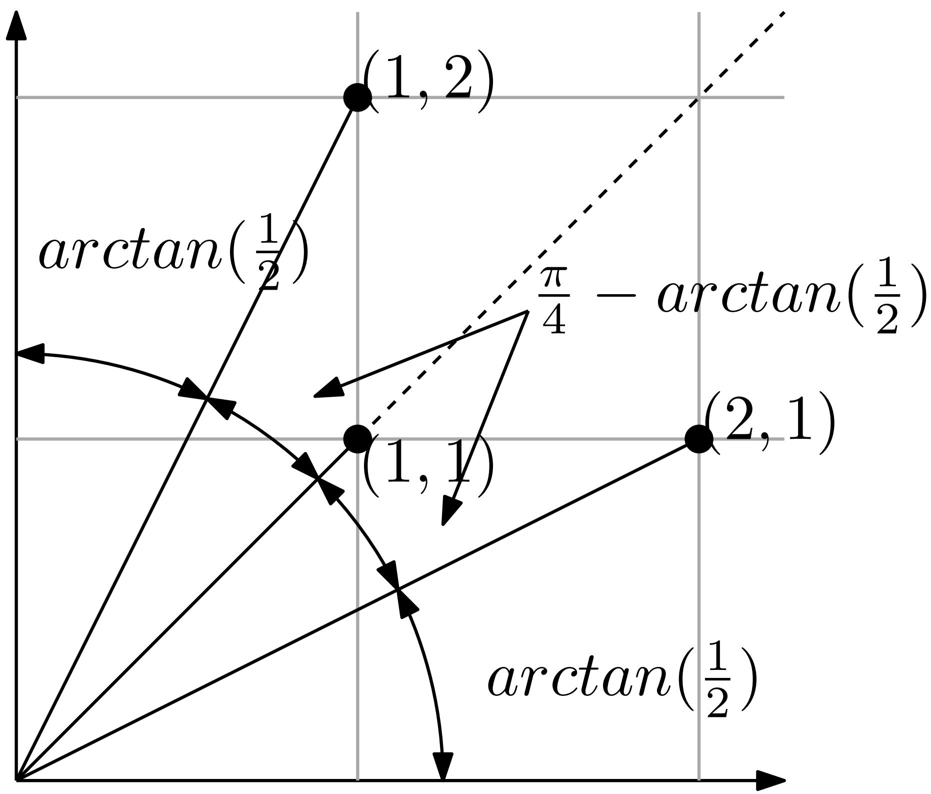

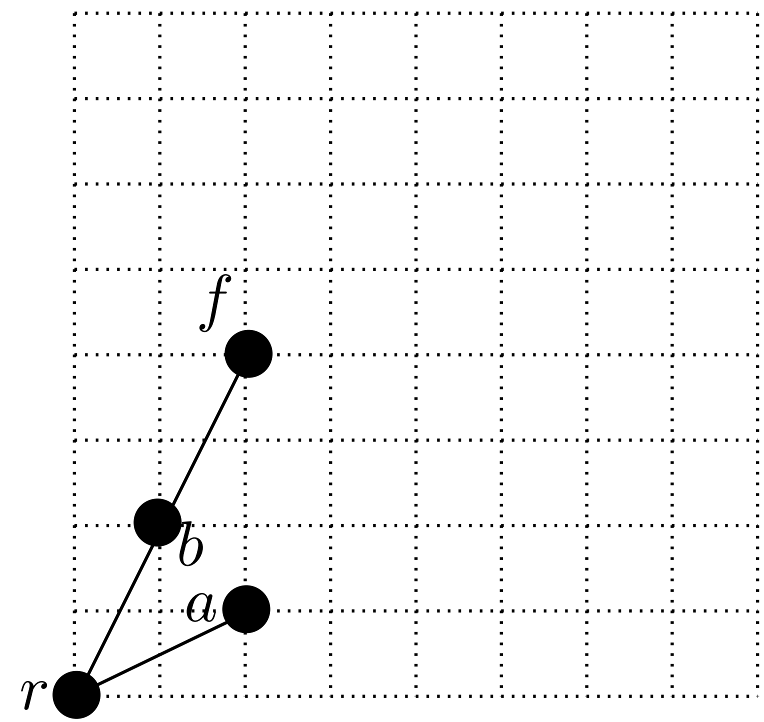

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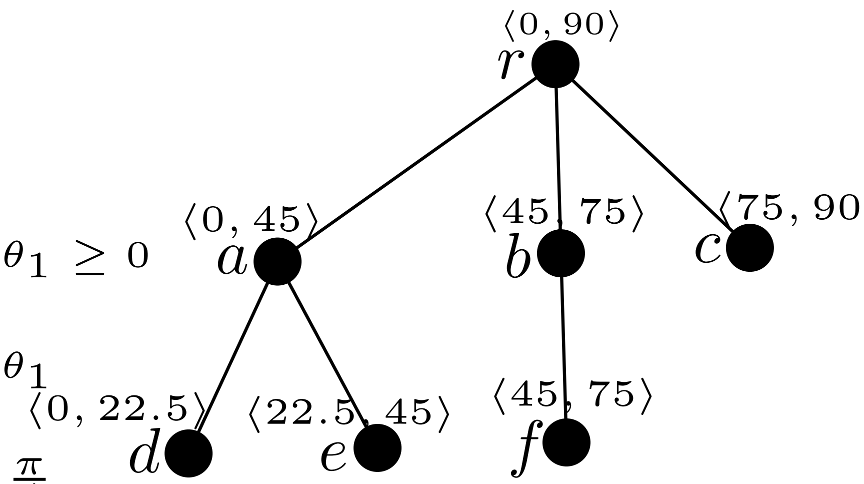

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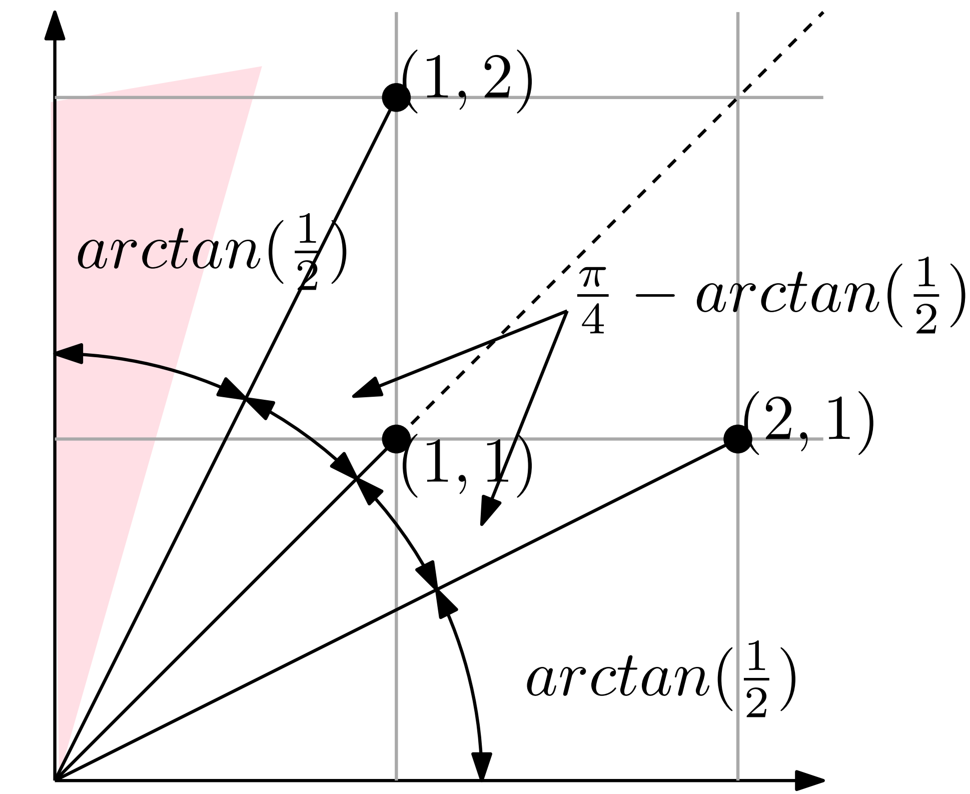

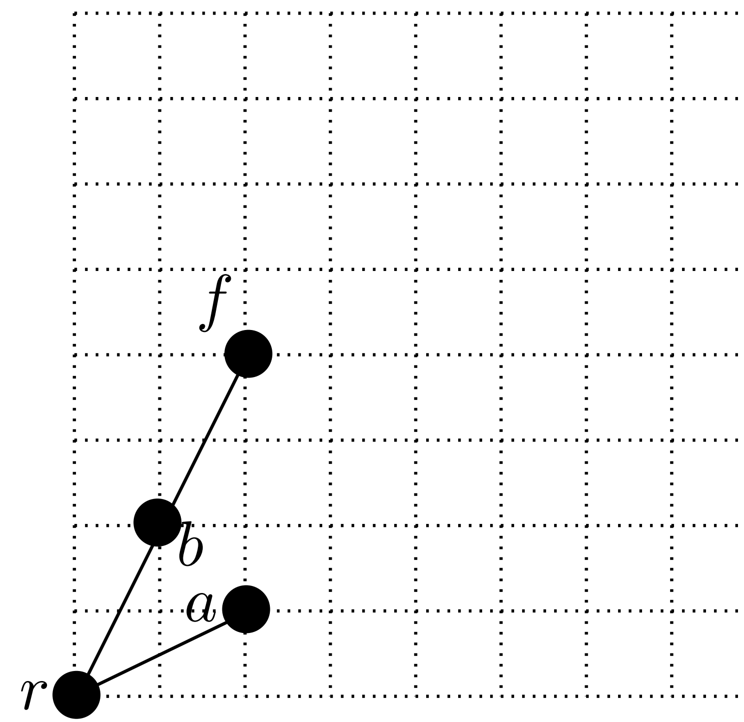

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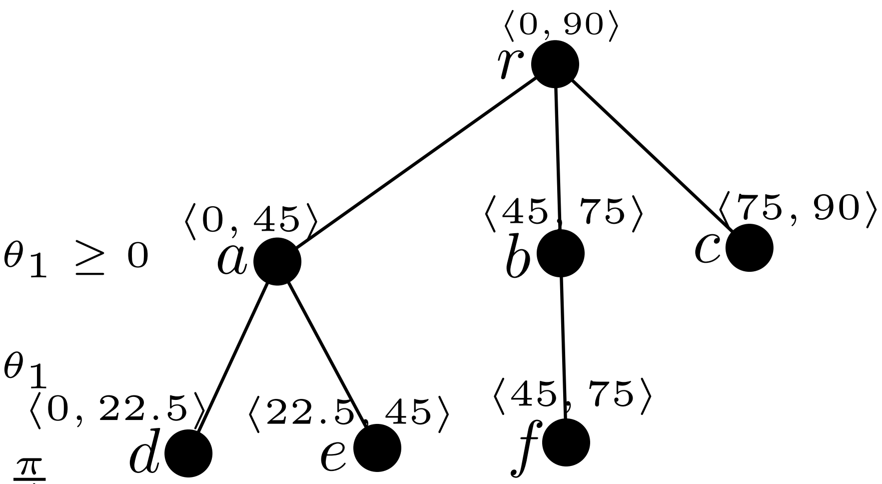

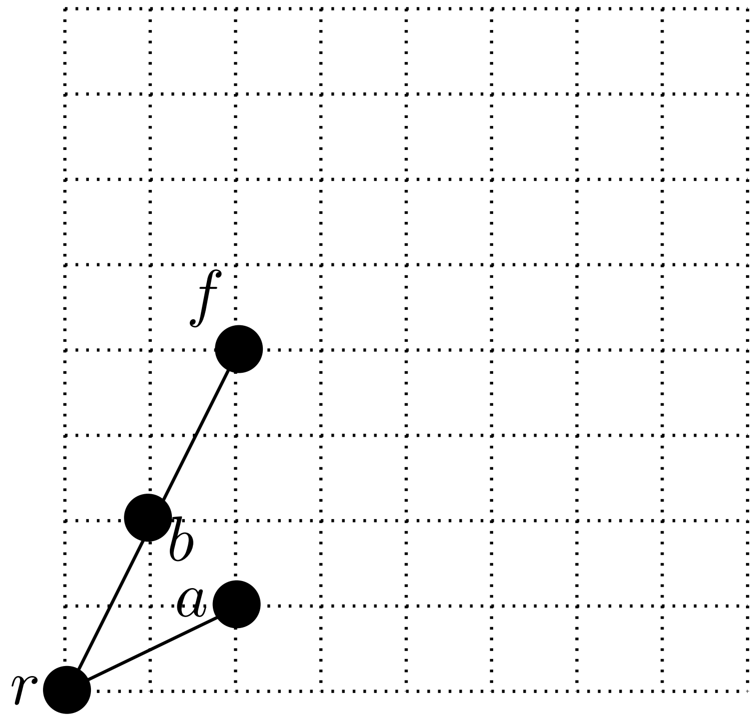

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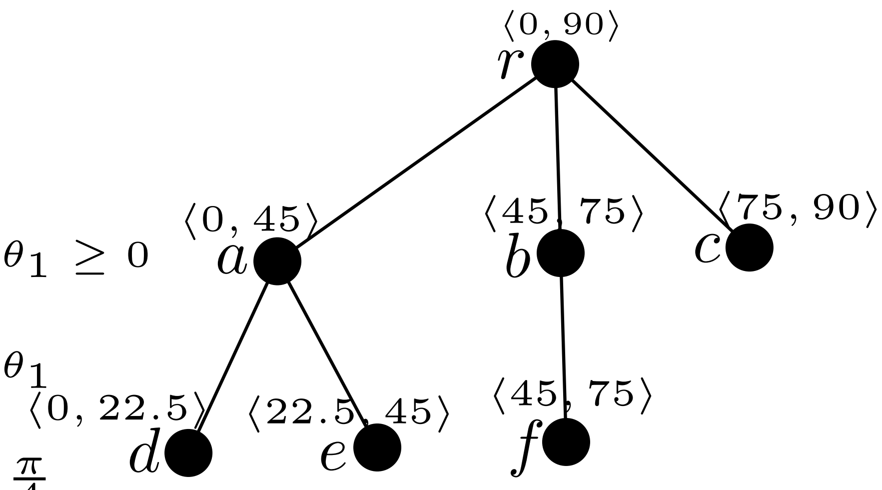

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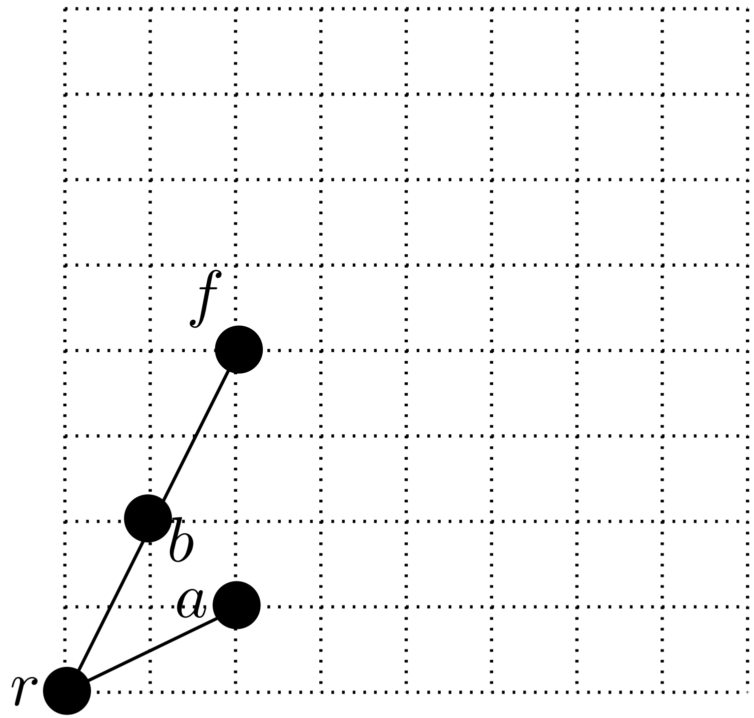

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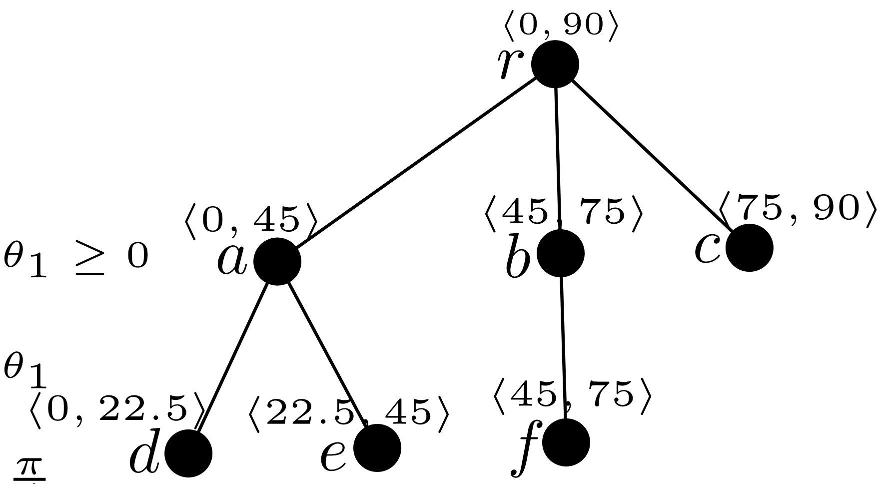

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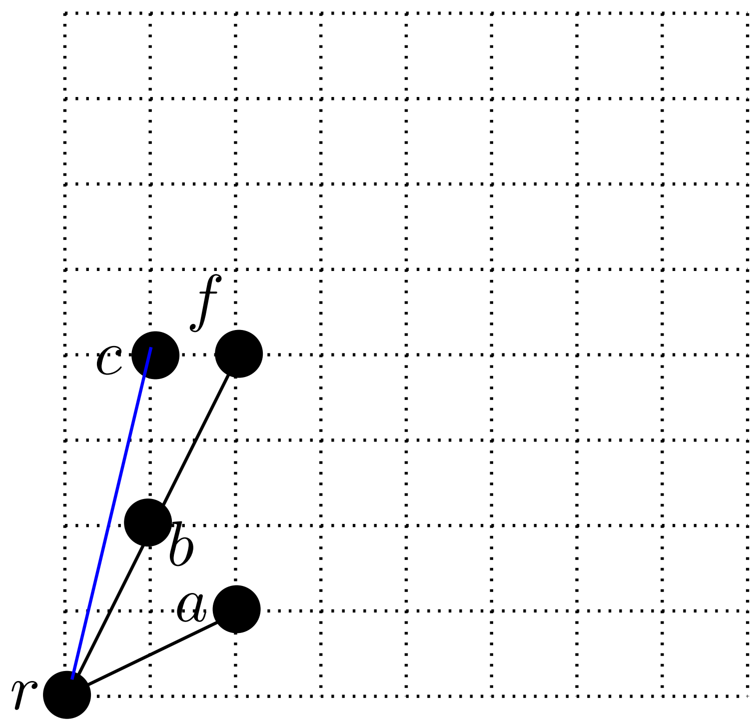
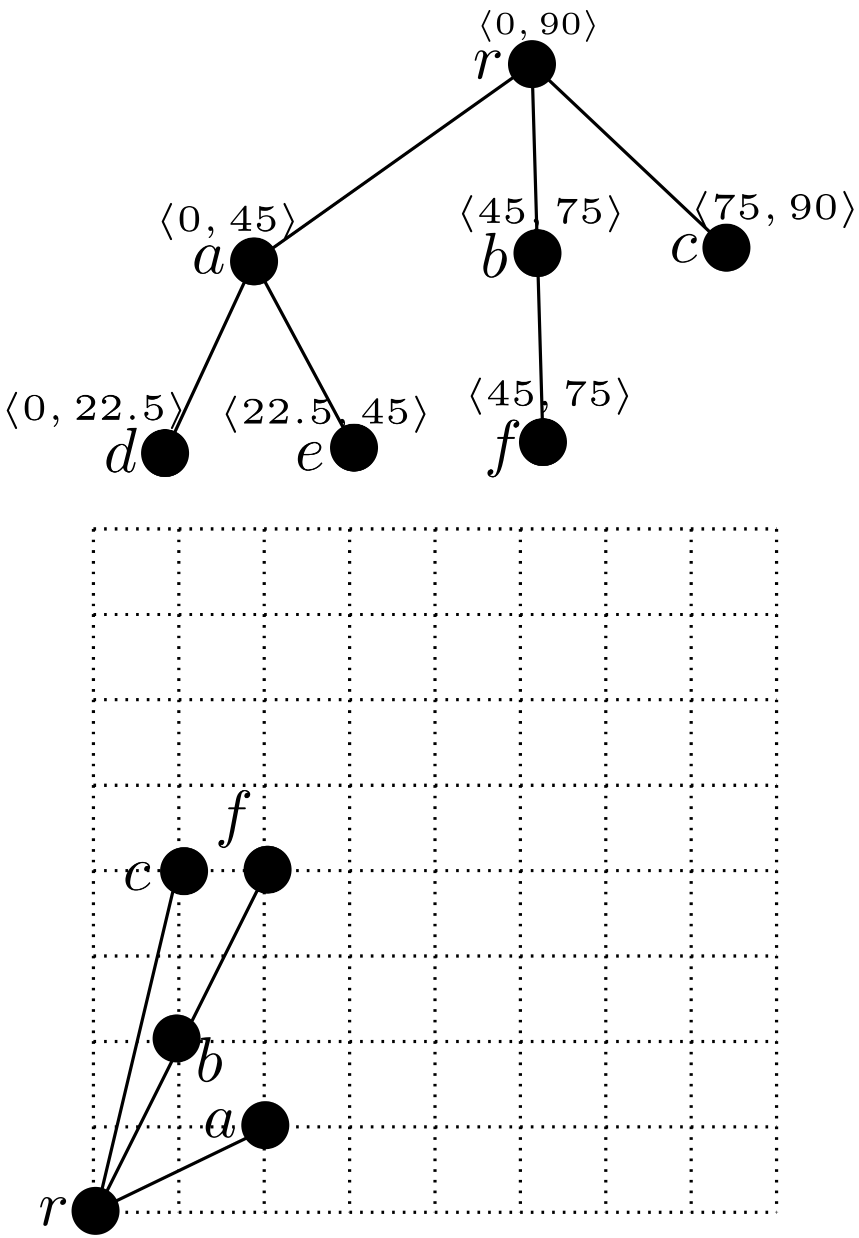


The tree drawing algorithm



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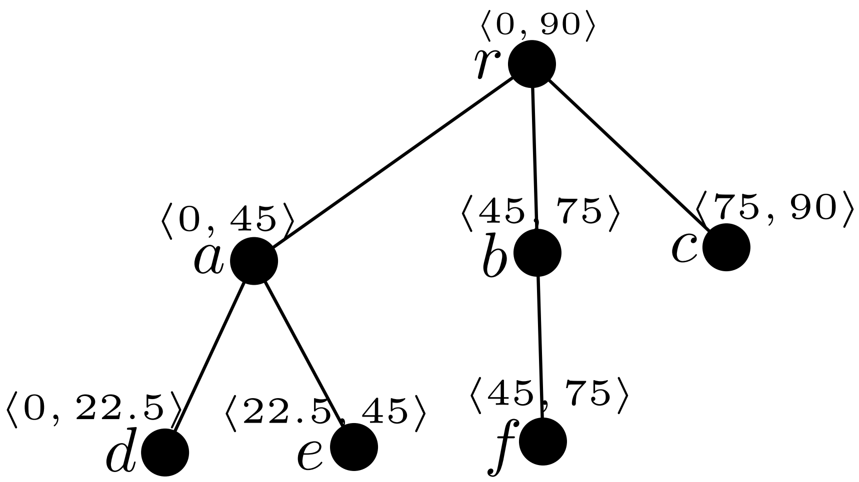

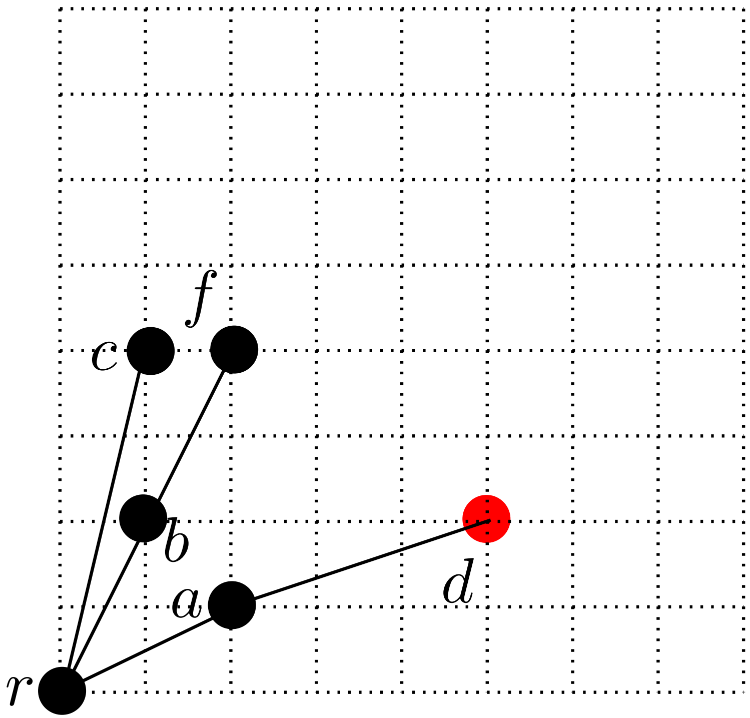

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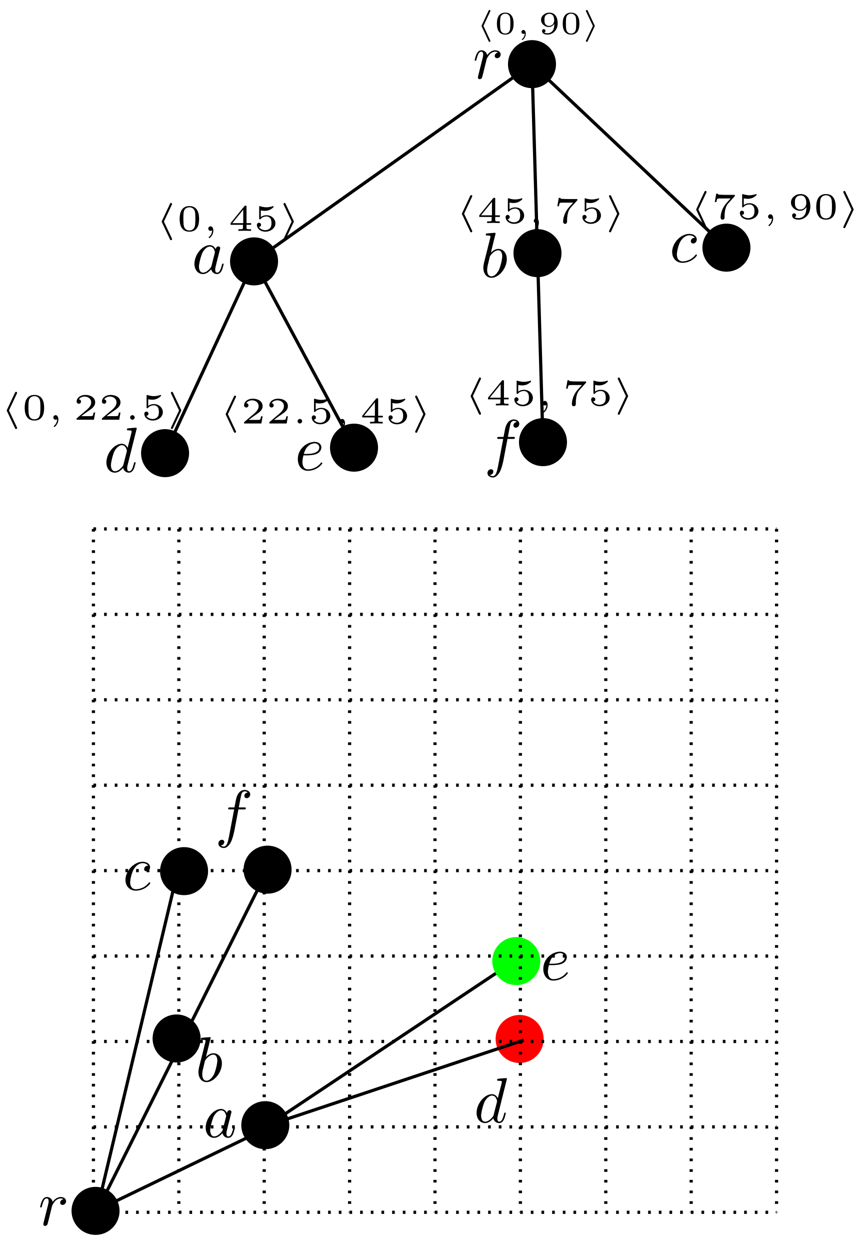

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Lemma

Let T be a rooted tree and Γ be the drawing of T produced by Algorithm-1. Let u be a node of T.

Then, the side of the sub-grid in Γ devoted to the drawing of the sub-tree T_u rooted at u is bounded by:

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Theorem
Given a rooted n-vertex Tree T, Algorithm-1 produces a monotone grid drawing using a grid of size at most $n \times n$.

On-going work

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- All work on monotone tree drawings assumes:
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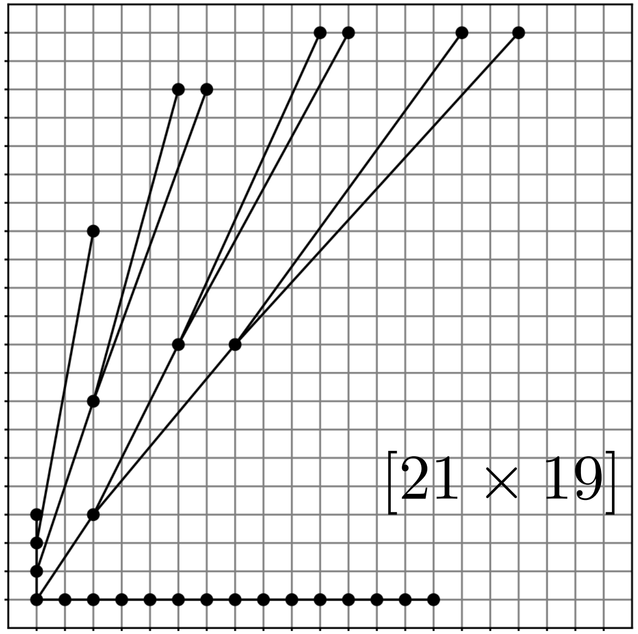

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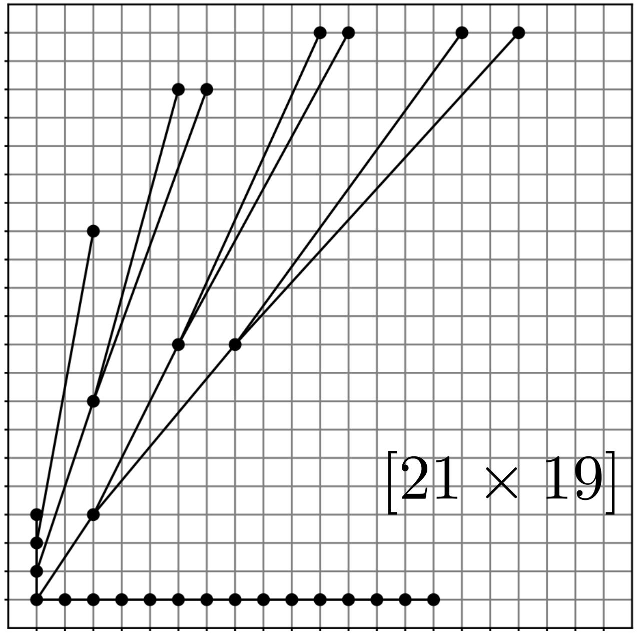

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  1. Drawings on smaller grids?
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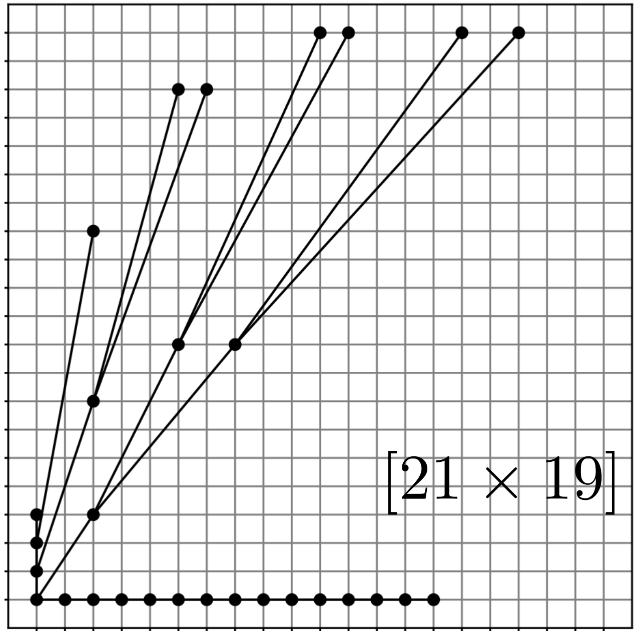

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Thank you!