Julius-Maximilians-
UNIVERSITÄT
WÜRZBURG

Chair for
INFORMATICS I
Efficient Algorithms and
Knowledge-Based Systems

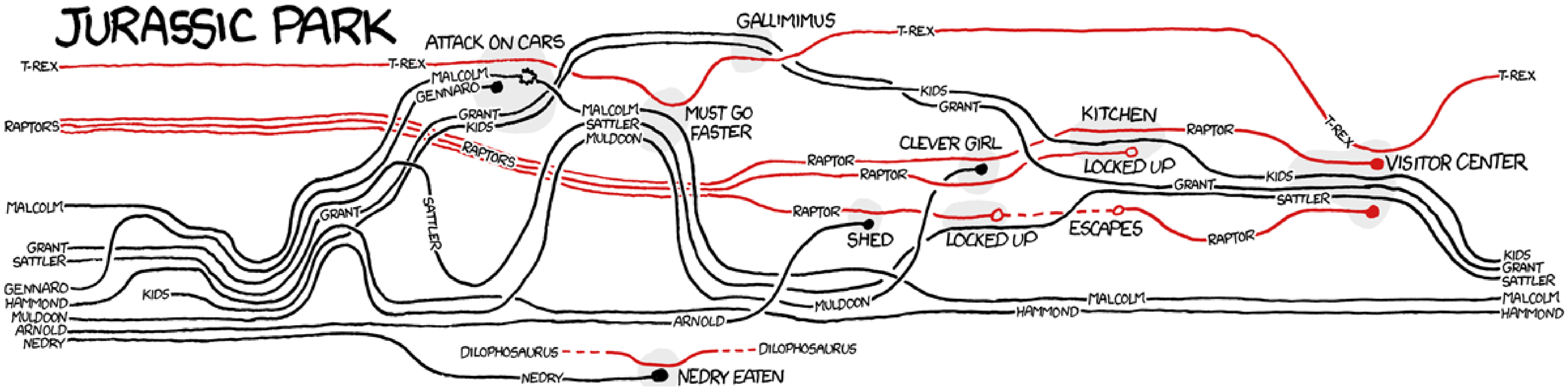
Institute for Informatics

Computing Storyline Visualizations with Few Block Crossings

Thomas C. van Dijk
Peter Markfelder

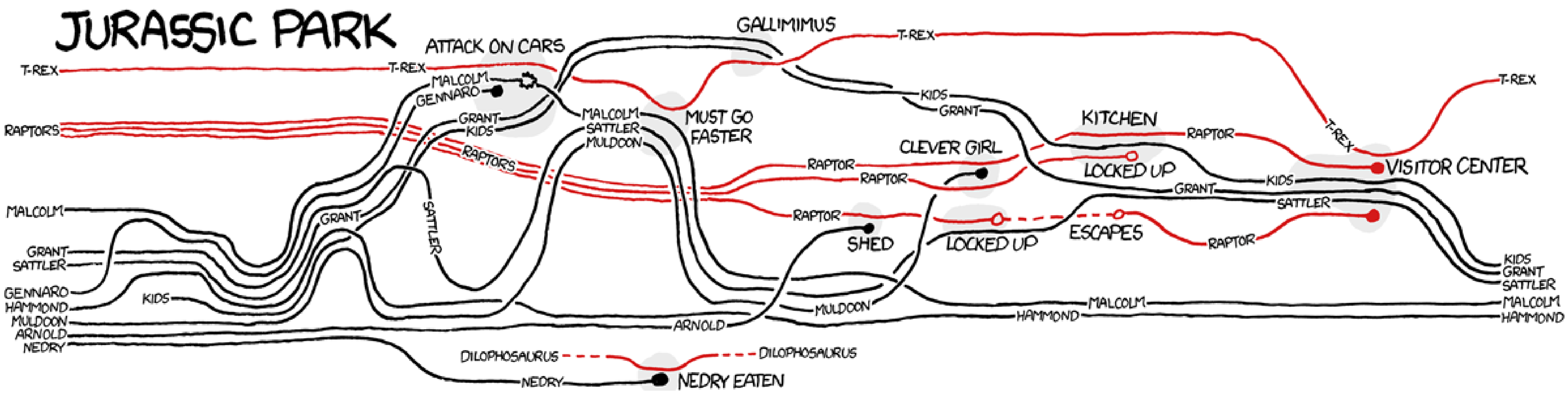
Fabian Lipp
Alexander Wolff

Storyline Visualizations



[https://xkcd.com/657/]

Storyline Visualizations



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Design considerations for storyline drawings:

- Line wiggles
- White space gaps
- Line crossings

[Tanahashi & Ma, TVCG12]

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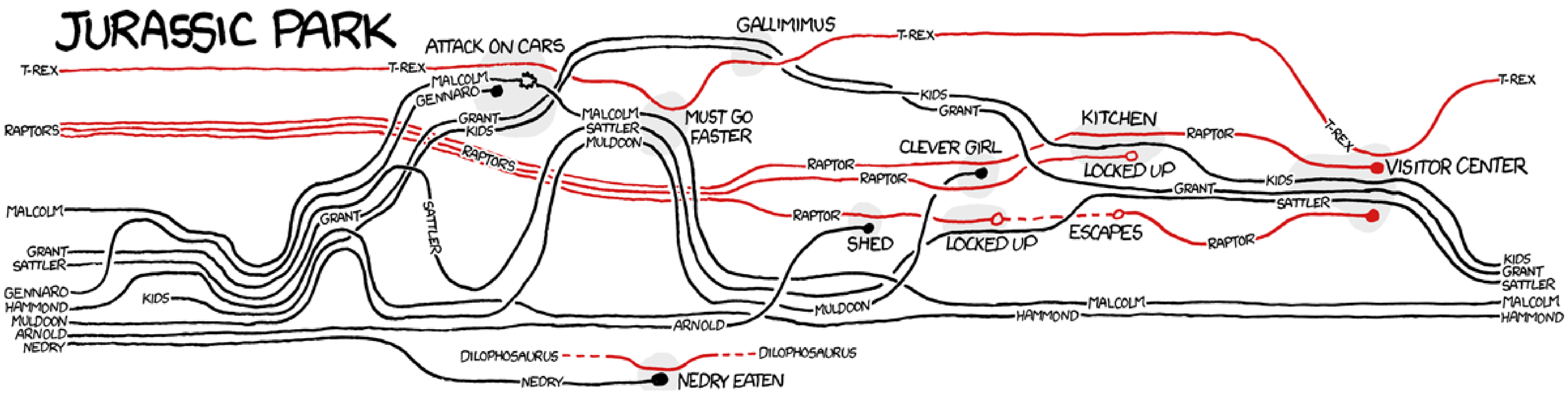

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Related Work: Simple Crossings

Aim: Drawing with a minimum number of crossings

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[Kostitsyna et al., GD'15]

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- upper and lower bounds for some cases with pairwise meetings

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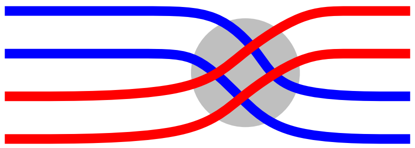
- upper and lower bounds for some cases with pairwise meetings

- ILP solving the problem [Gronemann et al., GD'16]

Related Work: Block Crossings



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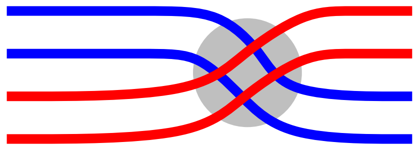
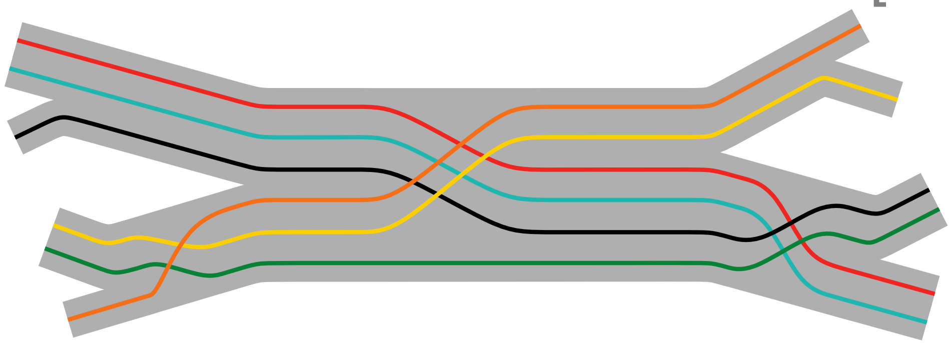


- Block crossings for metro lines [Fink, Pupyrev, Wolff; 2015]



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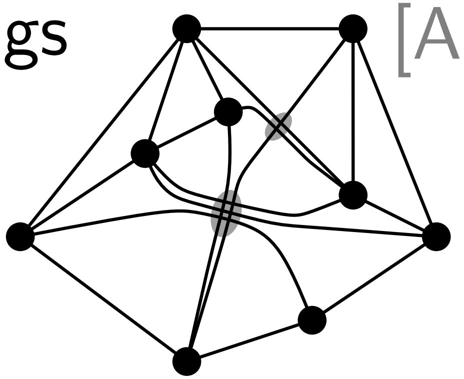

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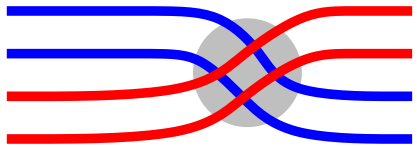

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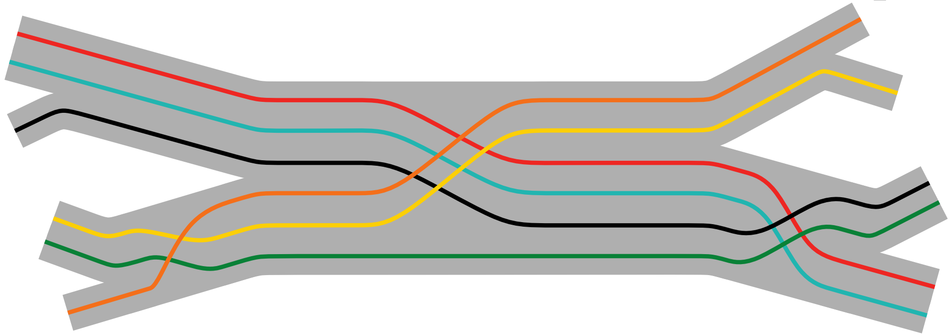

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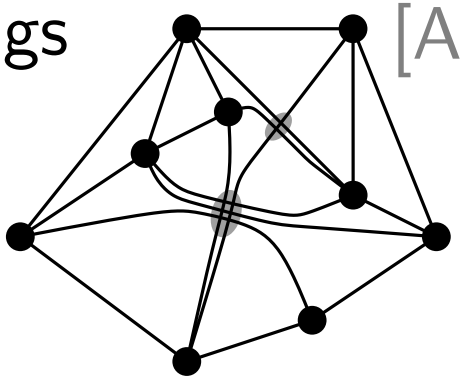

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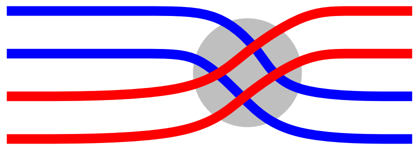

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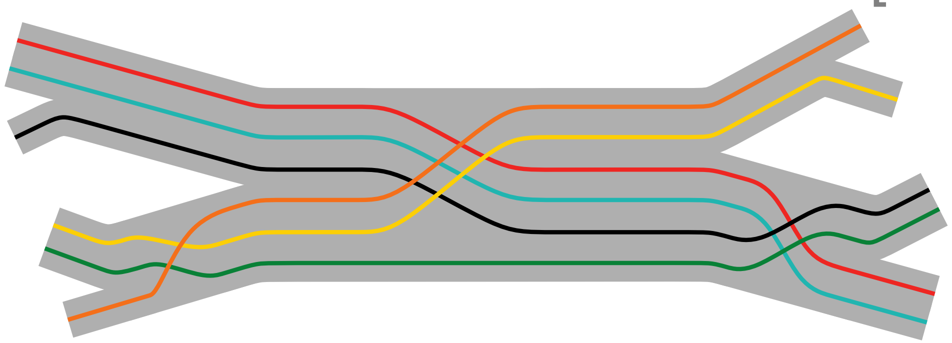

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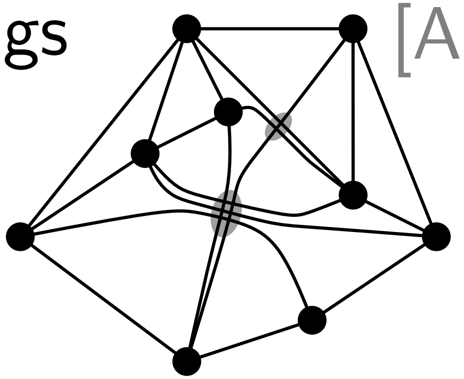

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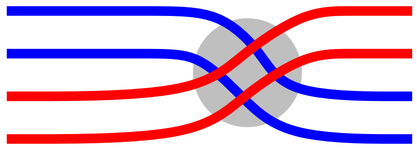

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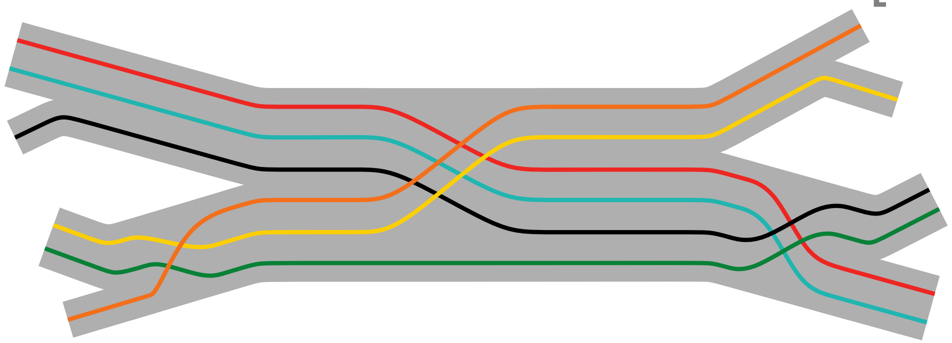

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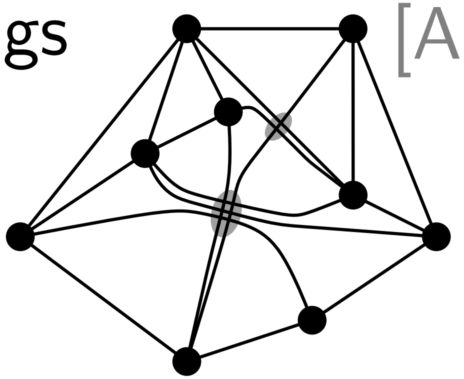

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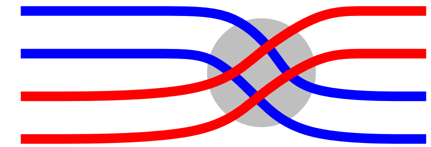

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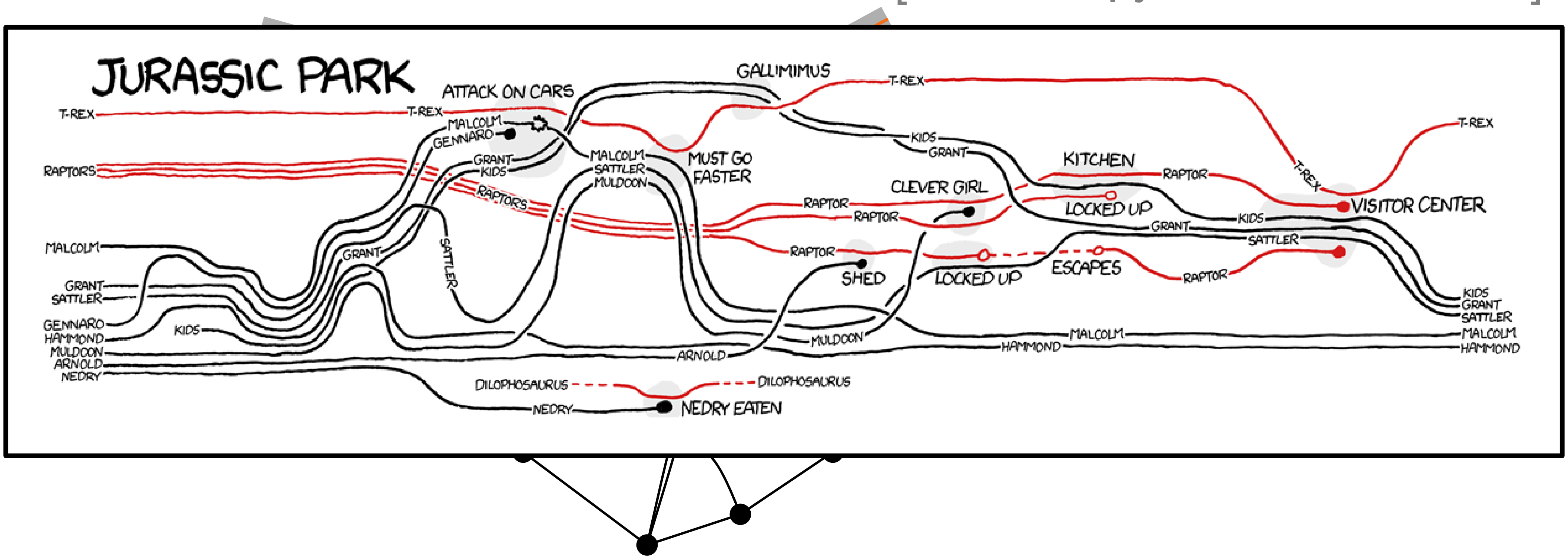

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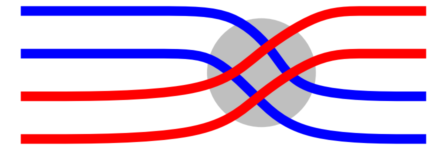

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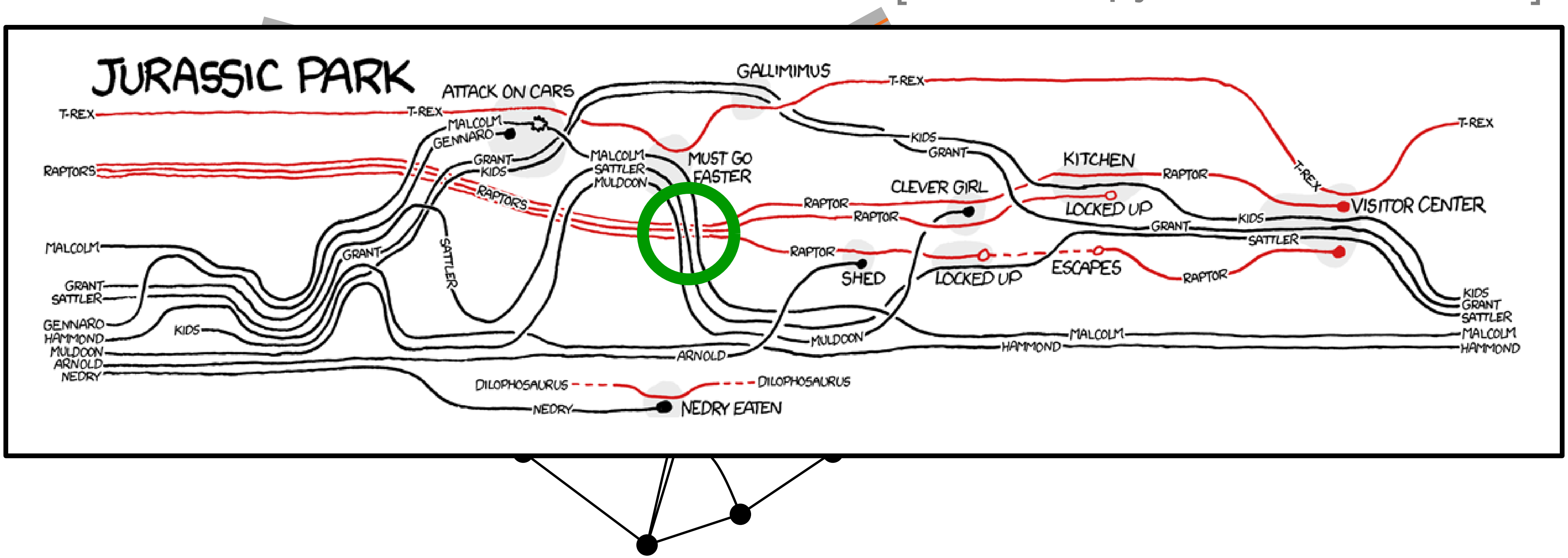

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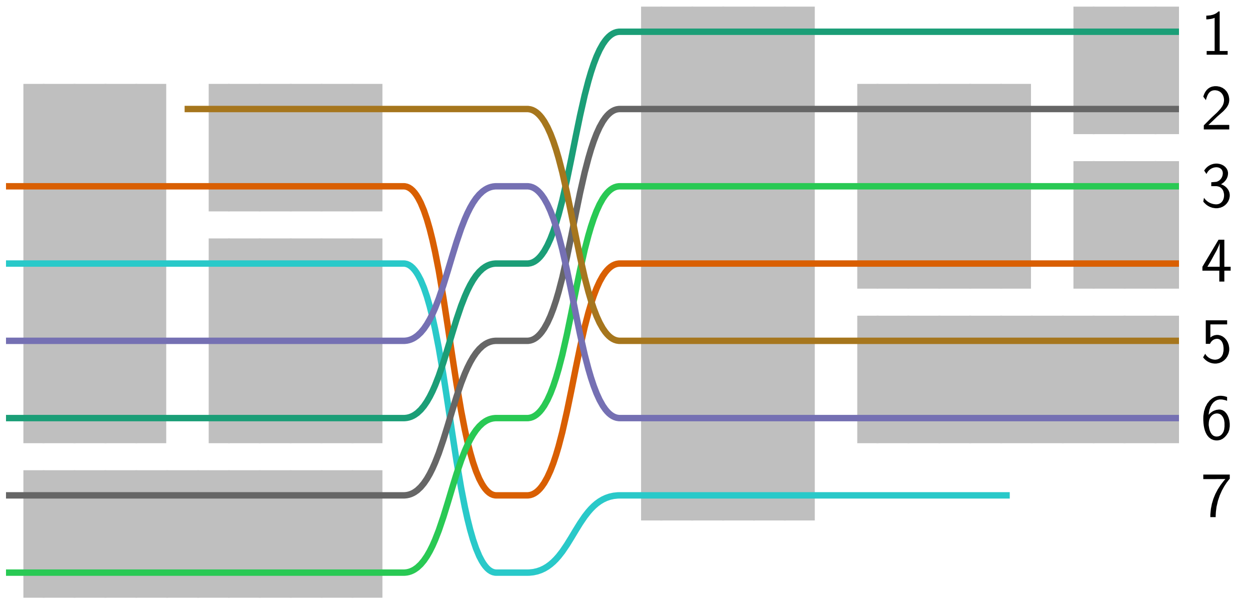
Meeting: set of characters, start and end time



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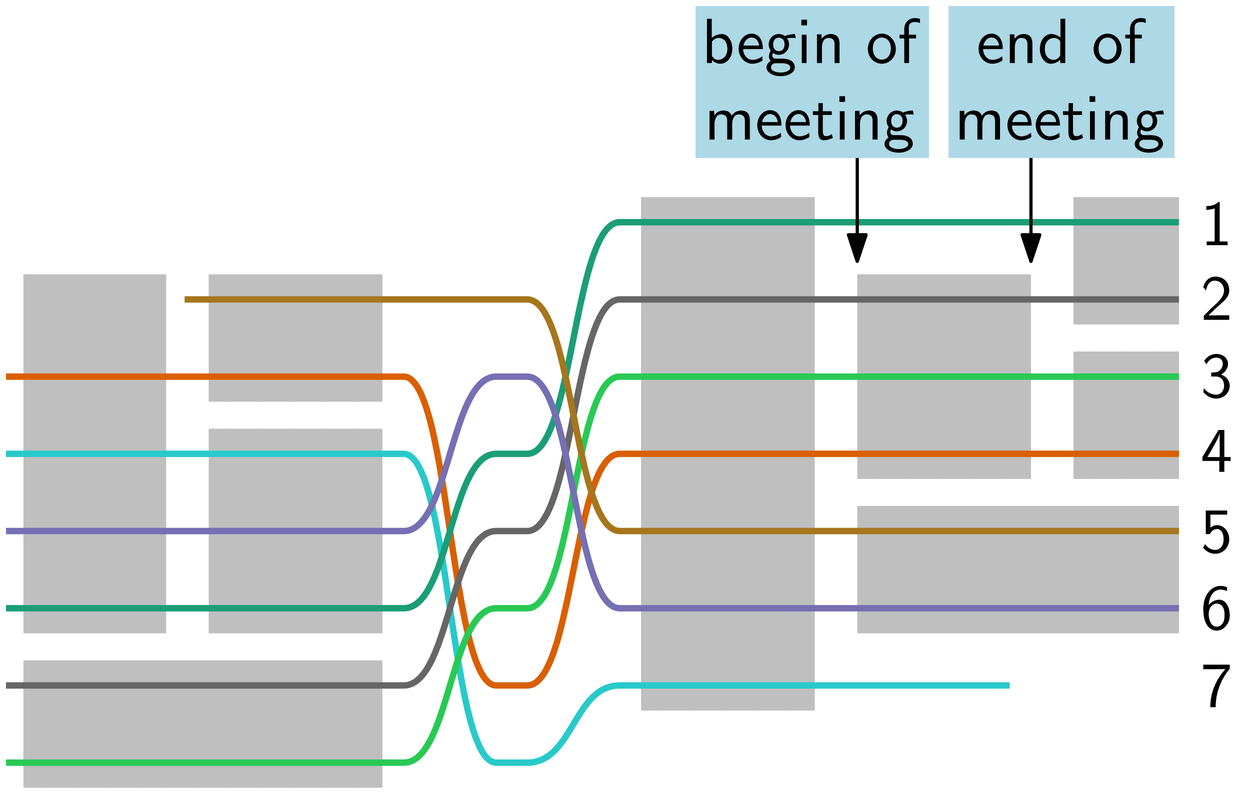

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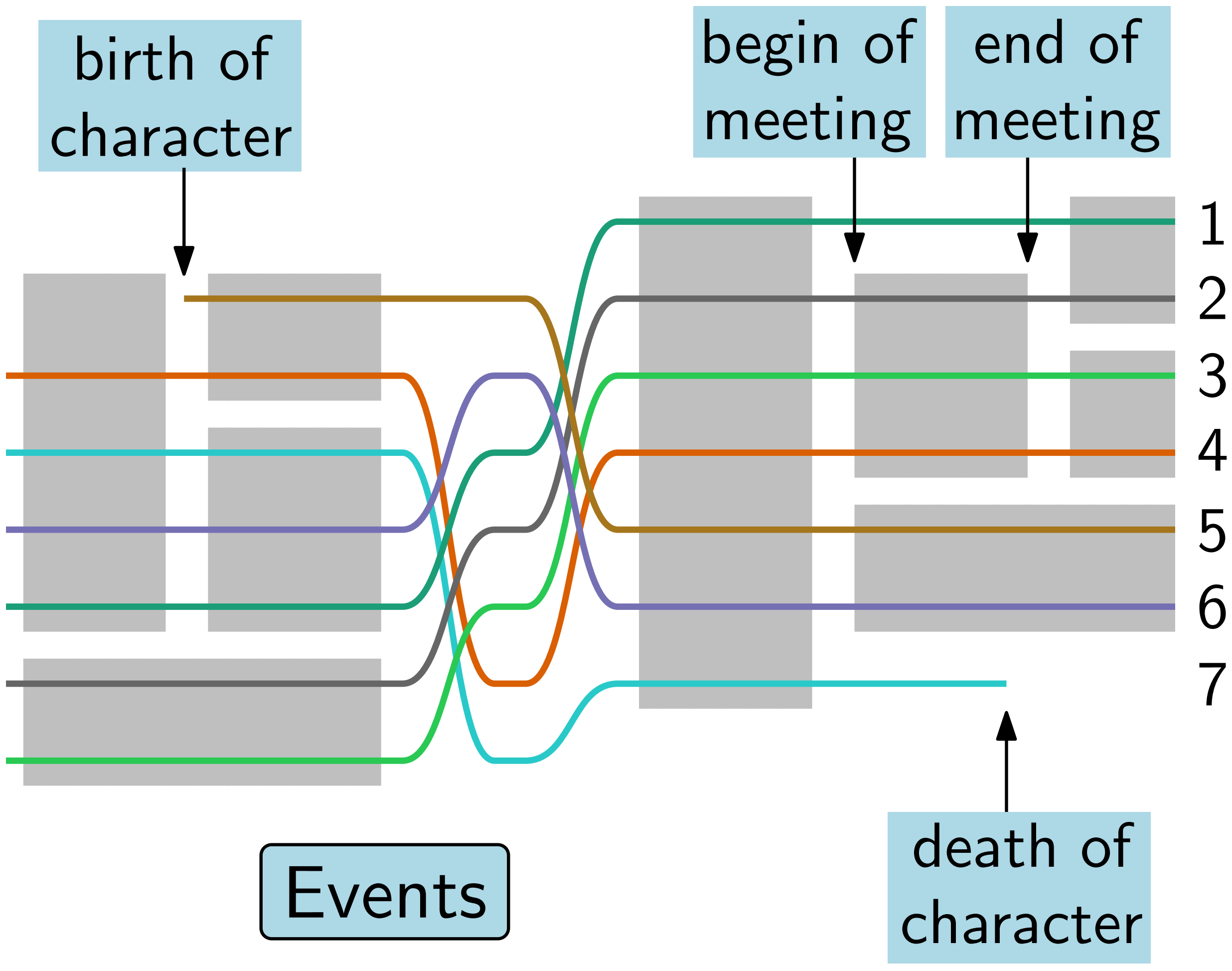

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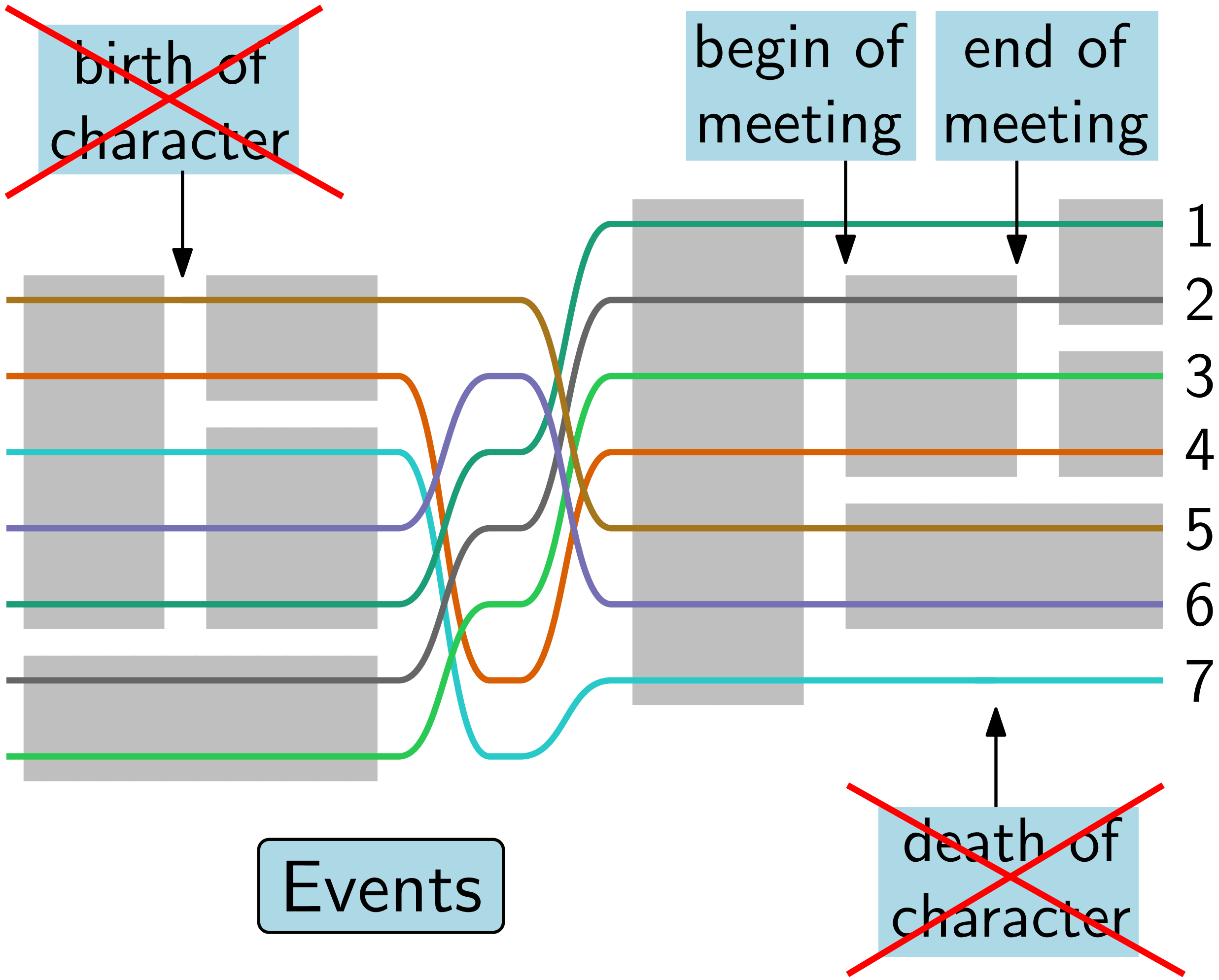

No births and deaths in this talk – but in the paper.

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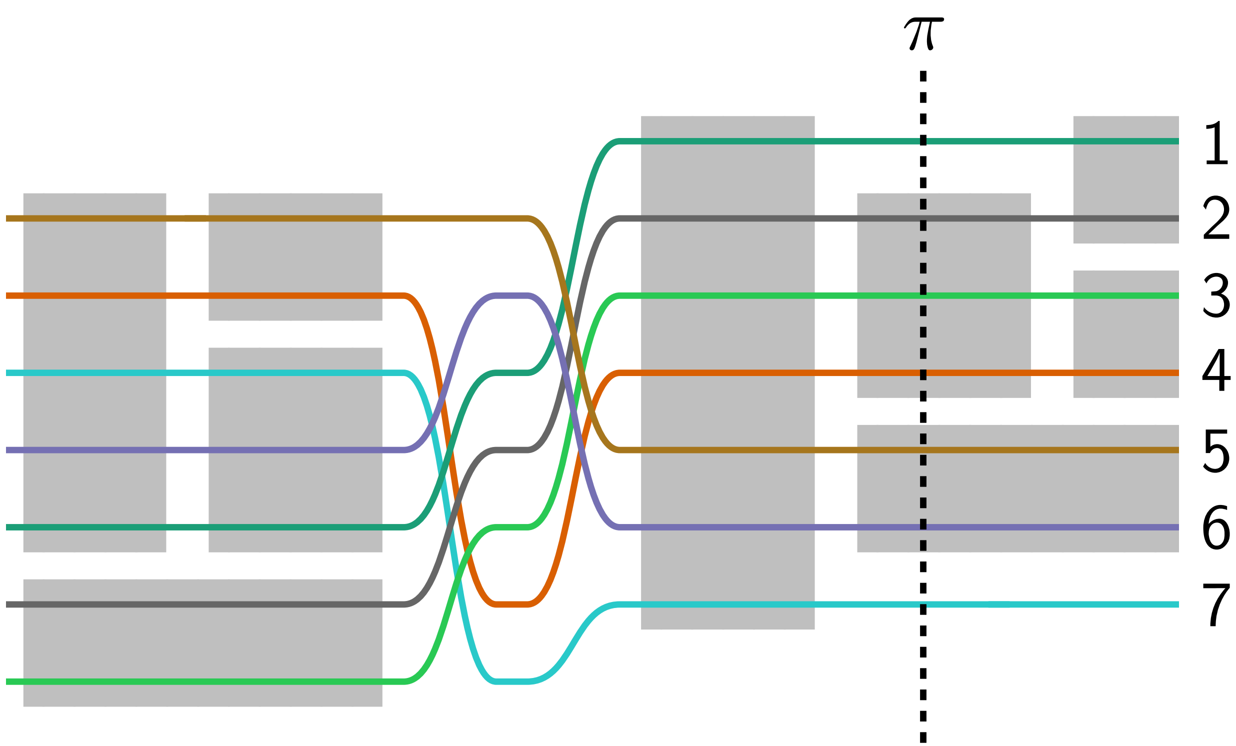

Permutation π supports meetings $\{2, 3, 4\}$ and $\{5, 6\}$

$=$

Sets $\{2, 3, 4\}$ and $\{5, 6\}$ each form a contiguous group in π

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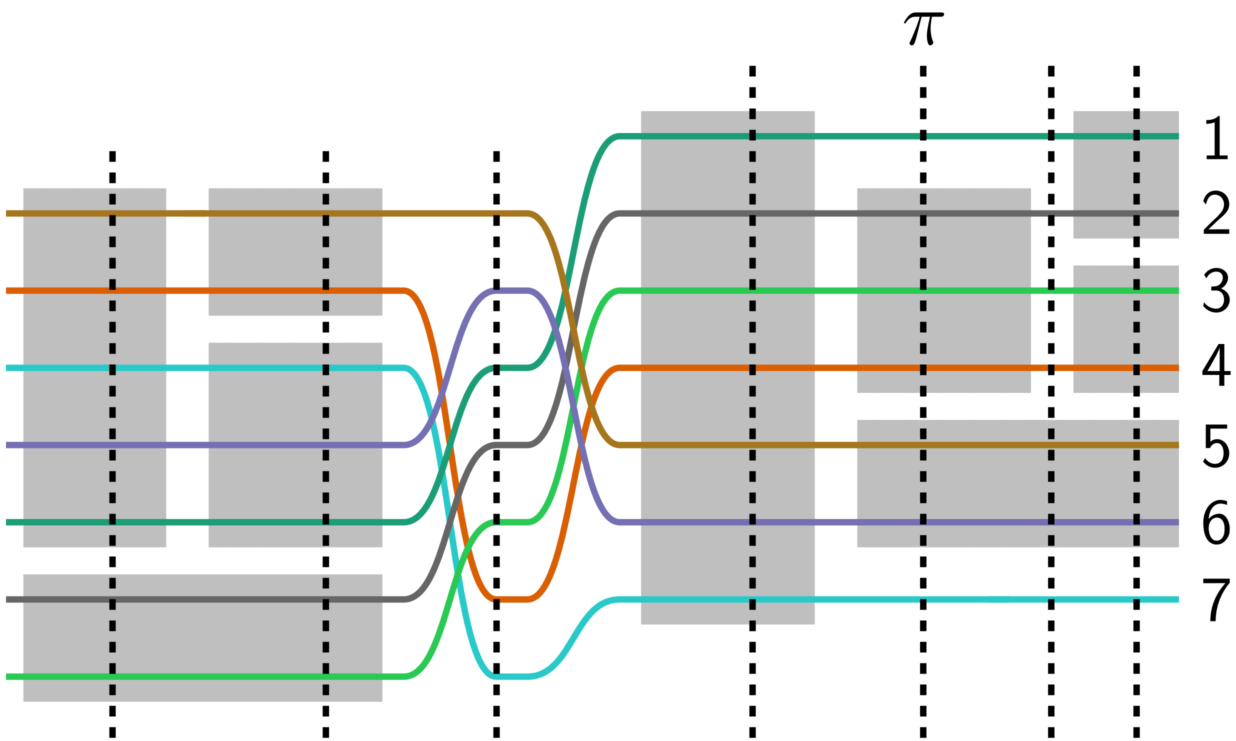

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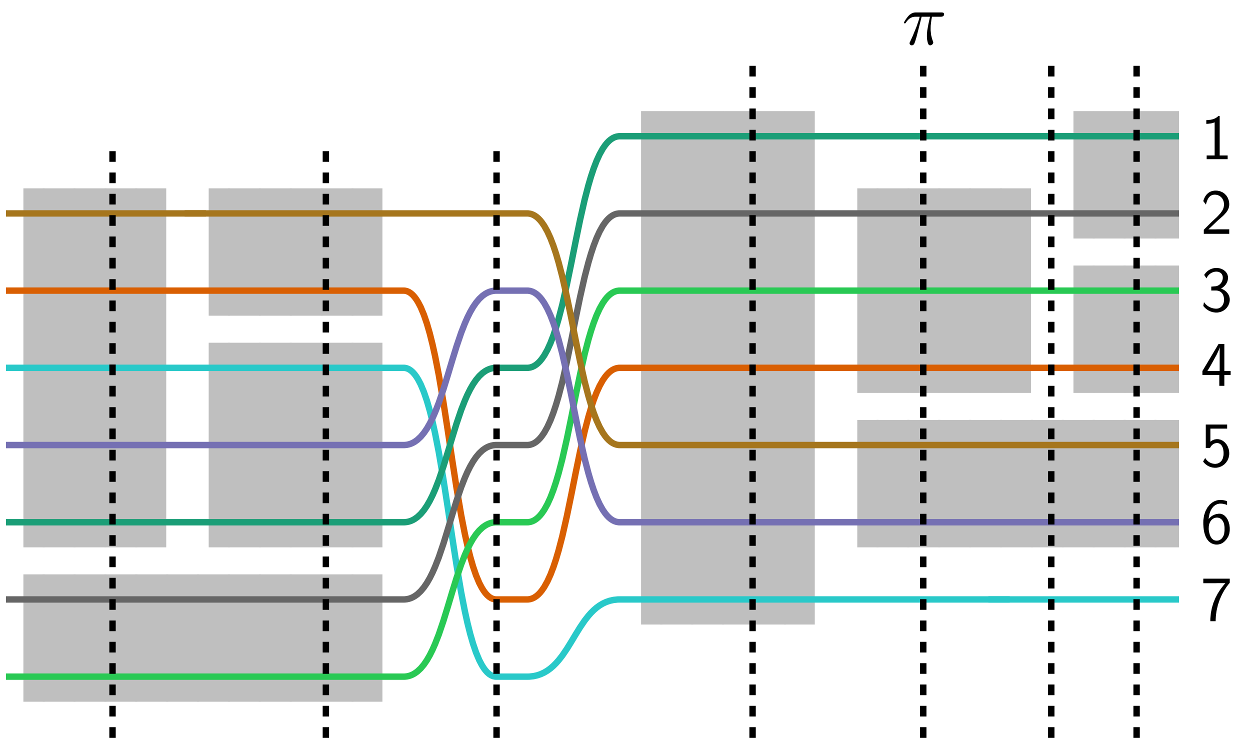

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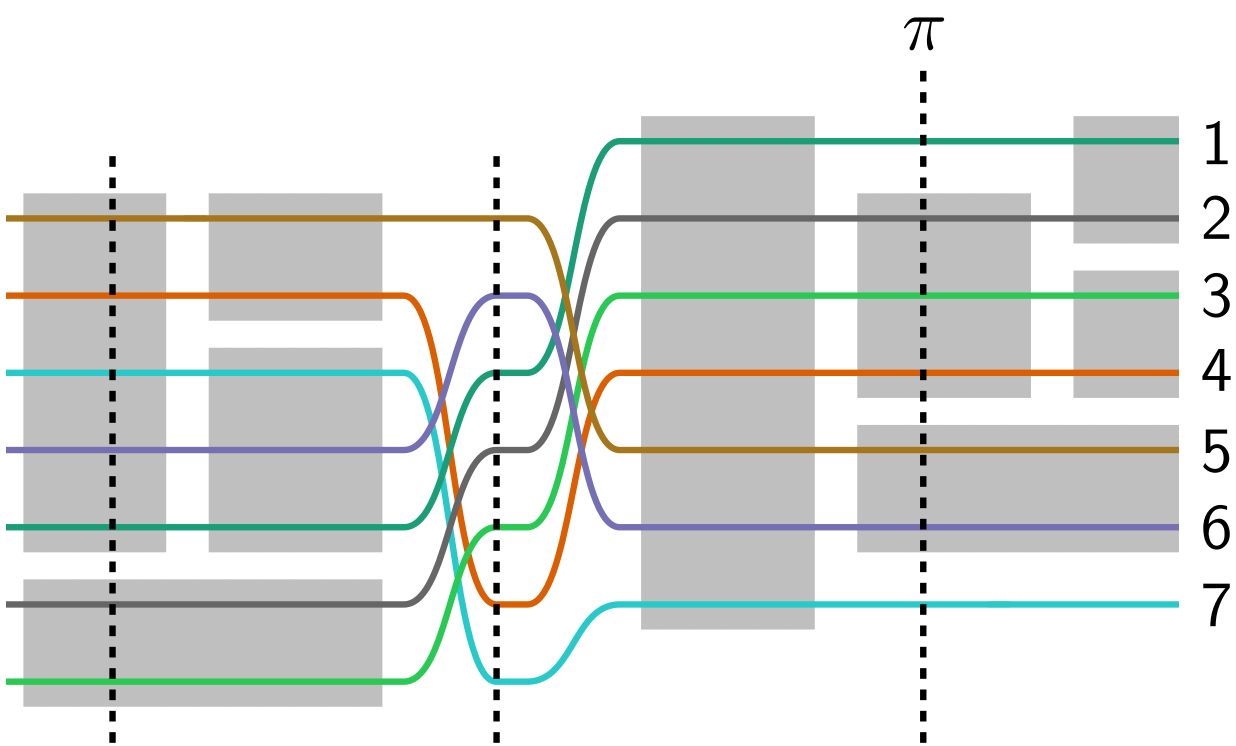

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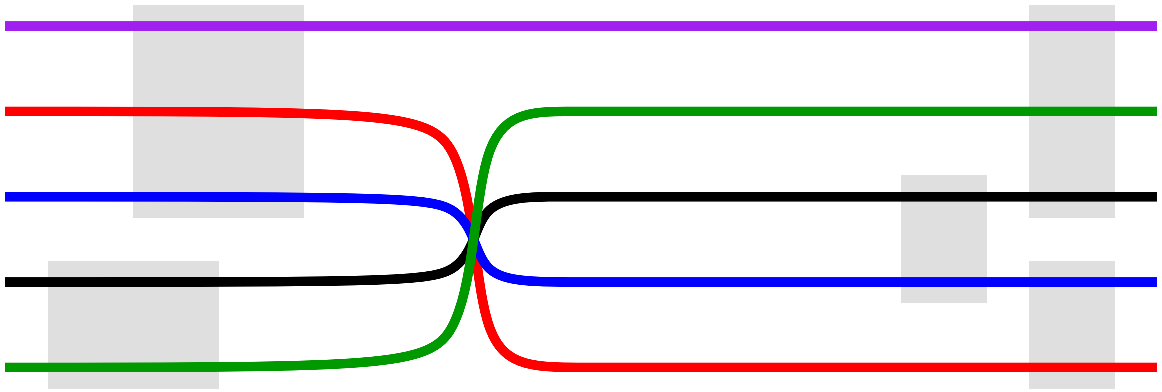
Approach of Gronemann et al.

Reduce to MLCM-TC (Multi-Layer Crossing Minimization Problem with Tree Constraints)



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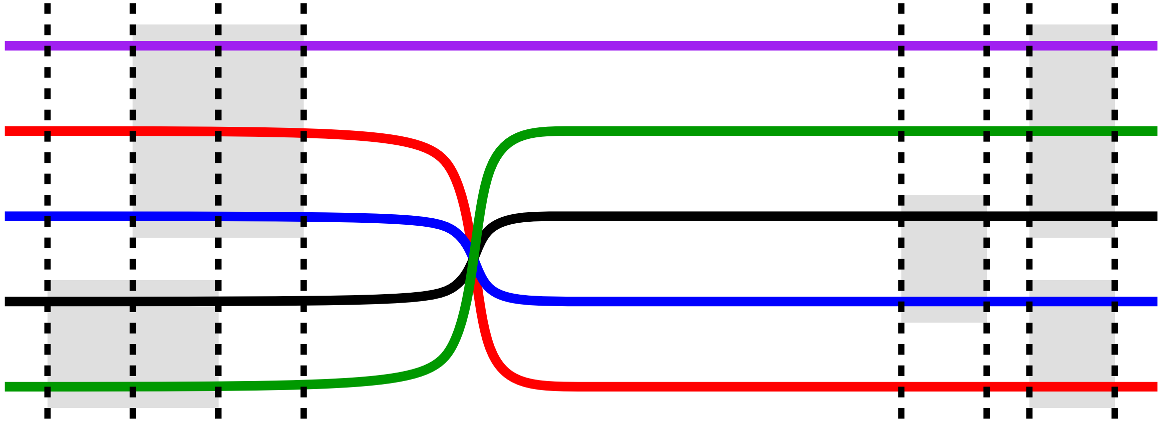

Each event corresponds to a layer

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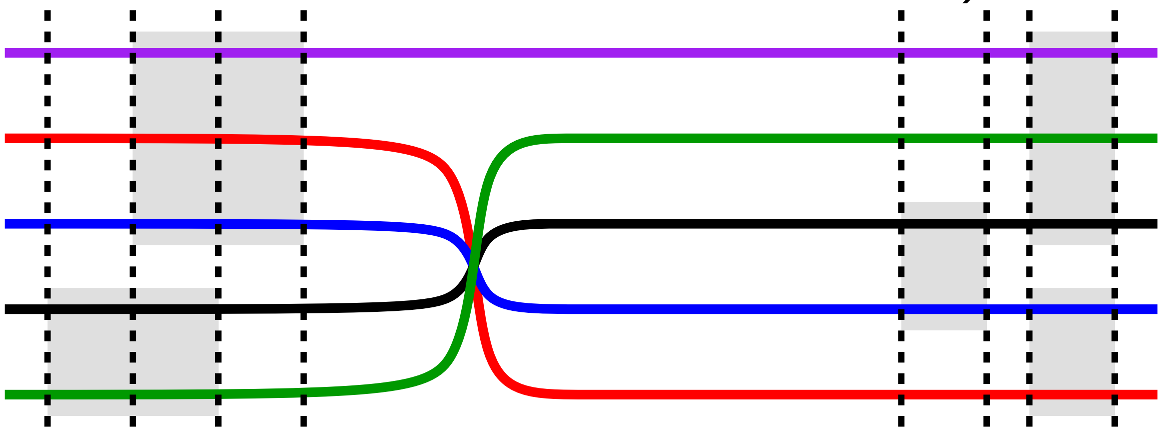

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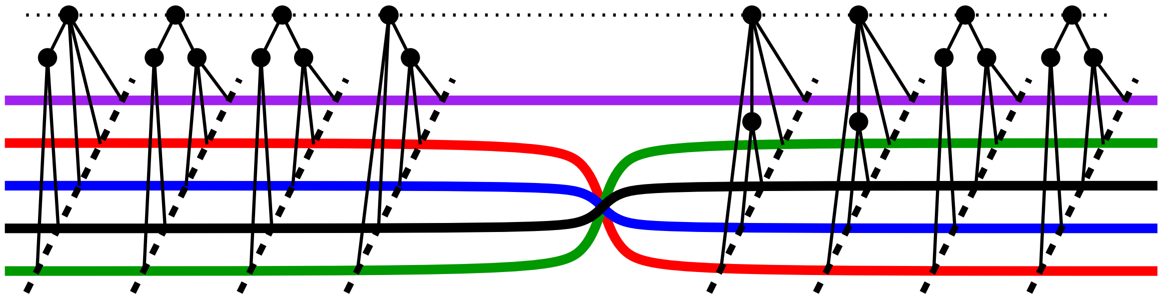

Trees describe neighborship between characters

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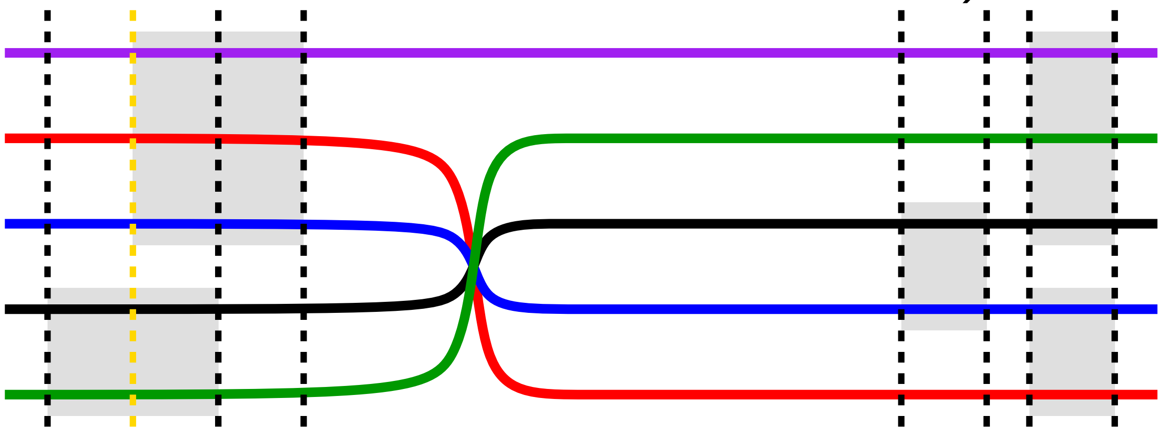

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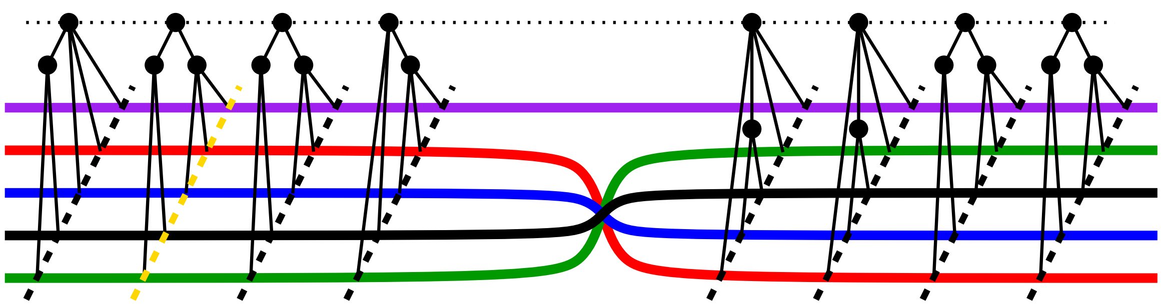

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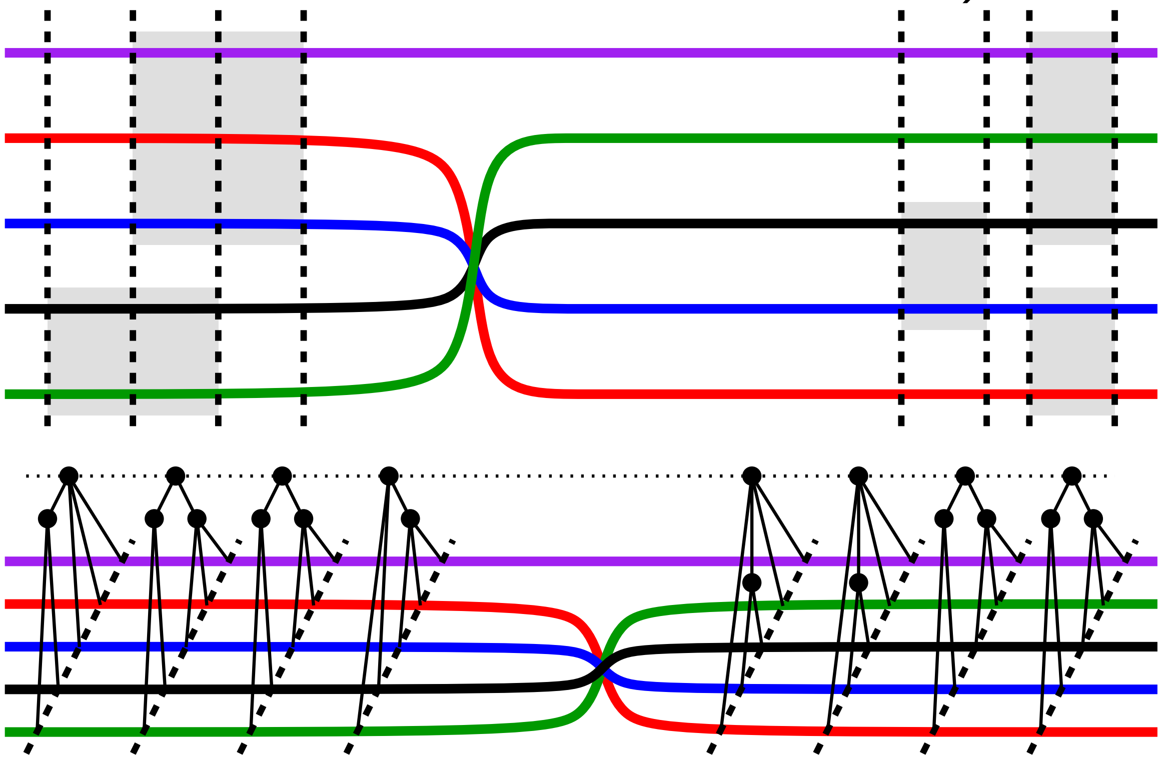

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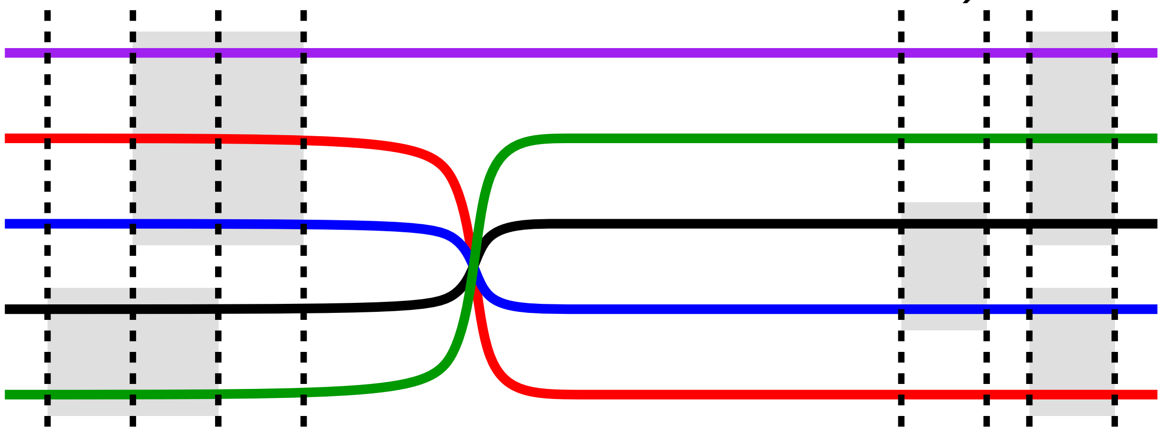

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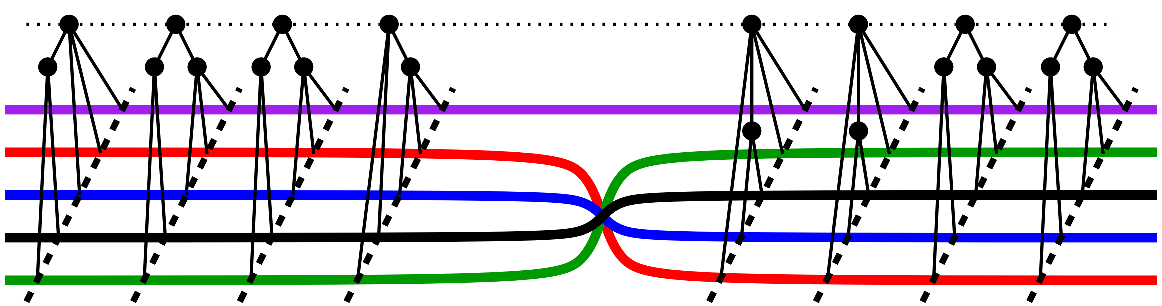

Trees describe neighborship between characters

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Use ILP to solve MLCM-TC

Describing Permutations

Variables describing the order in each permutation: x_{ij}^{r}

layer

characters

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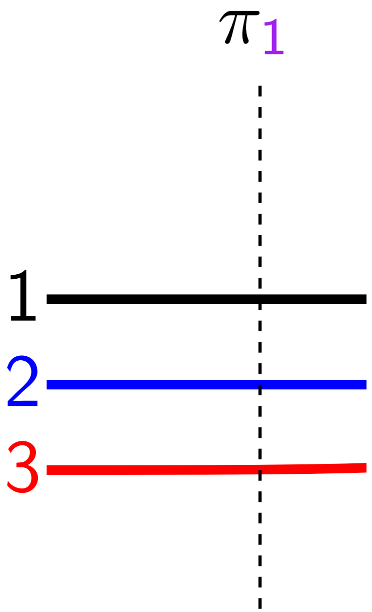

Describing Permutations

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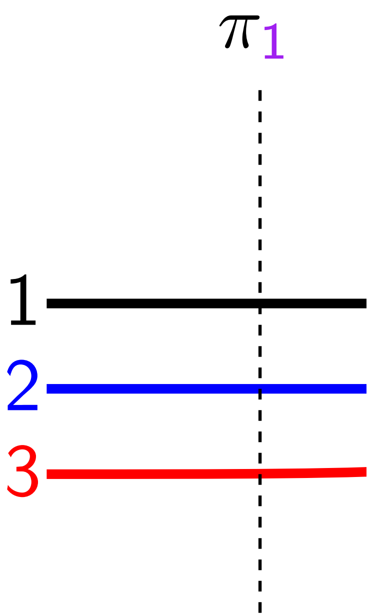

$$x_{12}^{1} = 1$$

Describing Permutations

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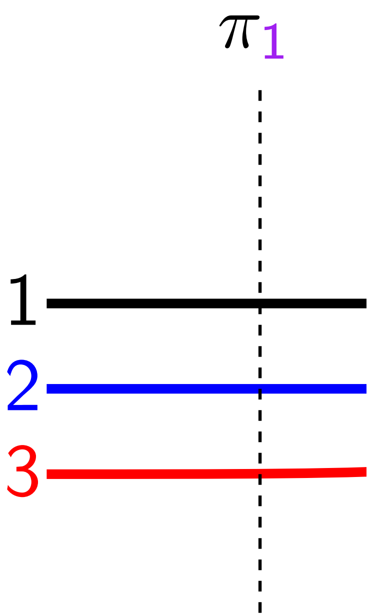

$x_{12}^{1} = 1$

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Describing Permutations

layer

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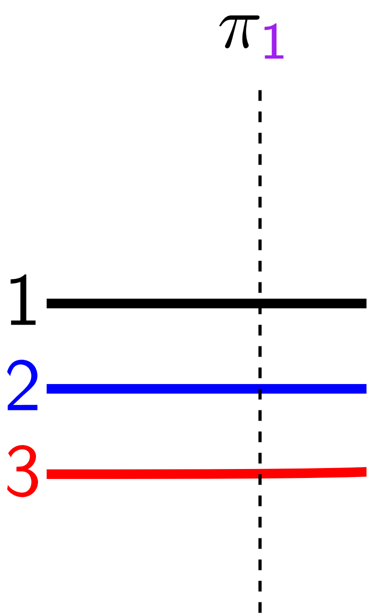

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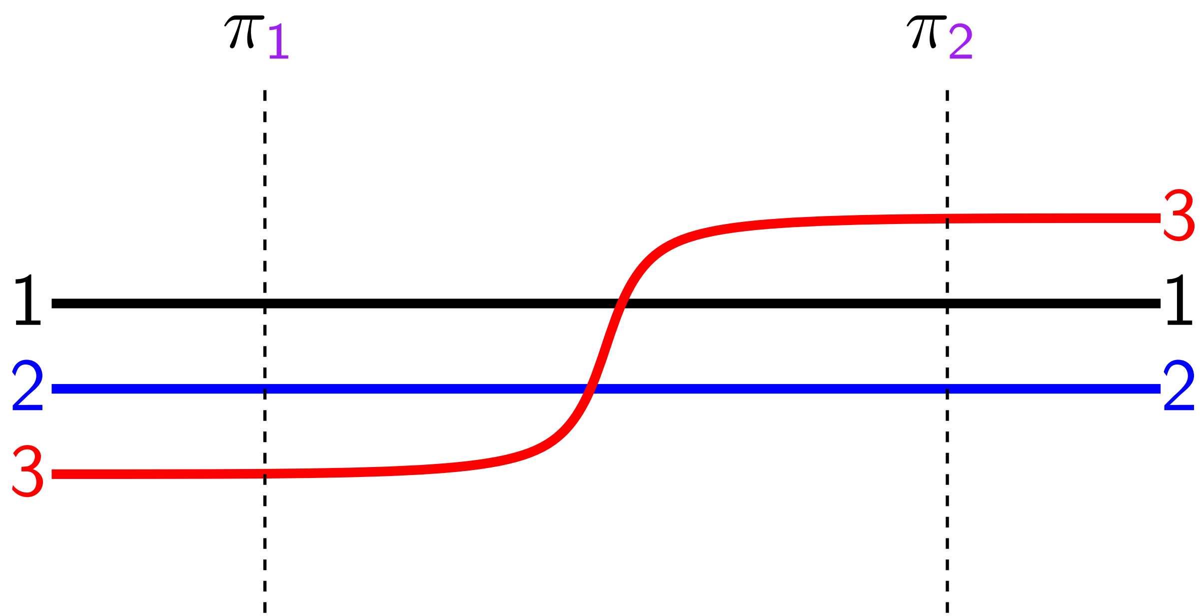

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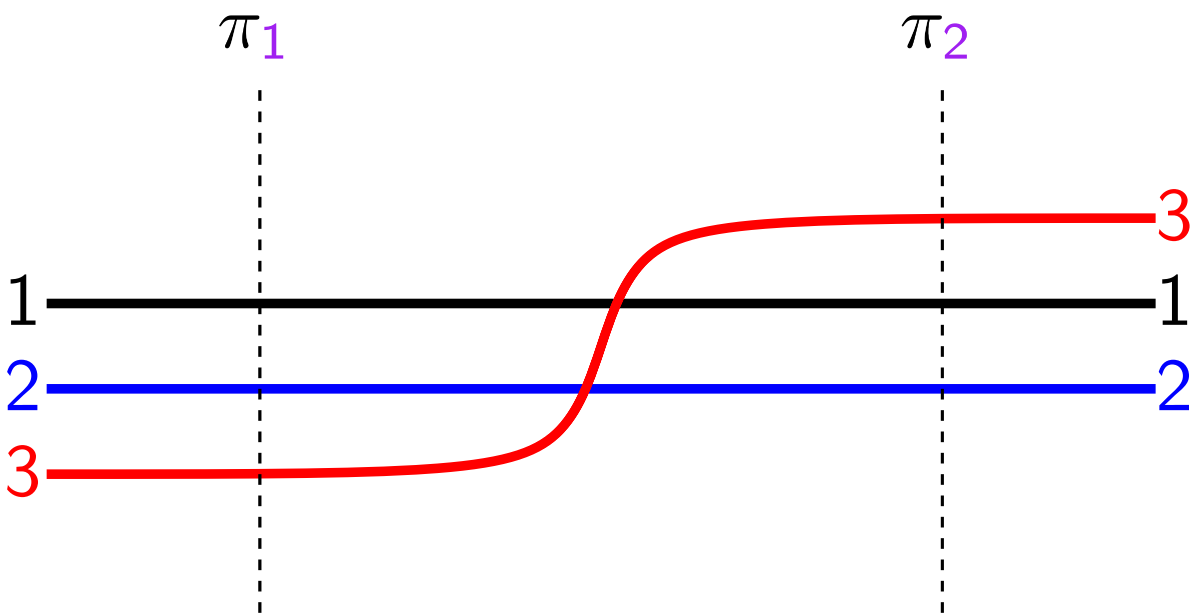

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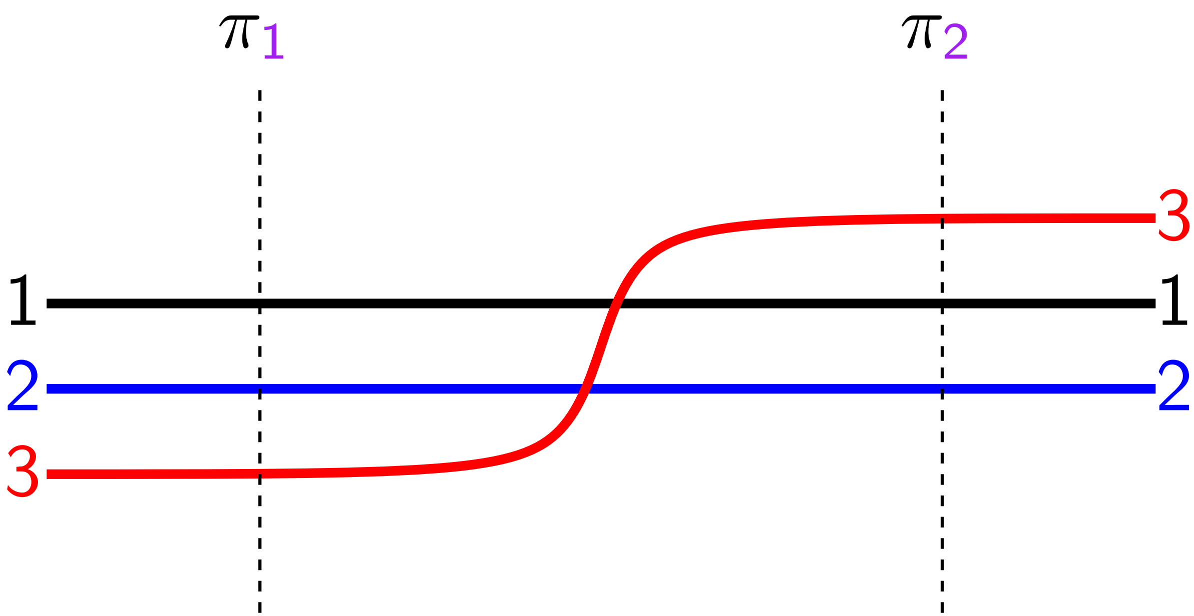

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Antisymmetry:

$x_{ij}^{r} \Leftrightarrow \neg x_{ji}^{r}$

Transitivity:

$x_{ij}^{r} \vee x_{jk}^{r} \vee x_{ki}^{r}$

$\neg x_{ij}^{r} \vee \neg x_{jk}^{r} \vee \neg x_{ki}^{r}$

Counting *Block* Crossings

Easy to count pairwise crossings using x_{ij}^r:

i and j cross after layer r: $\chi_{ij}^r \Leftrightarrow (x_{ij}^r \neq x_{ij}^{r+1})$

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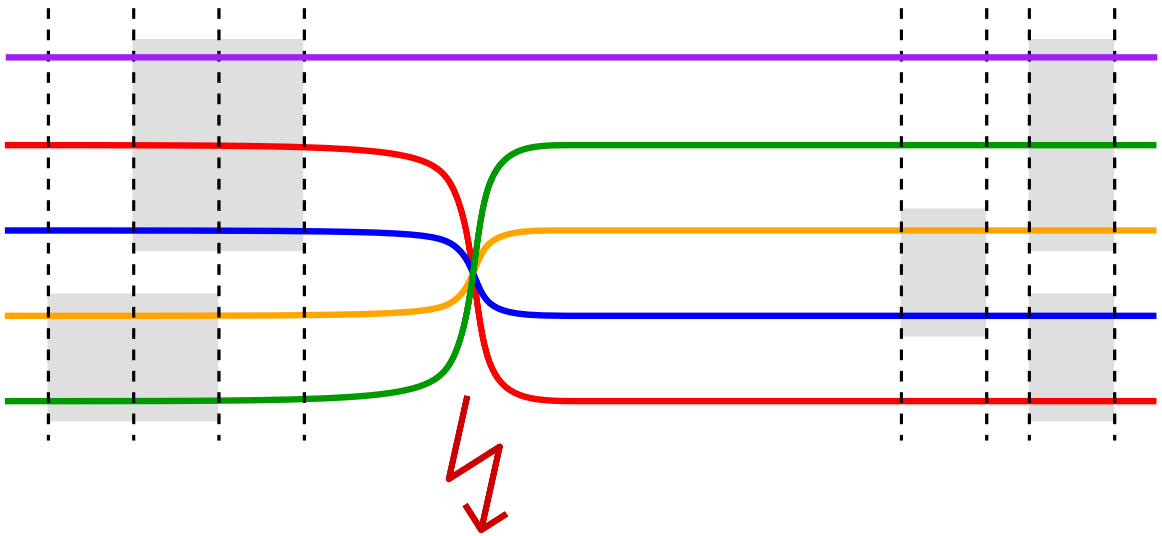
Allow at most one block crossing between two permutations



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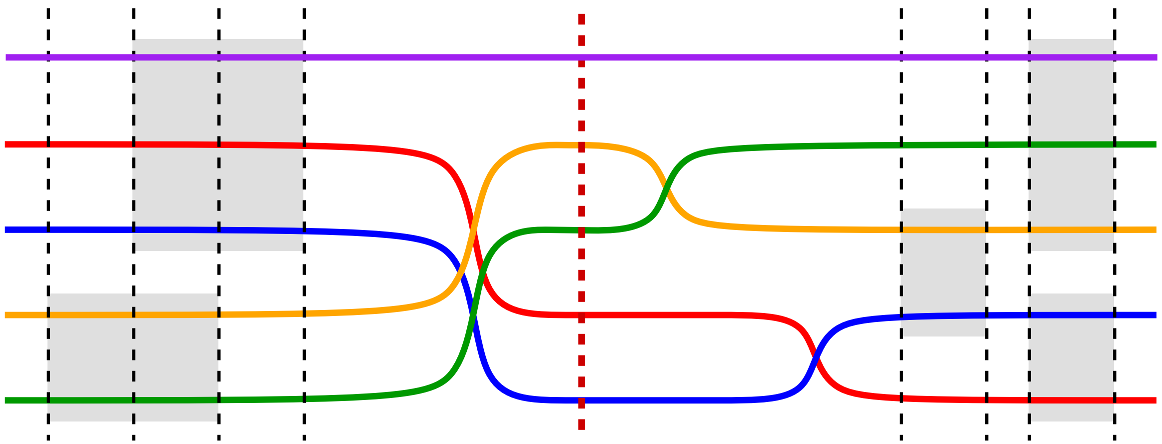

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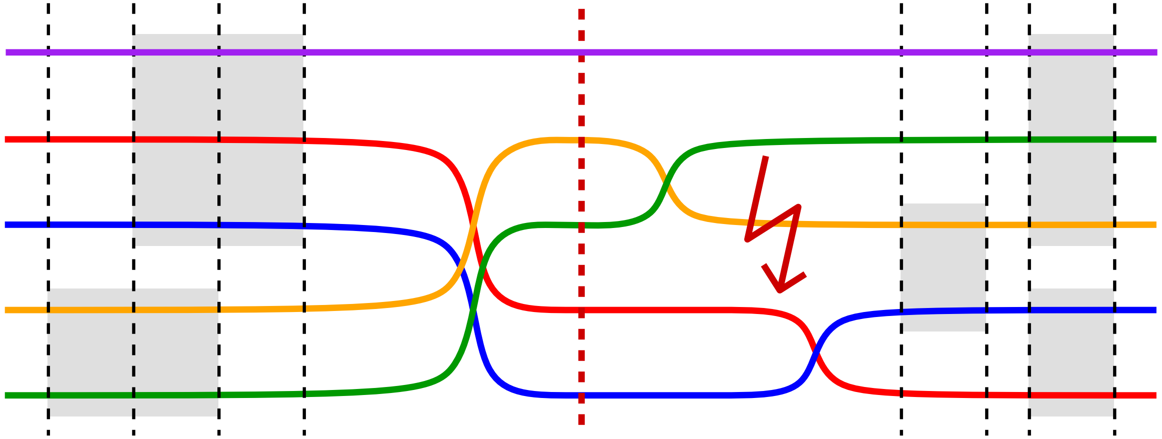

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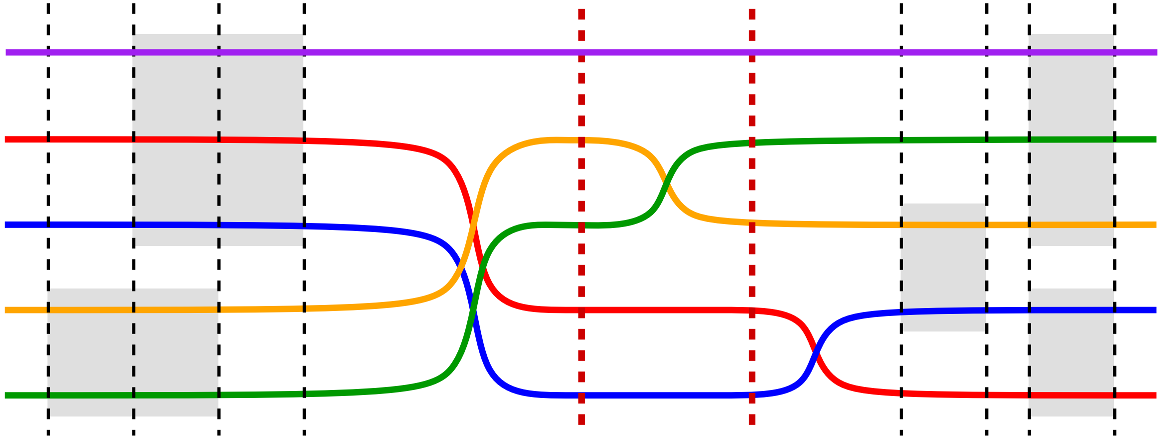

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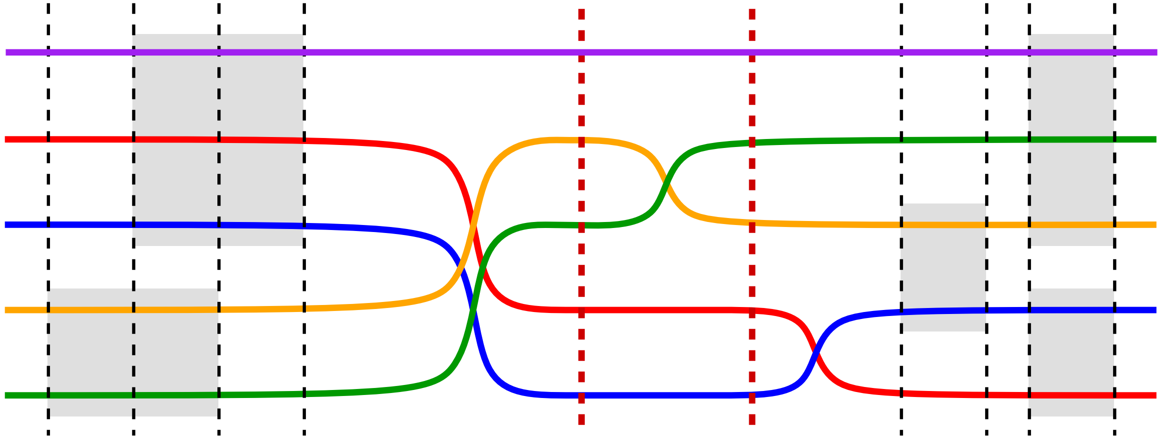
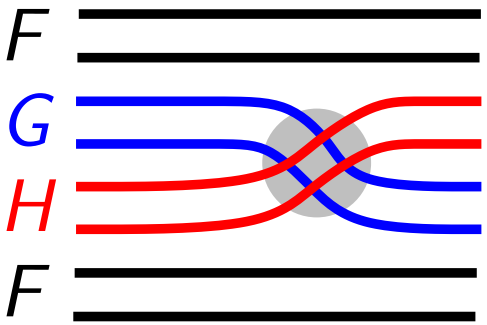


Add more permutations

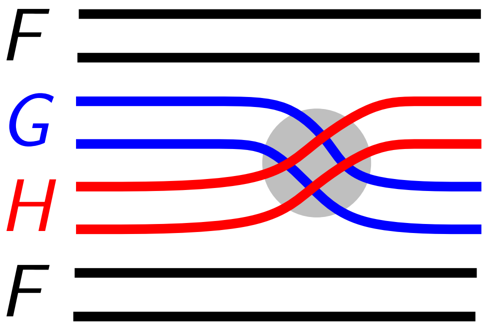
Prescribe maximum number of permutations

Describing Block Crossings



Blocks G and H cross

Describing Block Crossings



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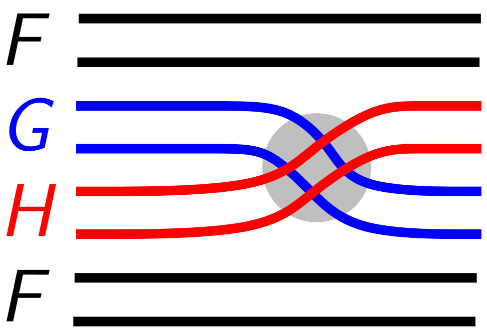

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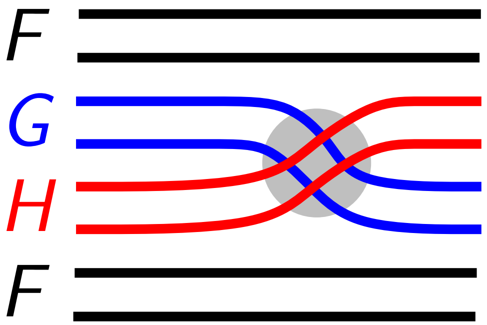

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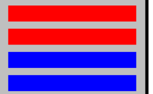

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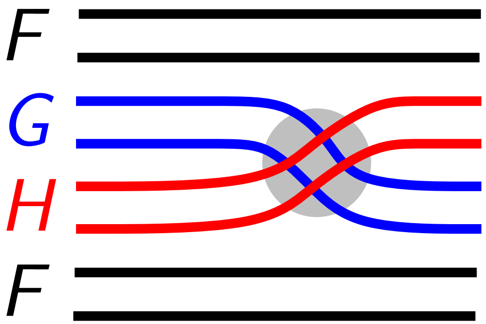

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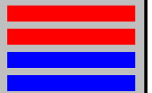

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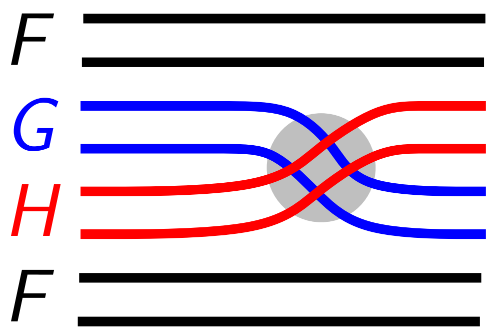

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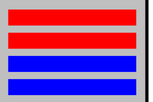

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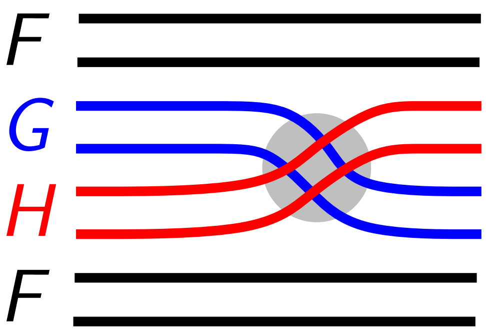

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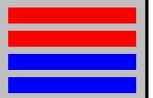

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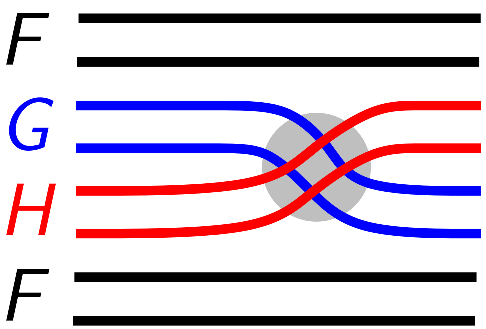

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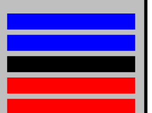

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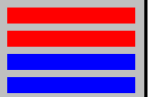

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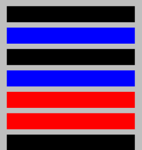
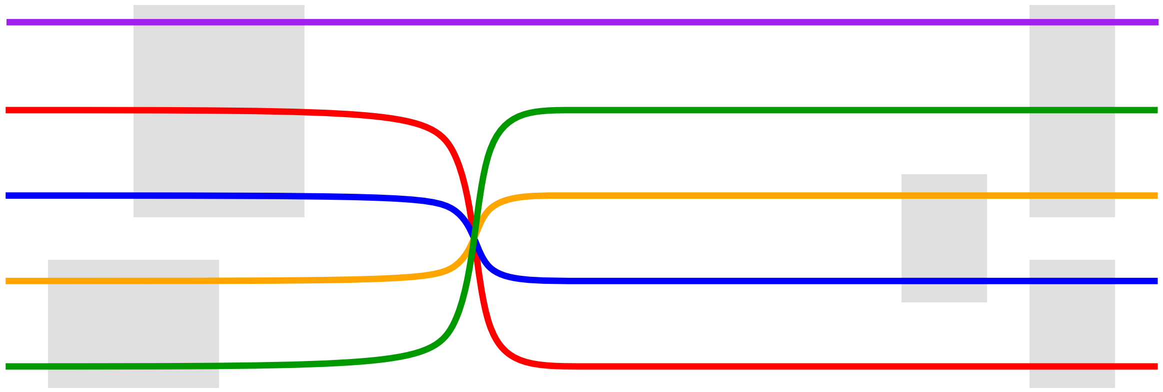


Meeting Groups



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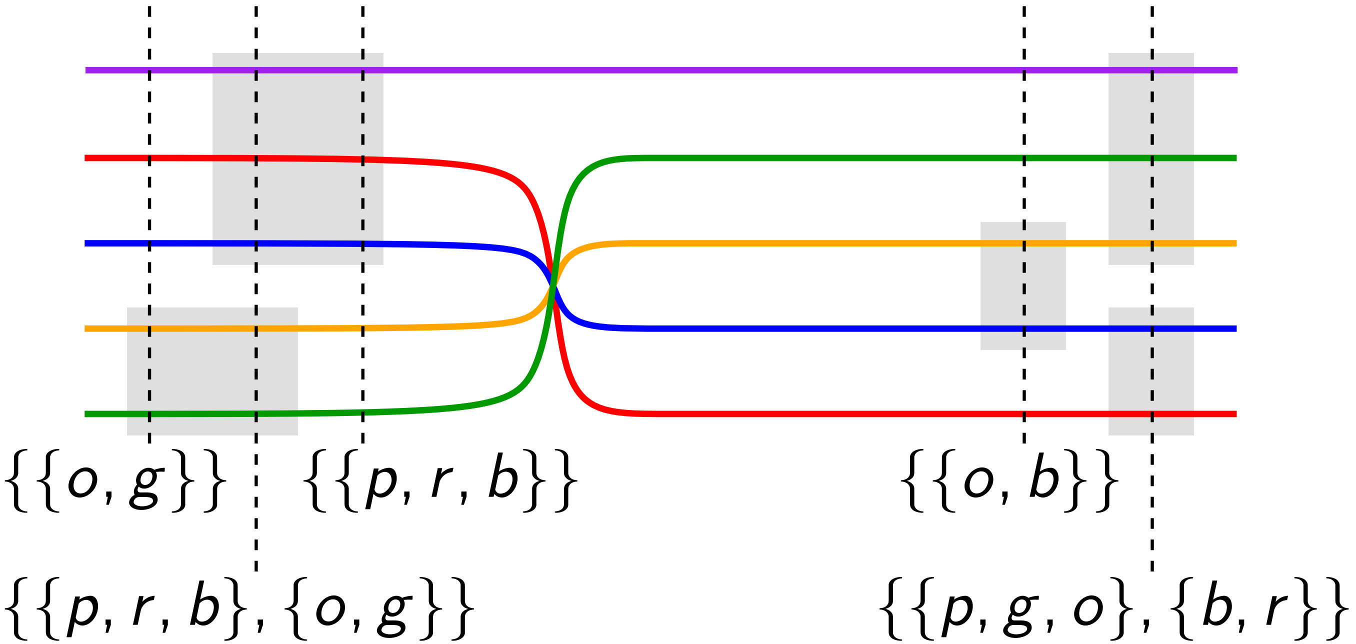

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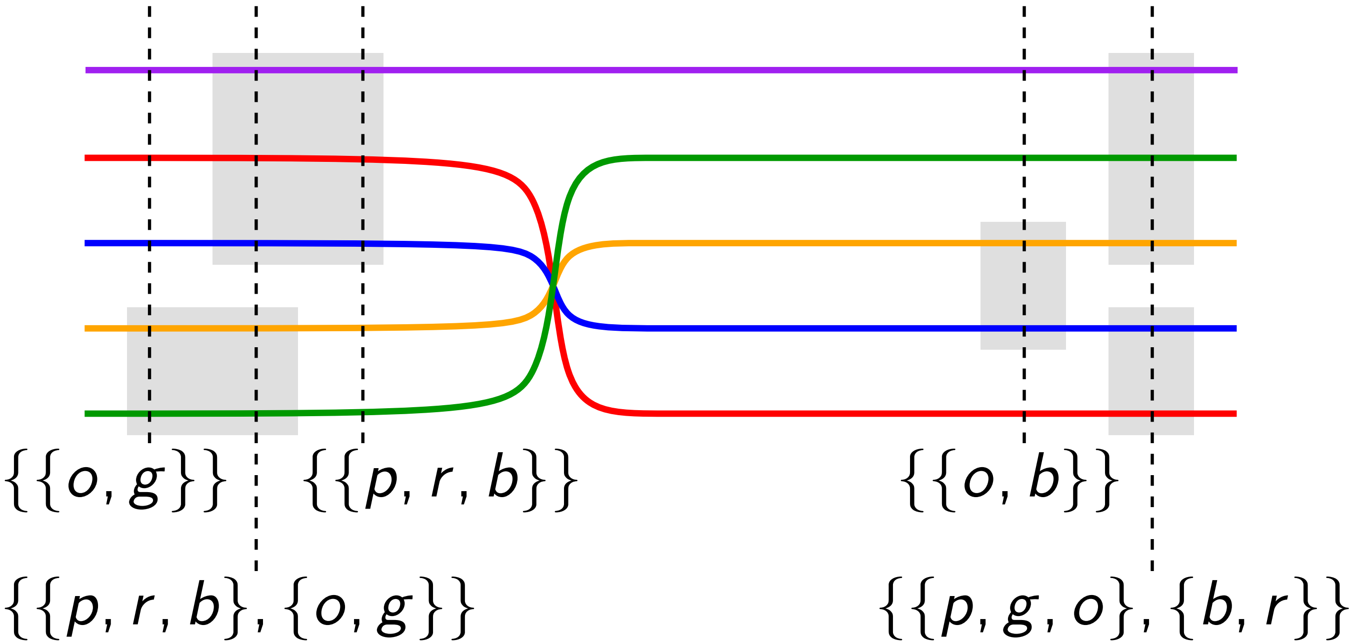

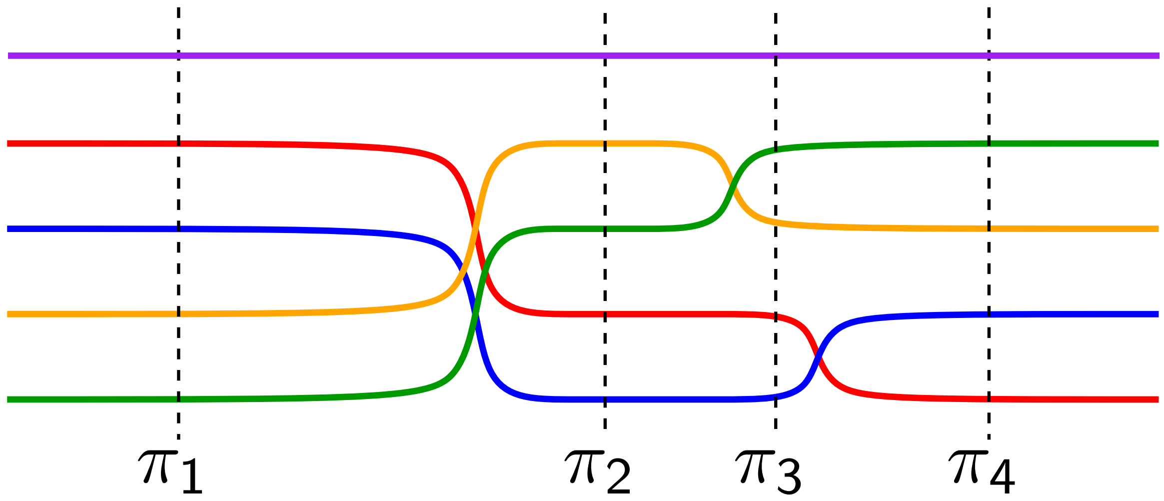

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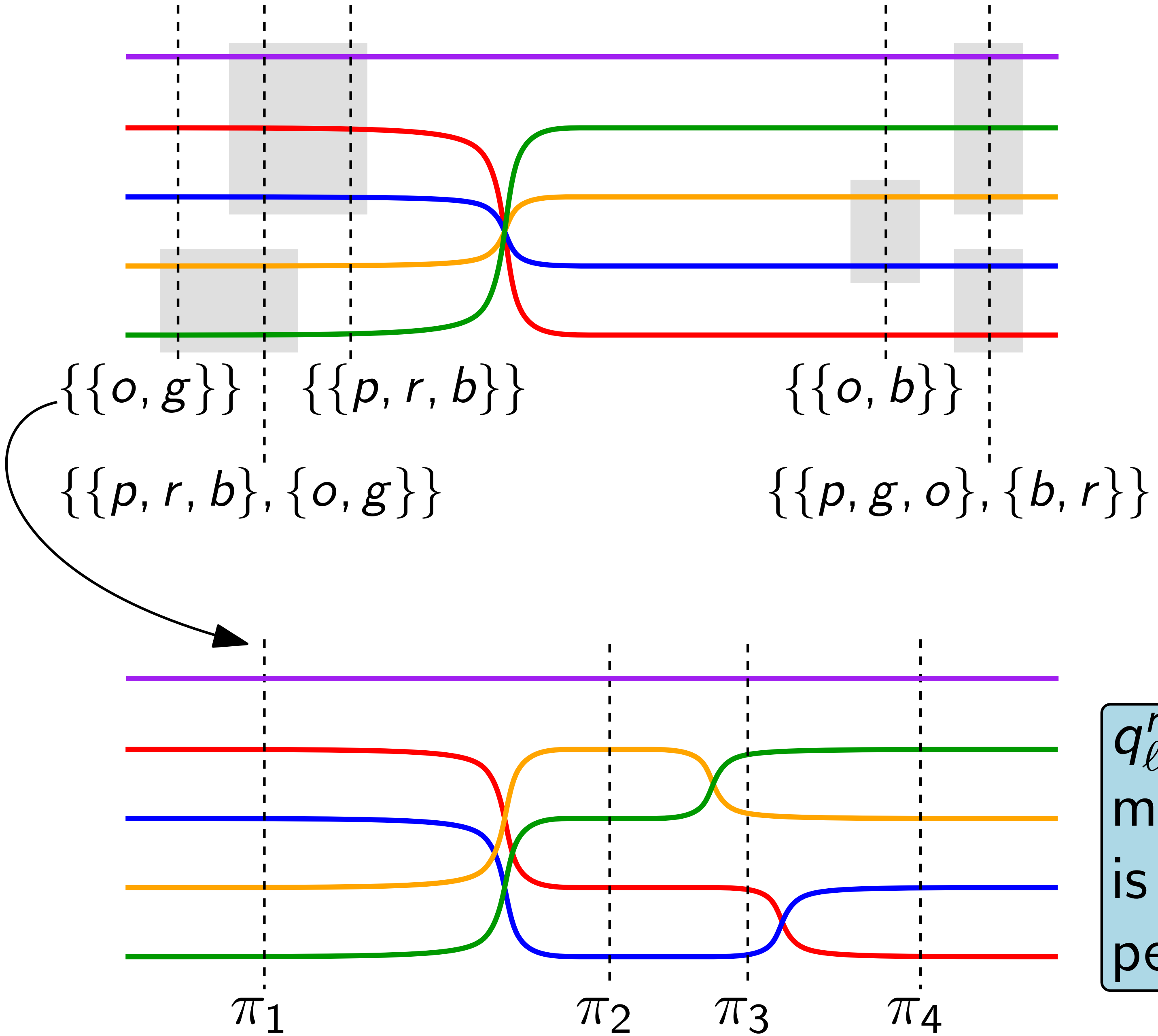

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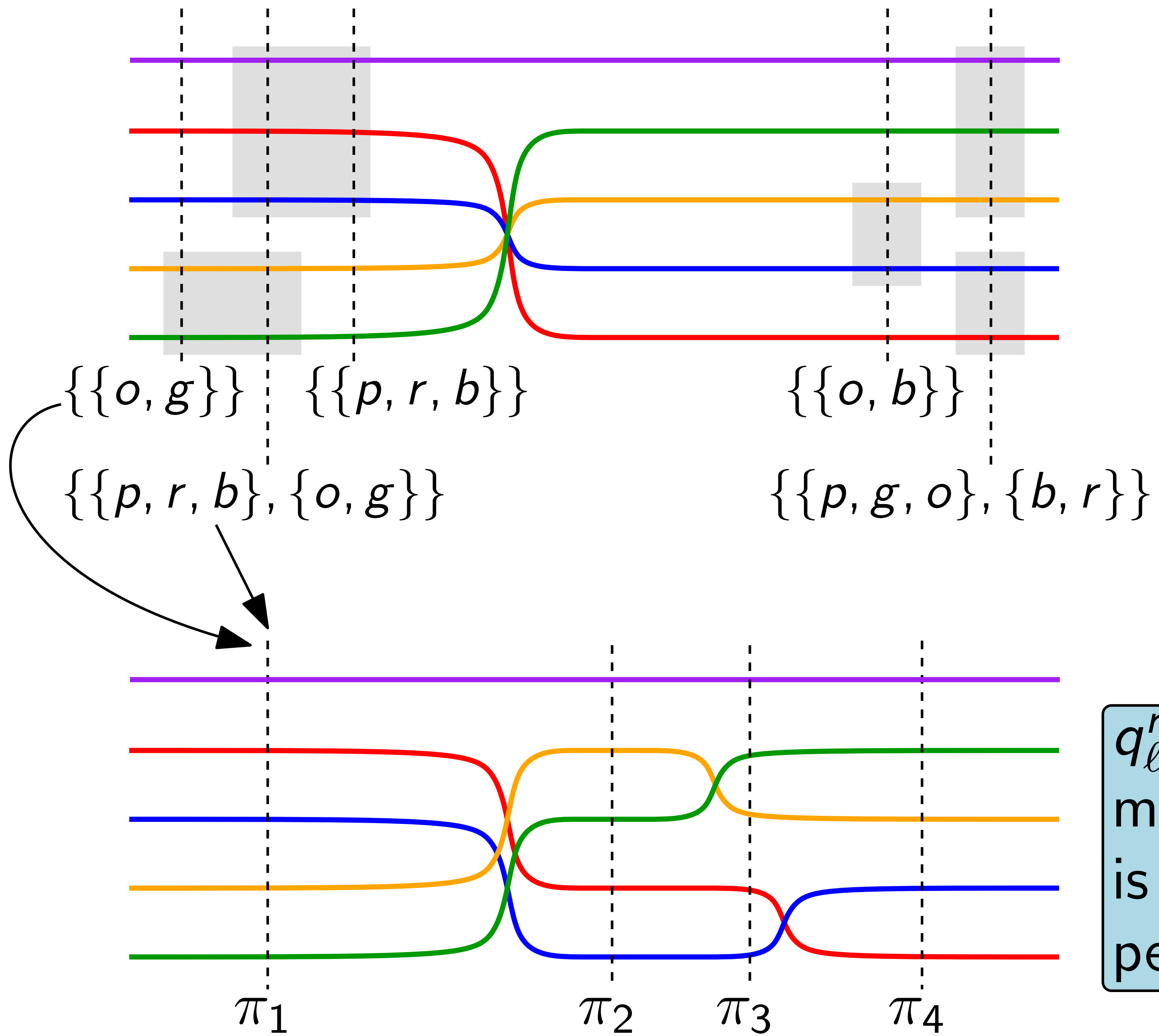

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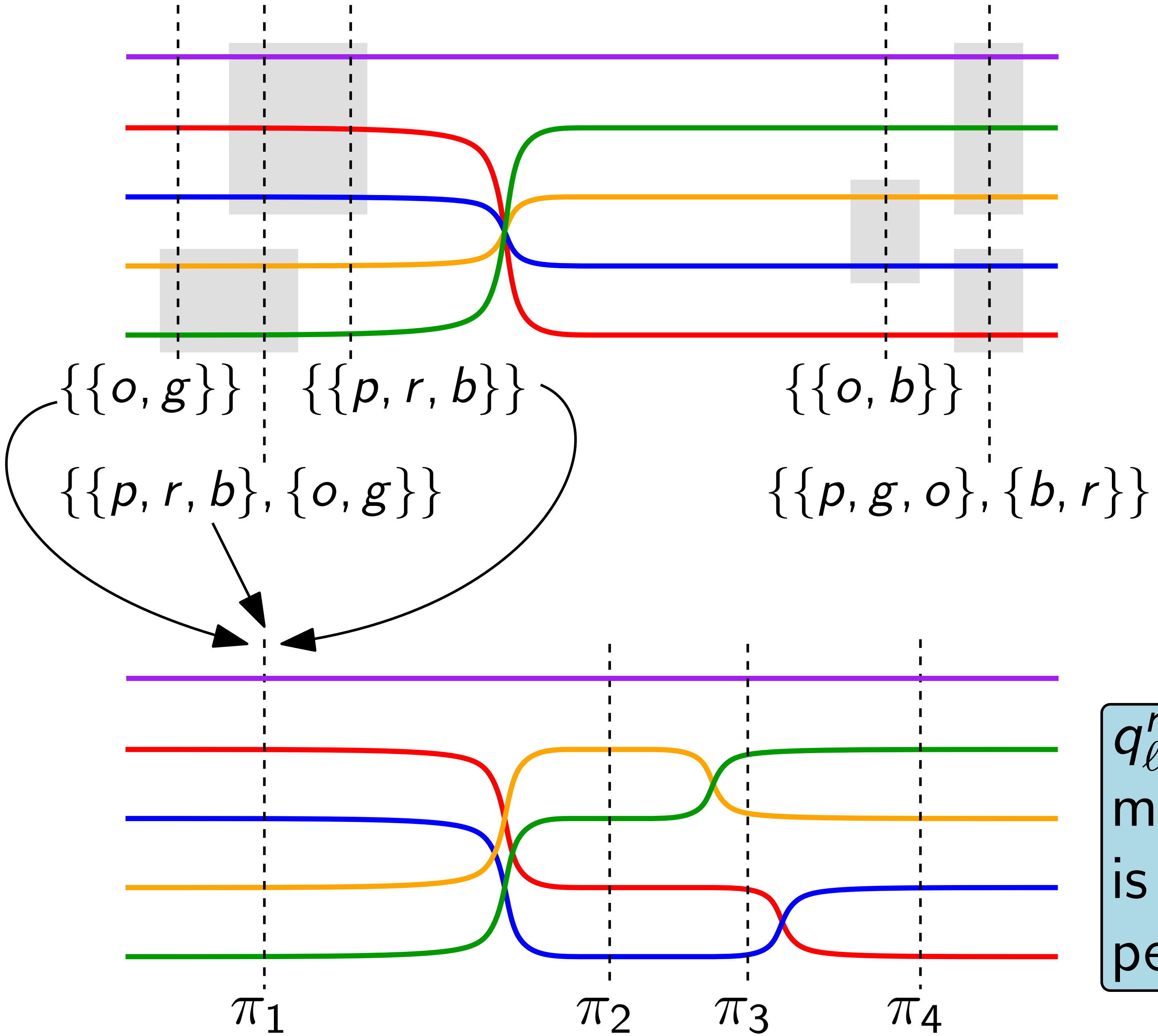

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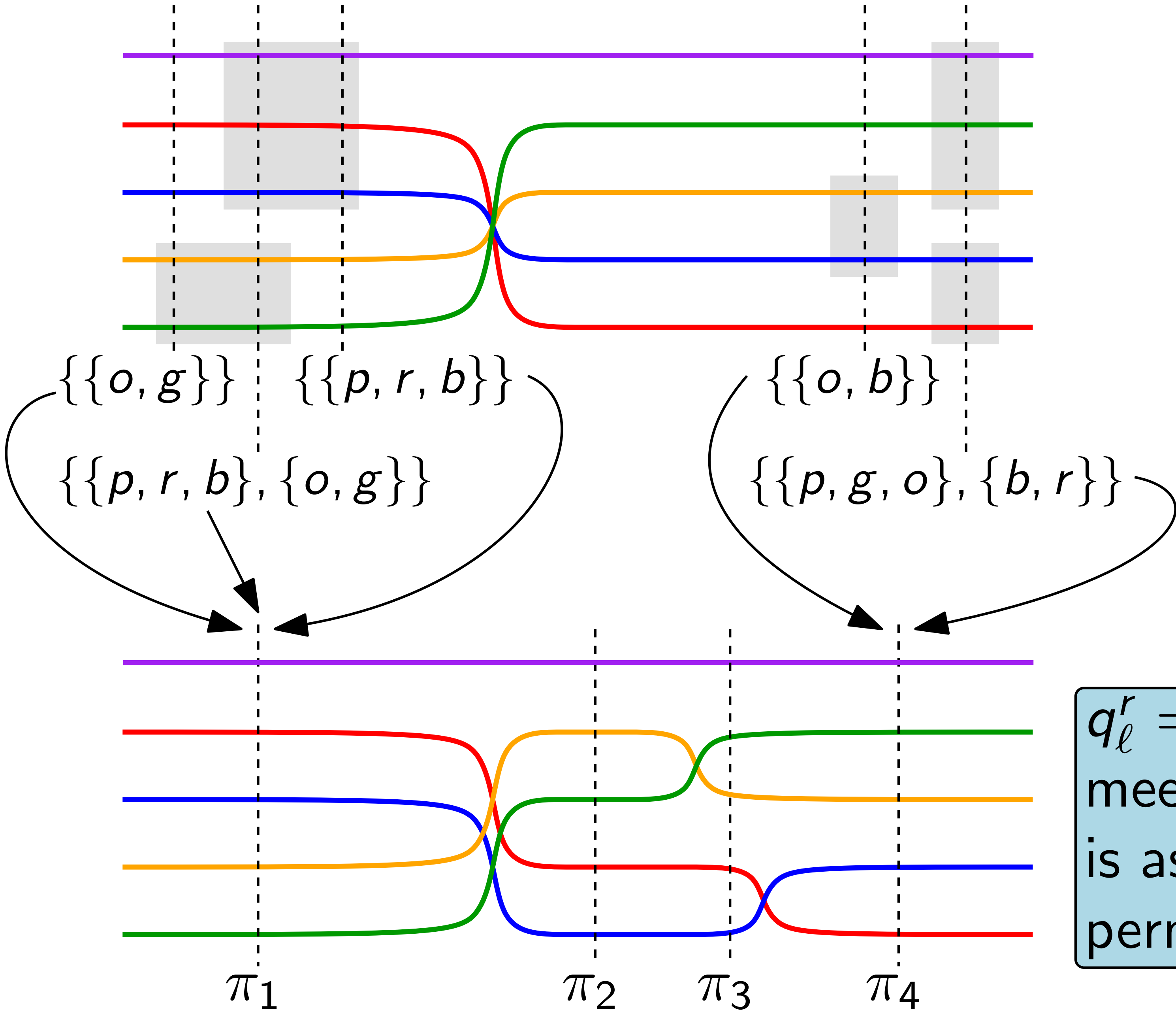

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Minimize the Number of Block Crossings

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Finding the optimum:

Repeatedly run the SAT solver with different values for λ (exponential search)

Experiments

FPT Breadth-first search a smarter state space; runtime: $O(k! \cdot k^3 \cdot n)$

- FPT are implemented in C++
- Concurrent meetings not implemented for FPT
- SAT clauses generated by Python and solved using MiniSat

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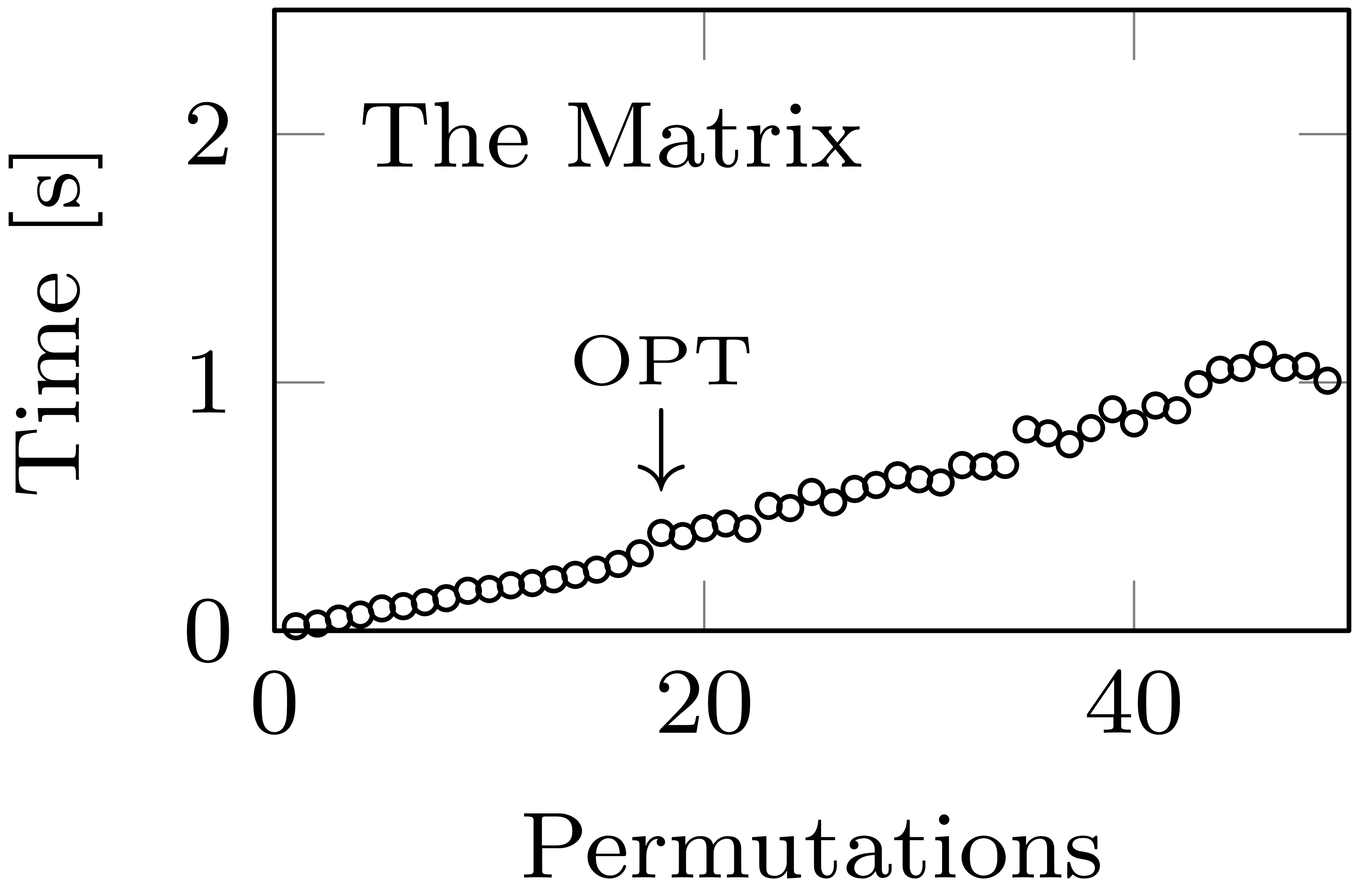
Test Data:

- Real-World instances (movies used by Gronemann et al.): The Matrix, Inception, Star Wars
- Random instances
- Random instances having a solution with few block crossings

SAT: Runtime vs Number of Permutations



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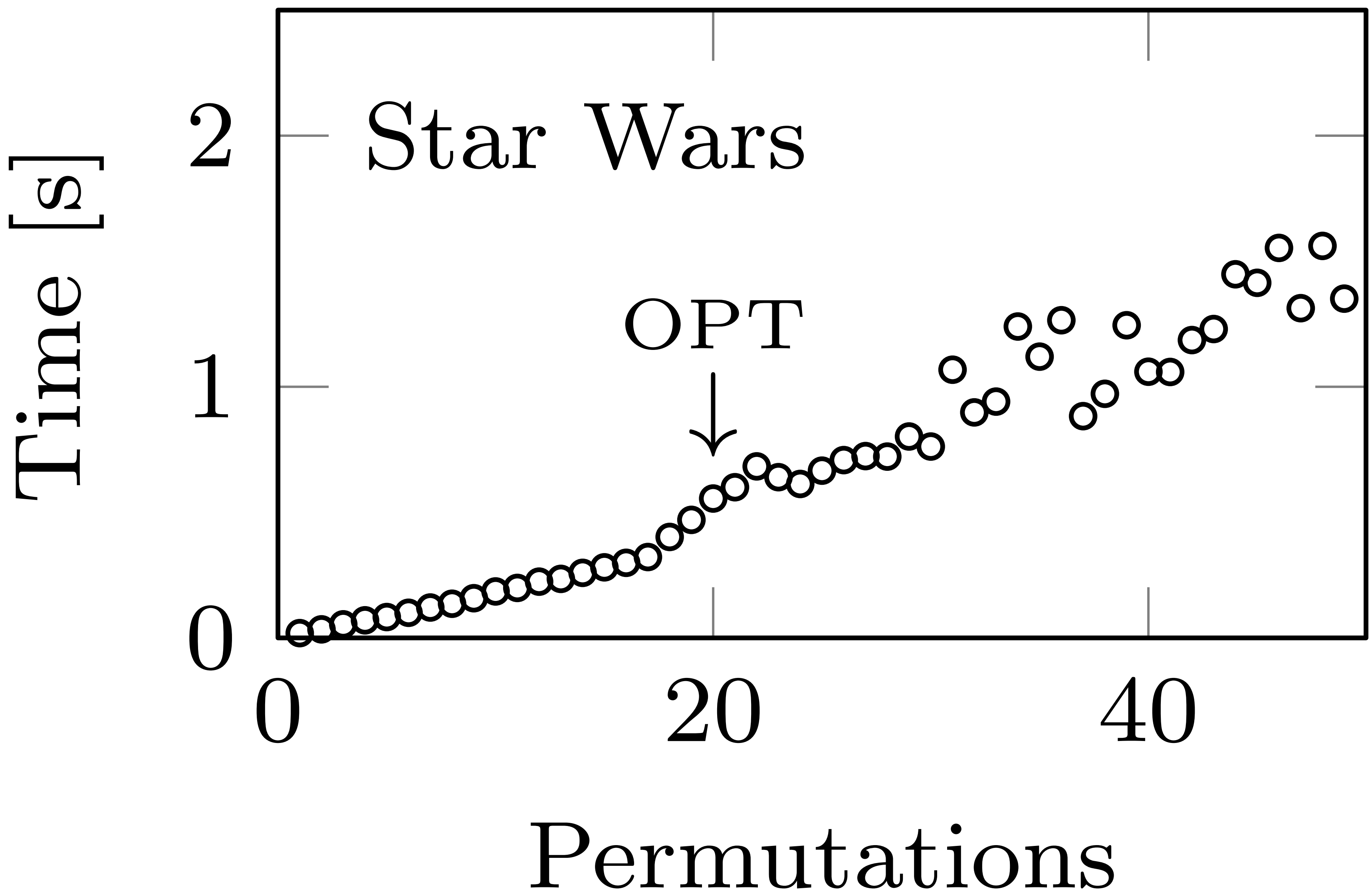

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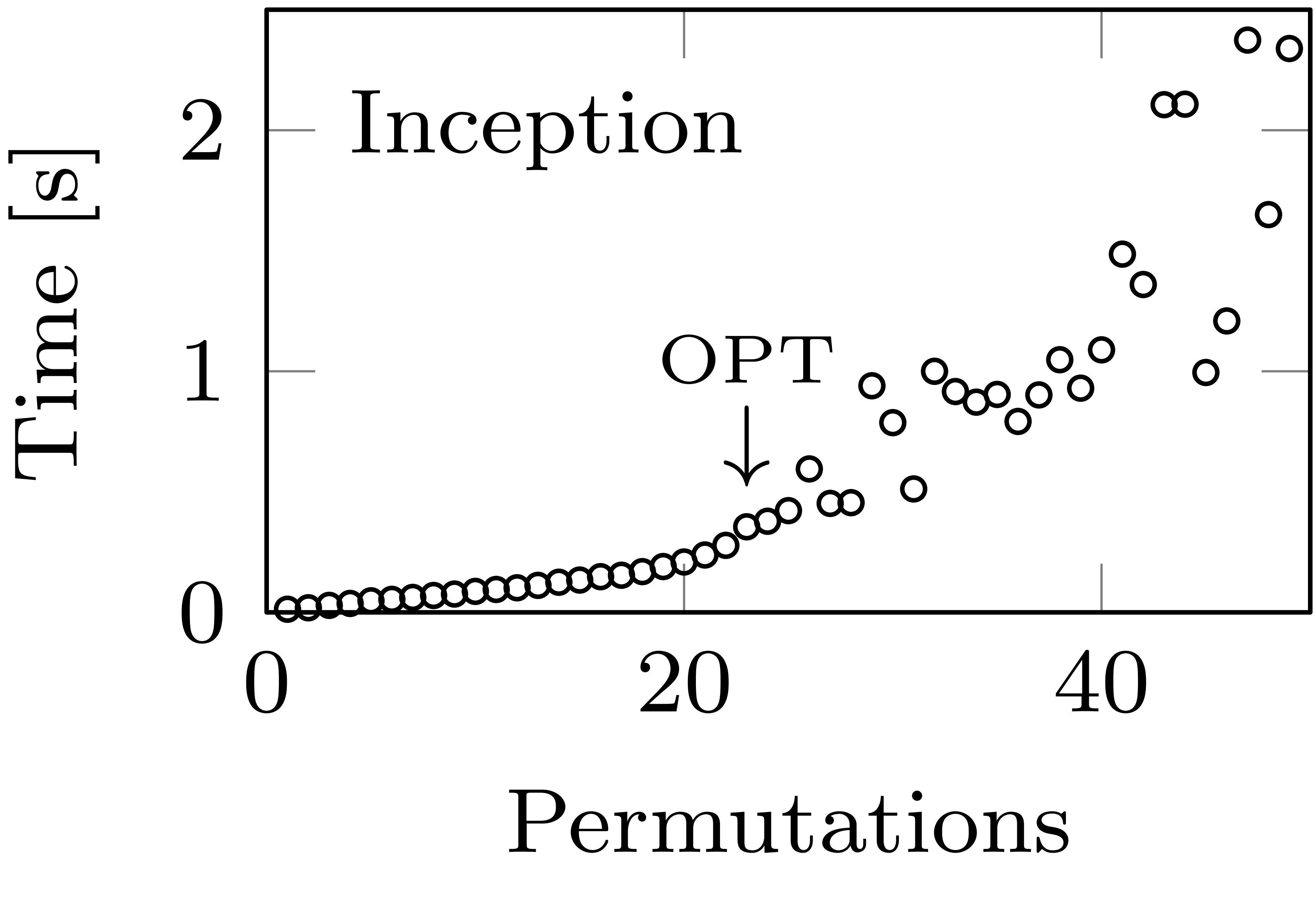
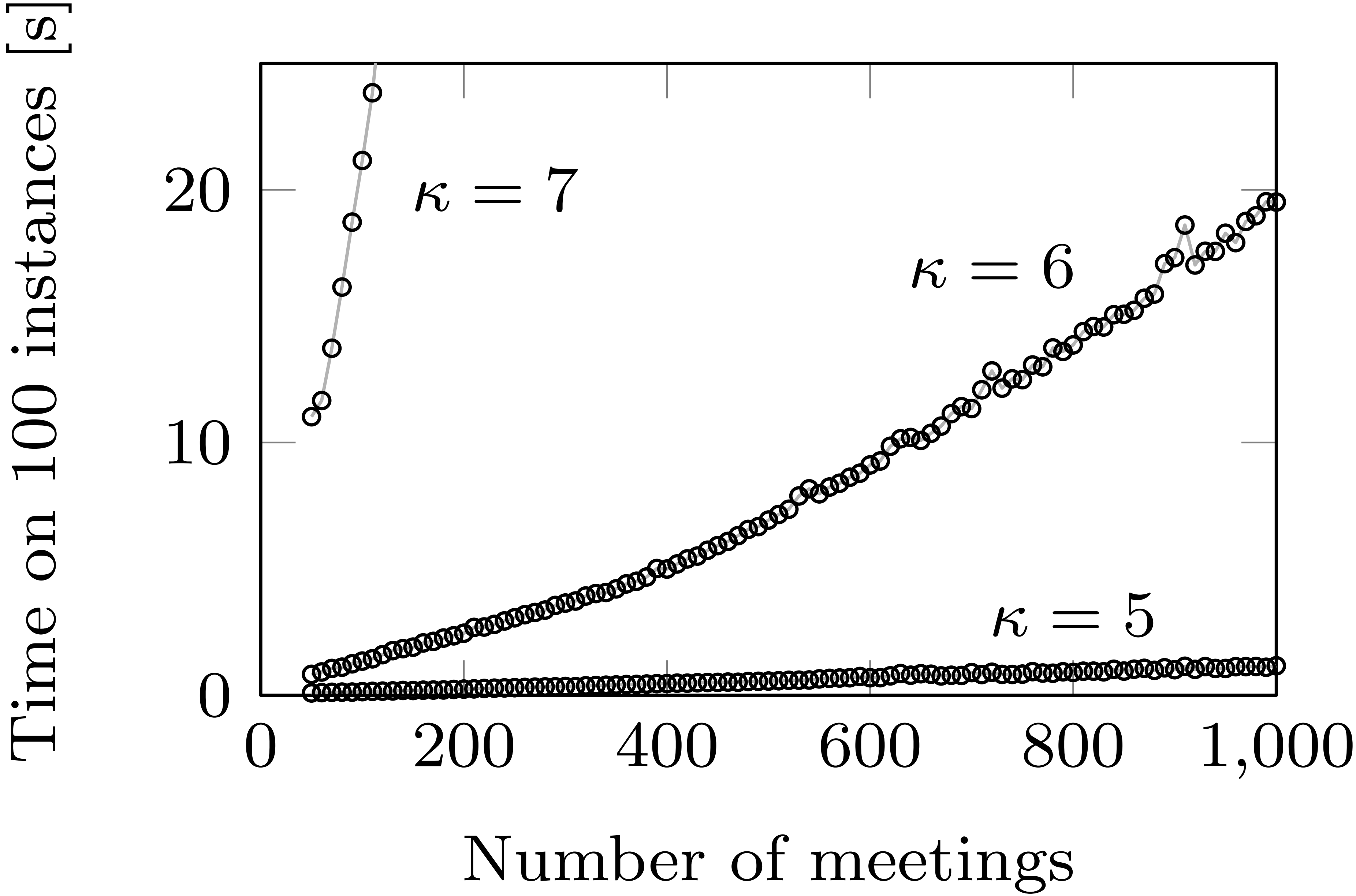


Uniform Random Instances: FPT



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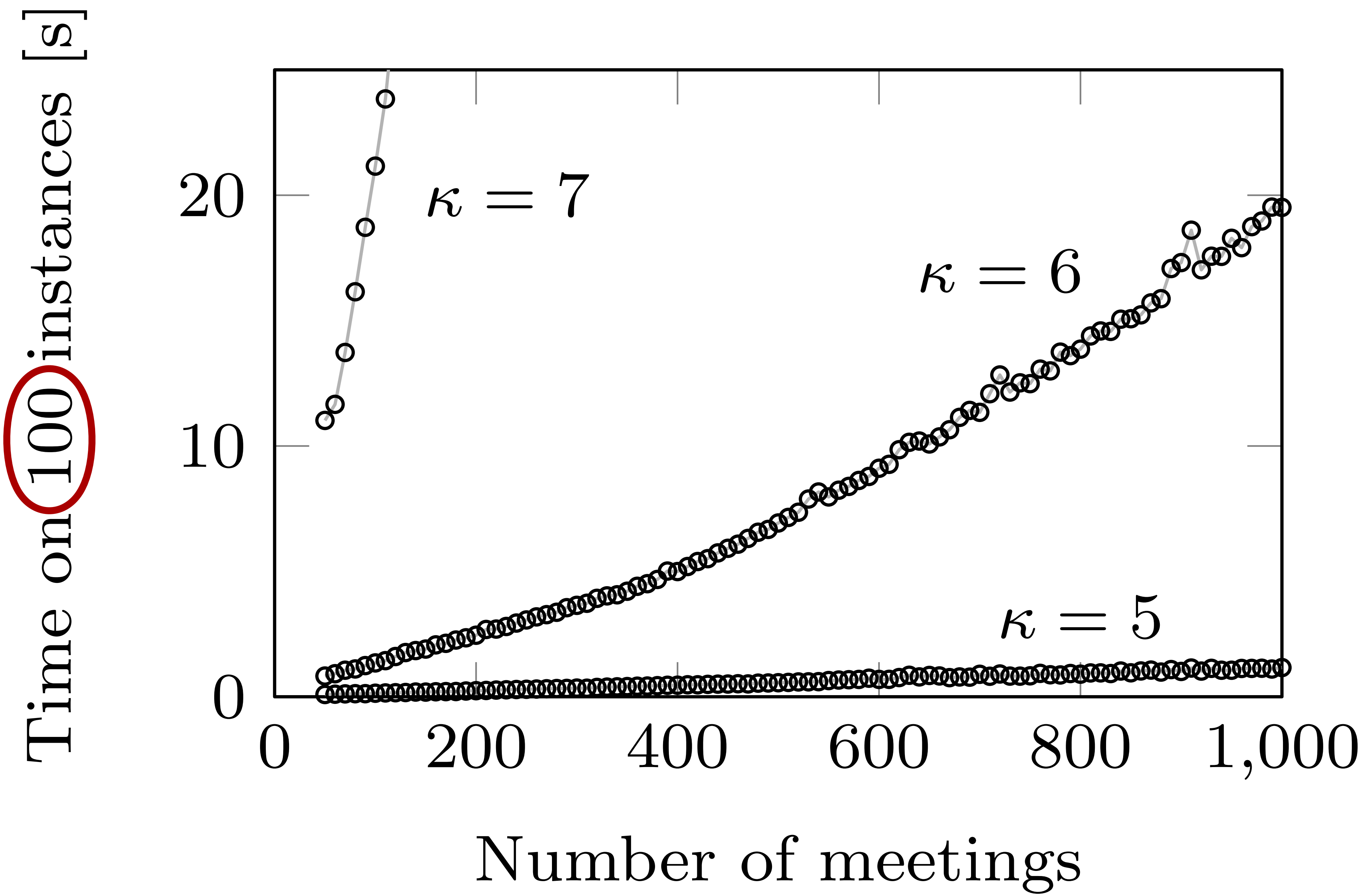
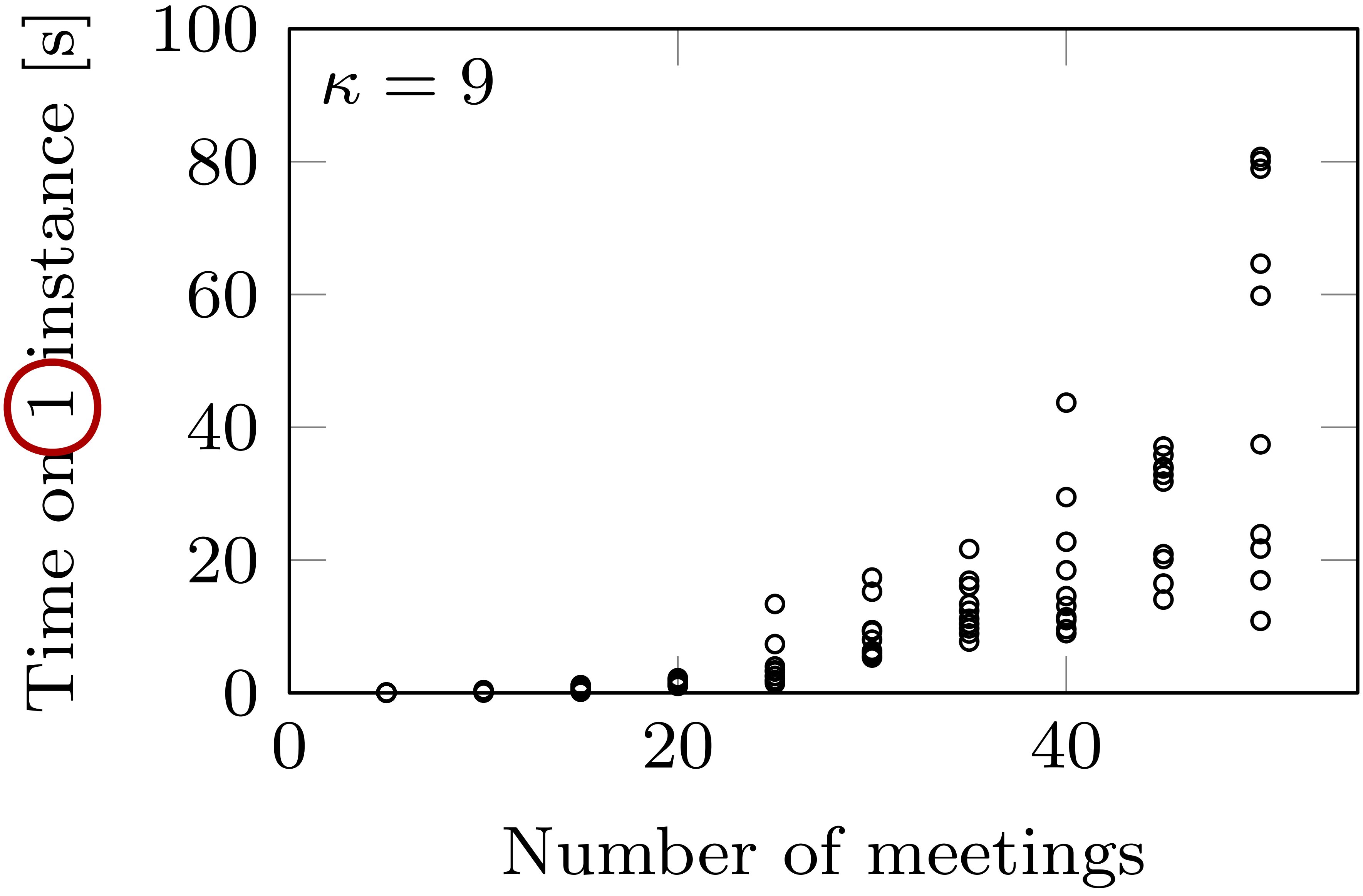
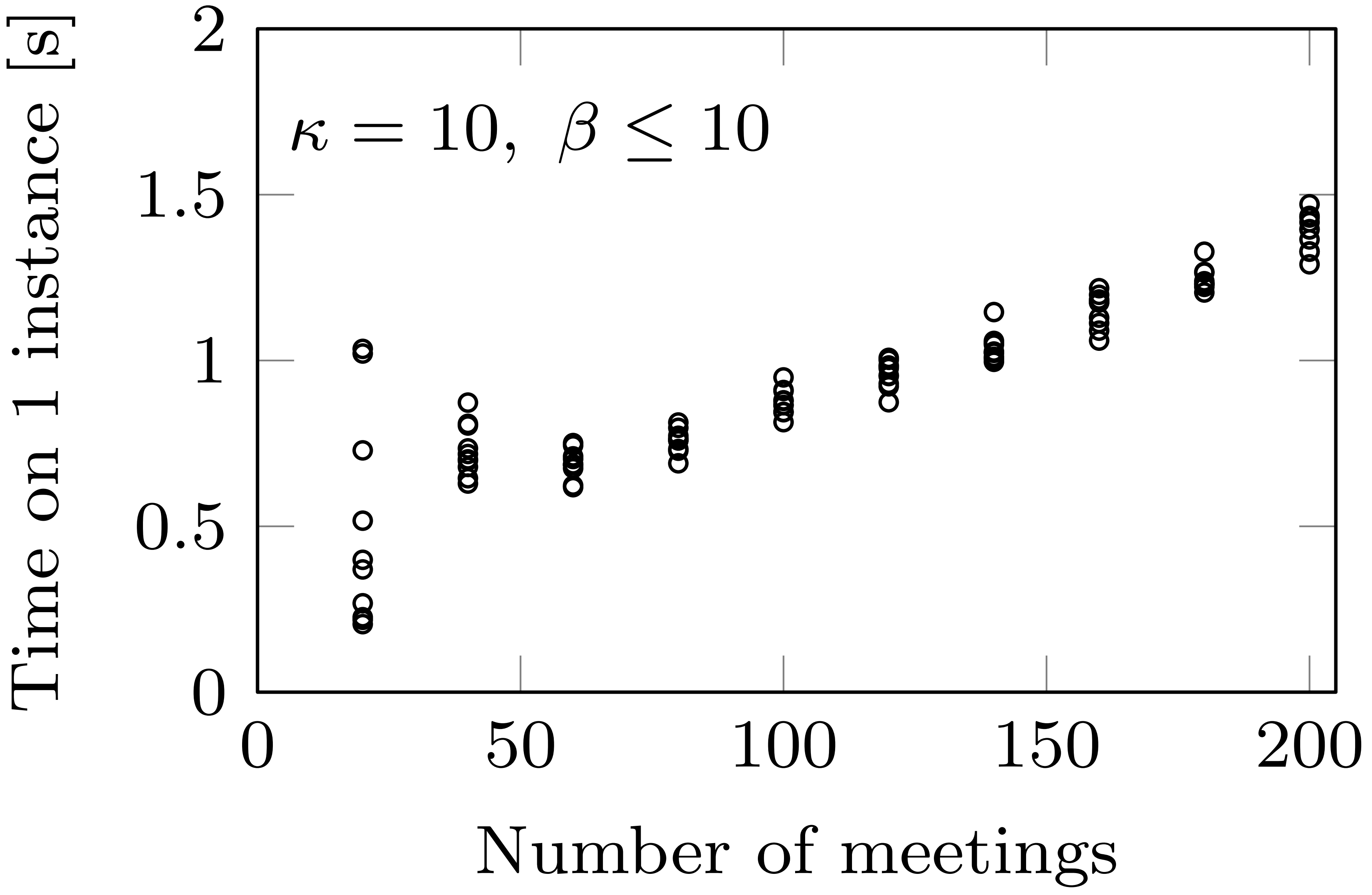


Uniform Random Instances: SAT



Small-OPT Random Instances: SAT



Results

Movie Instances:

	Our approach			Gronemann et al.		
Instance	cr	bc_{OPT}	Time [s]	cr_{OPT}	bc	Time [s]
Star Wars	54	10	3.77	39	18	0.99
The Matrix	21	4	2.86	12	8	0.77
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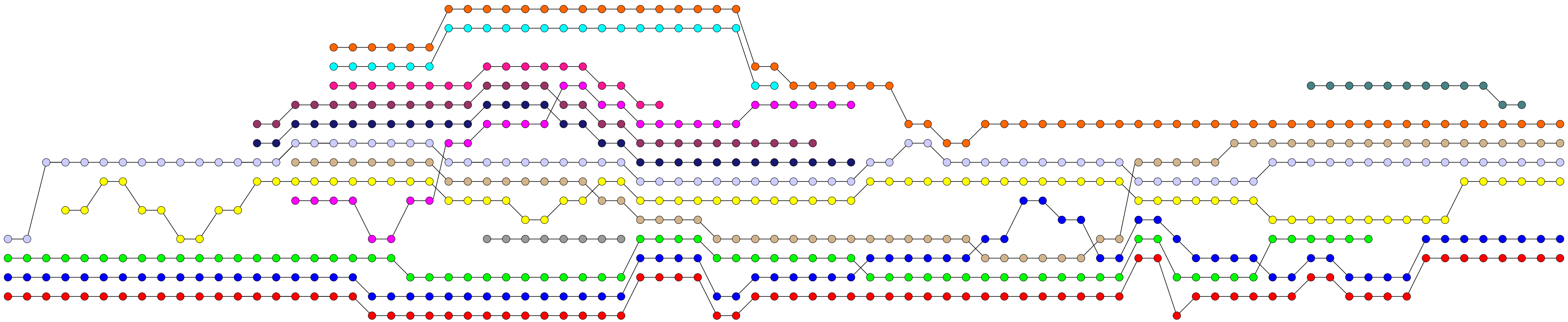
Gronemann et al. 12 crossings / 8 block crossings



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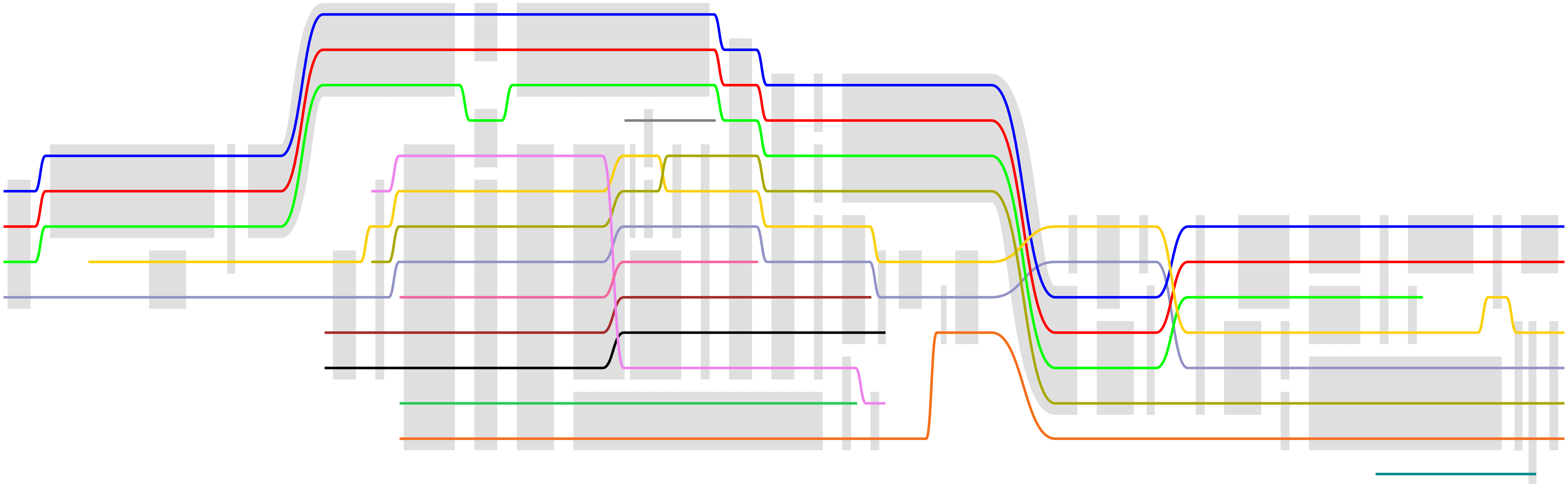

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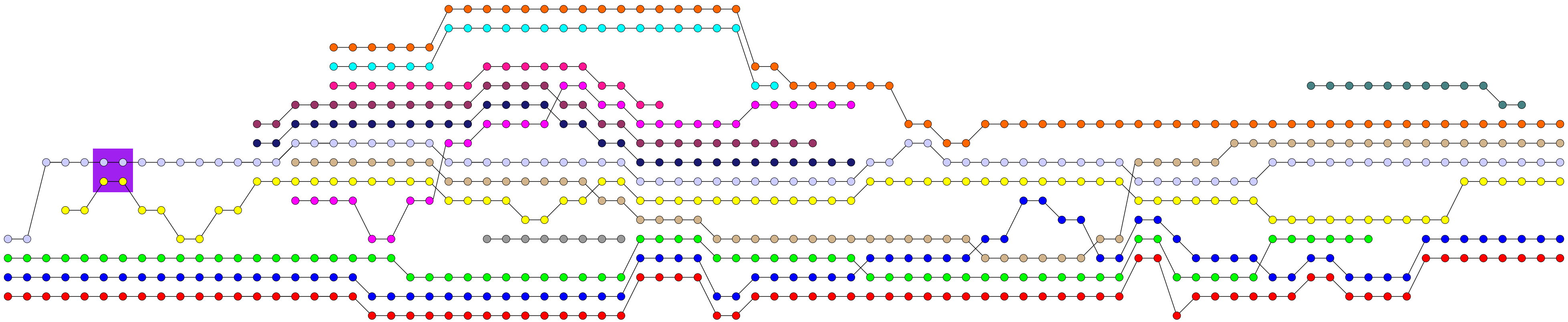

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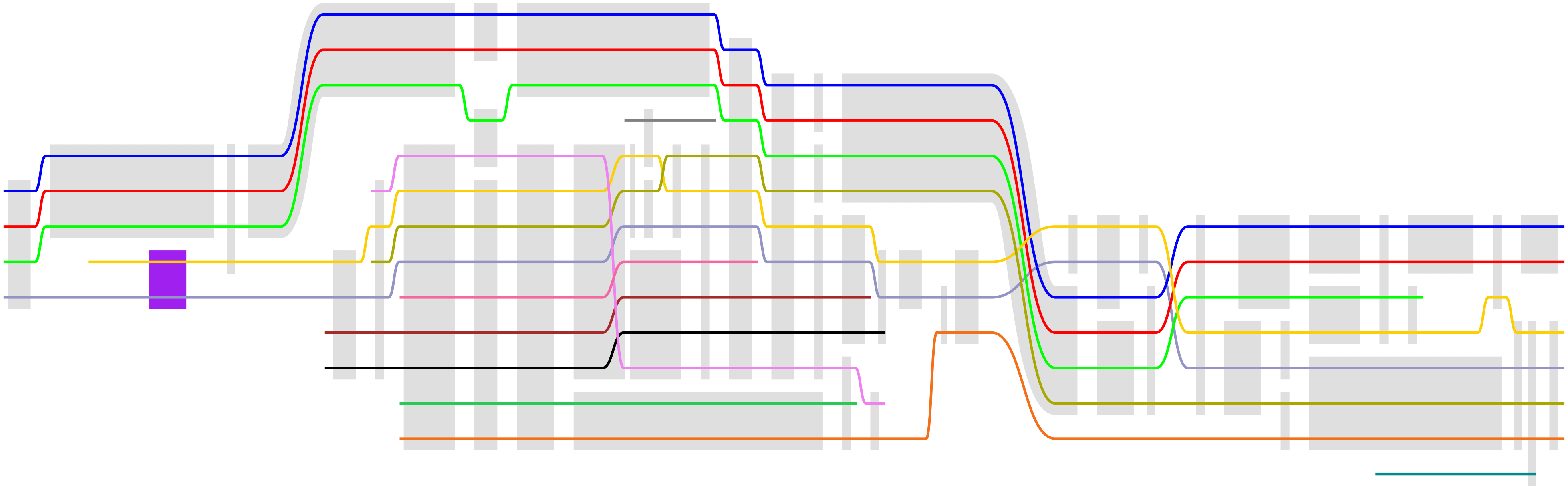

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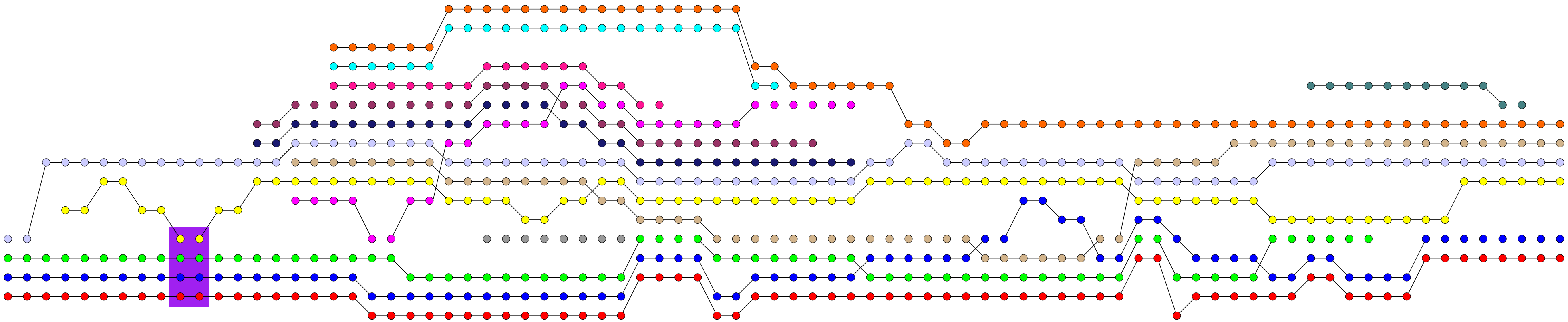

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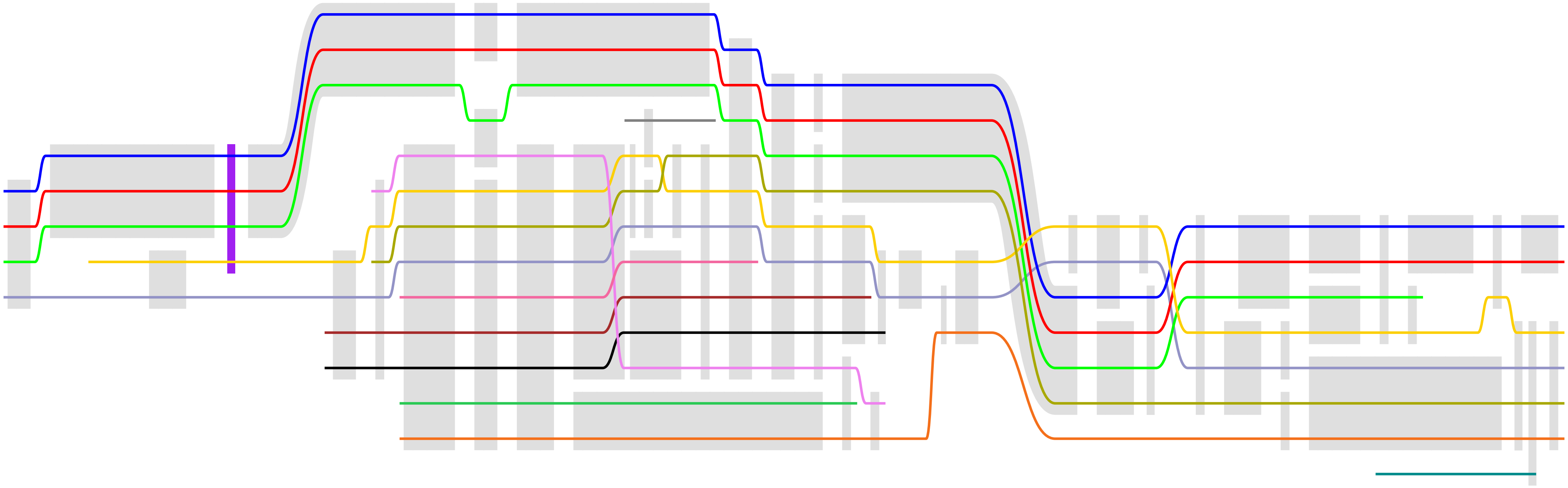

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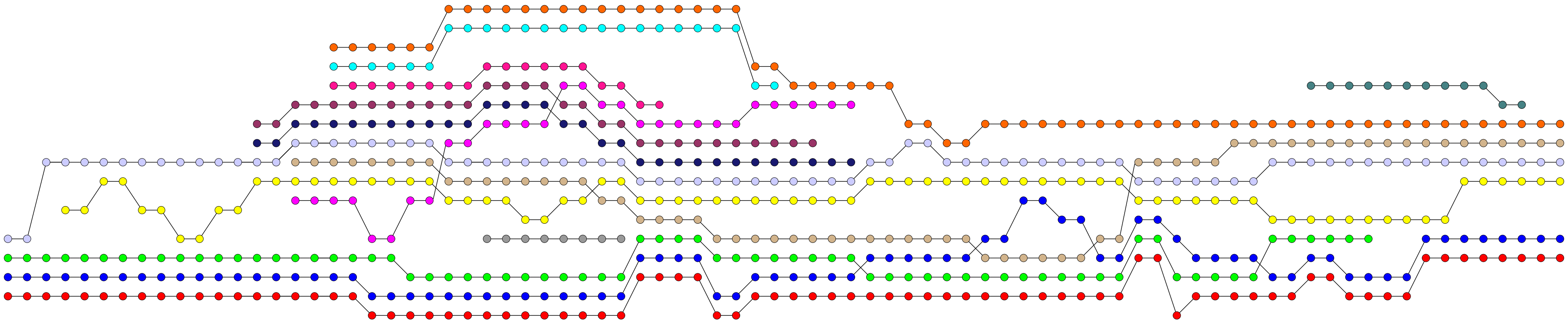

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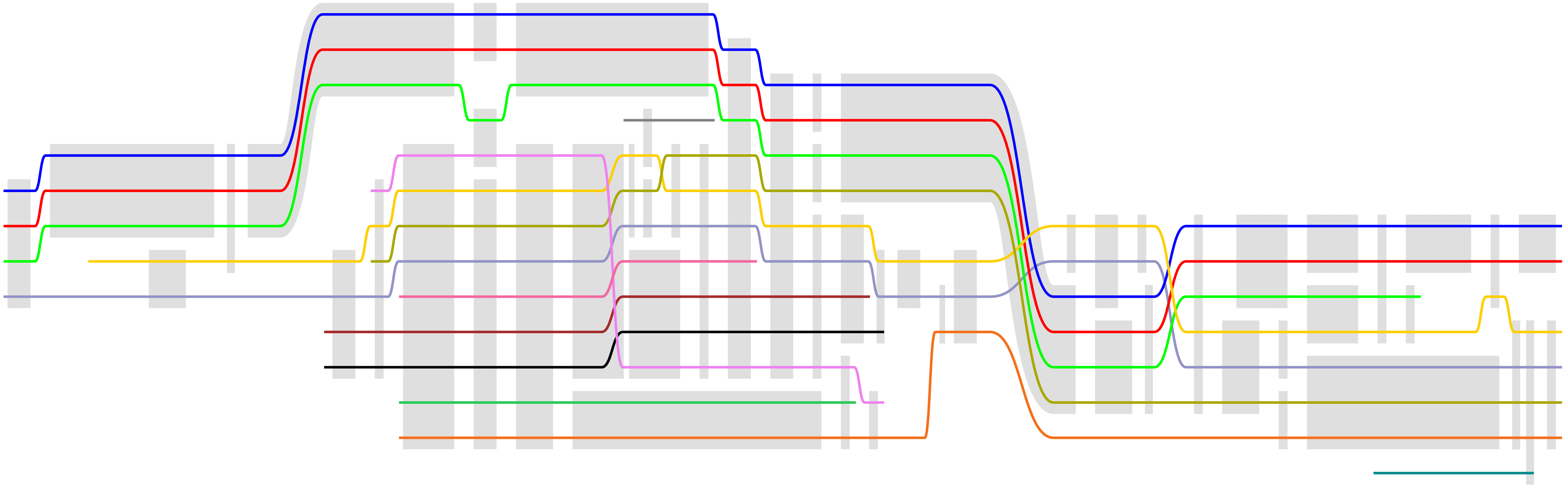

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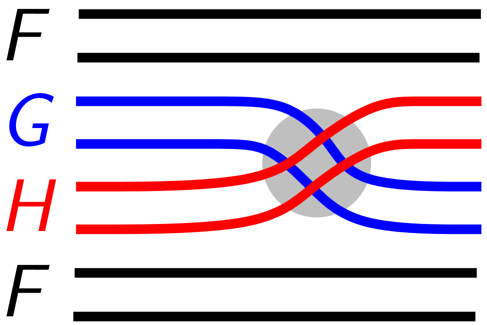
Future work

- Try other (parallel) SAT solvers.
- Find more efficient way to model lifespans.
- Consider additional quality criteria of the drawing, e.g., minimize wiggles. [Fröschl & Nöllenburg, GD17]
- Perform a user study on the effect of block crossings, especially for storyline visualizations.

Appendix

Describing Block Crossings



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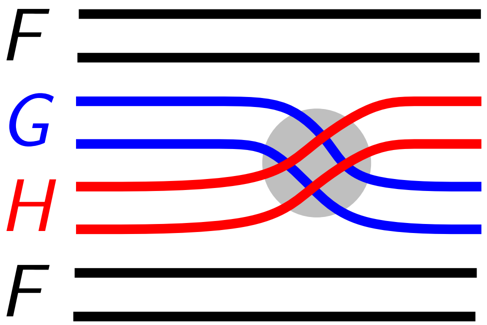

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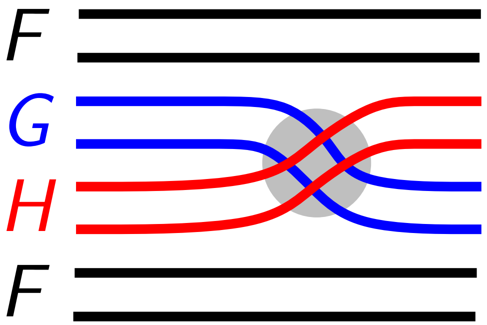

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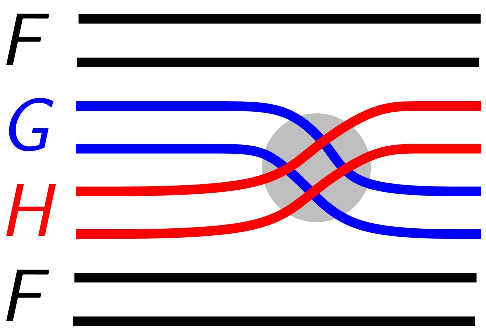

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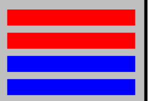

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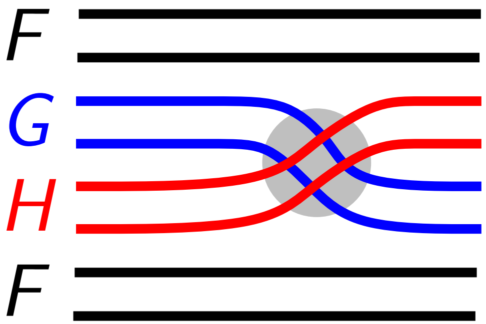

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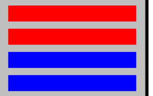

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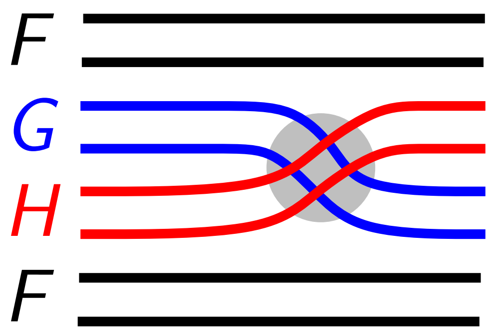

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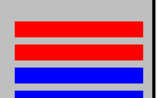

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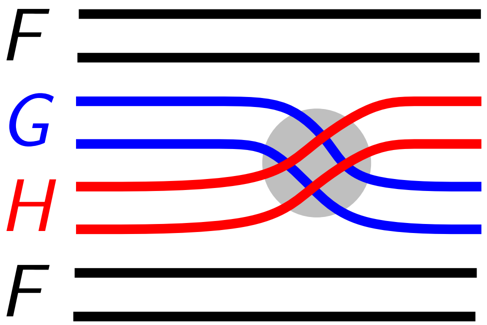

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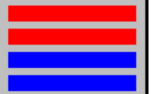

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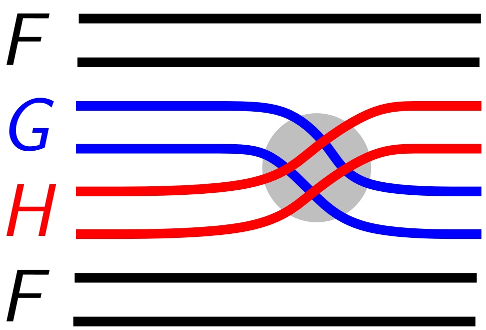

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G is above H
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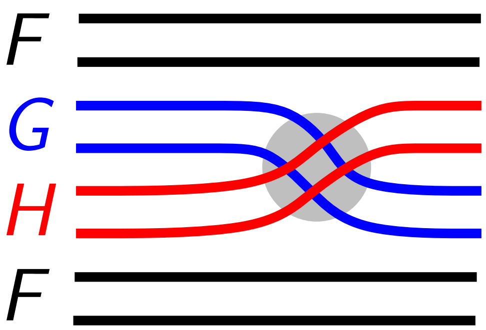

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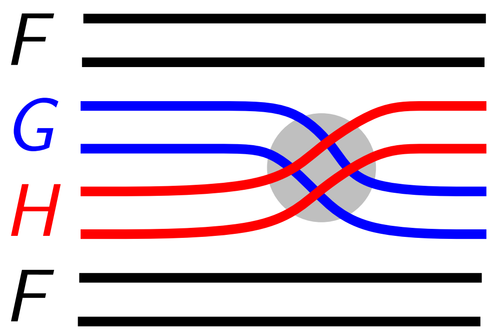

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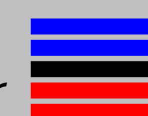

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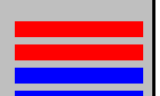
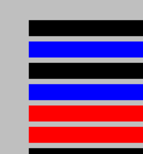
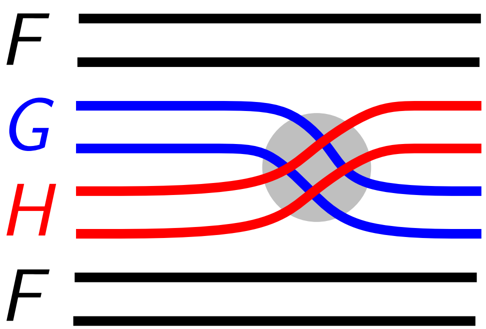


G and H are contiguous blocks



Describing Block Crossings



Blocks G and H cross

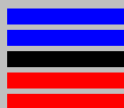
Constraints:

Exactly characters of G and H cross each other

$\chi_{ij}^r \Leftrightarrow g_i^r \wedge h_j^r$

G and H are adjacent

$x_{ij}^r \wedge x_{jk}^r \wedge g_i^r \wedge h_k^r \Rightarrow \neg f_j^r$



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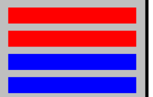

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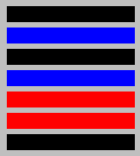

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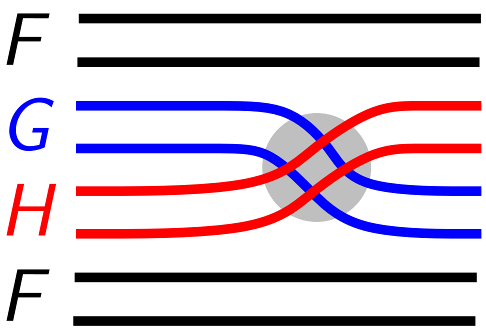

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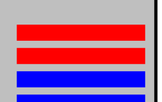

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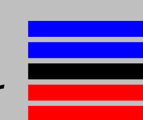

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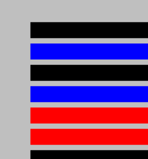

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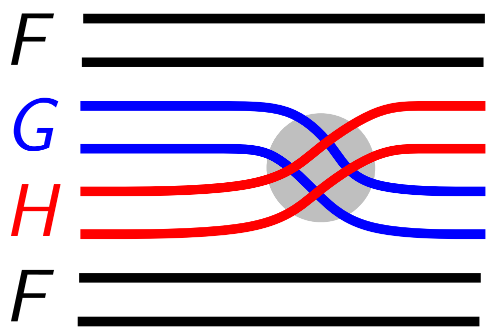

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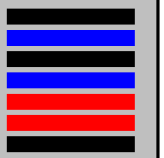
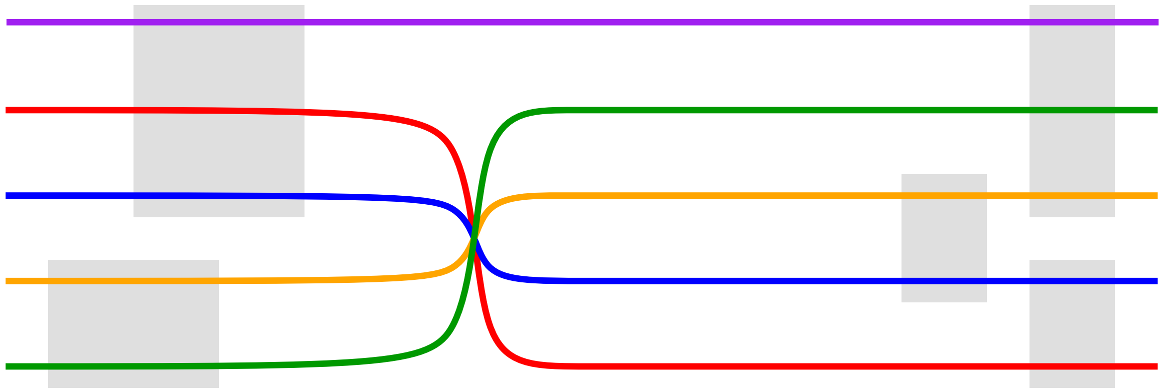


Meeting Groups



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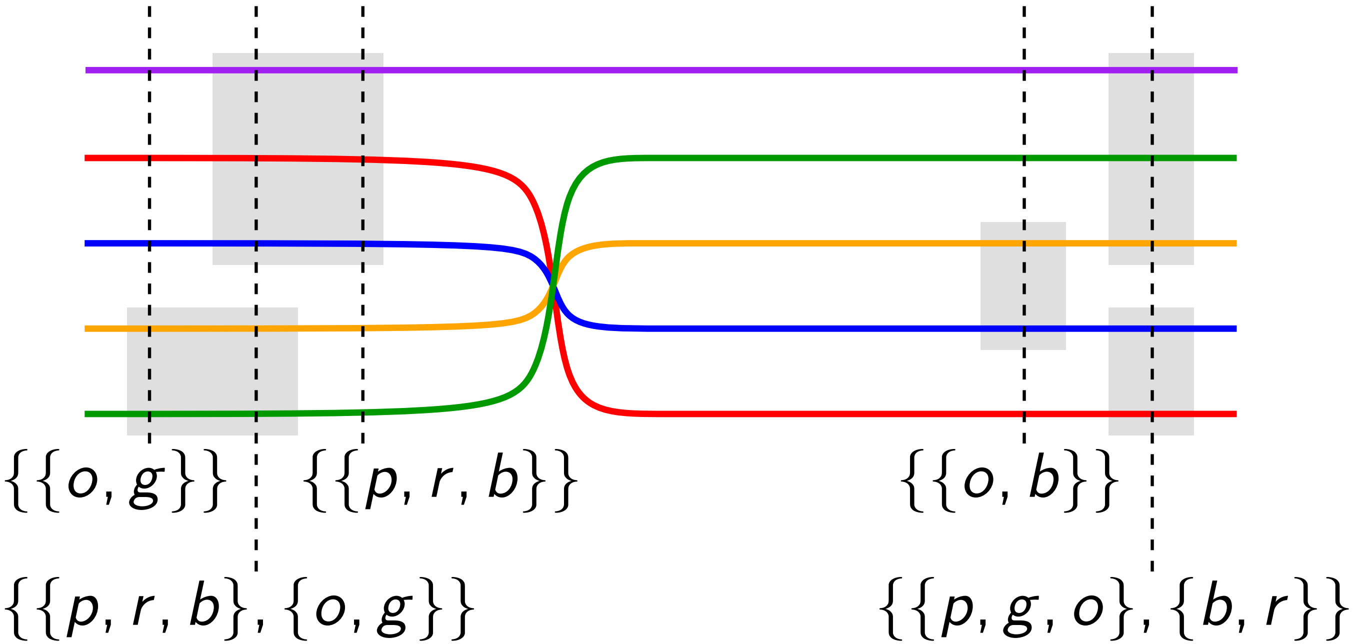

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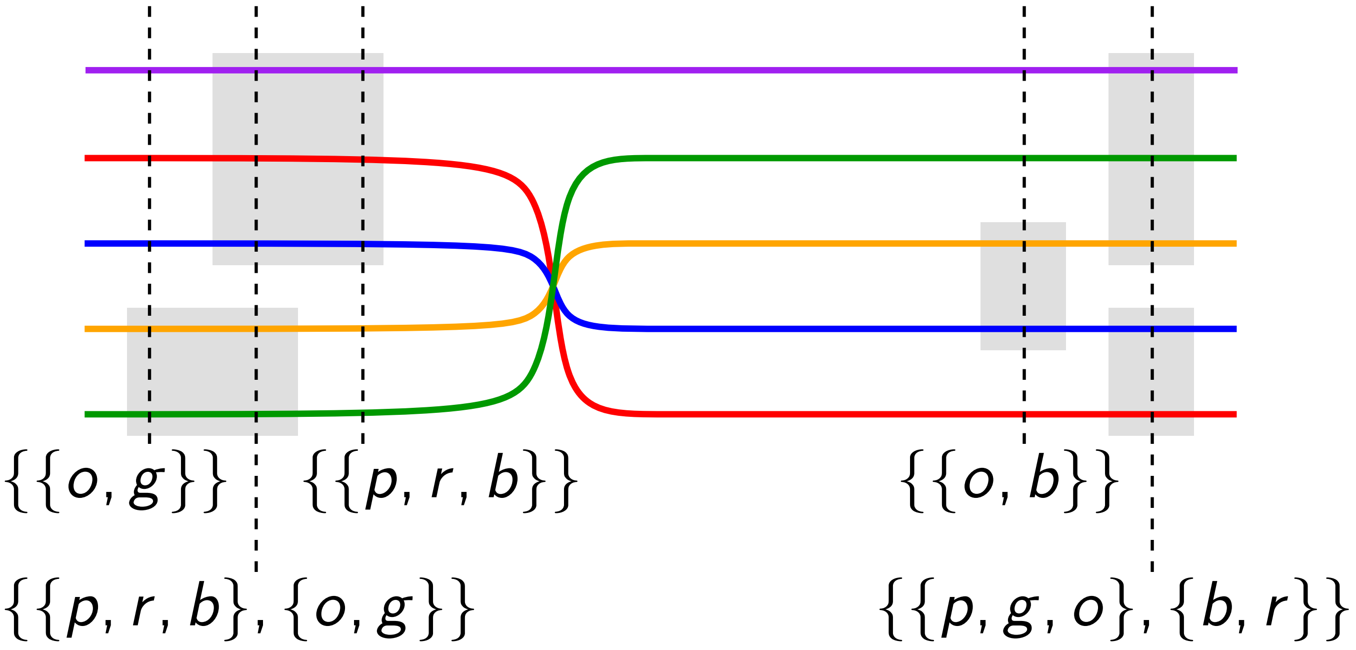

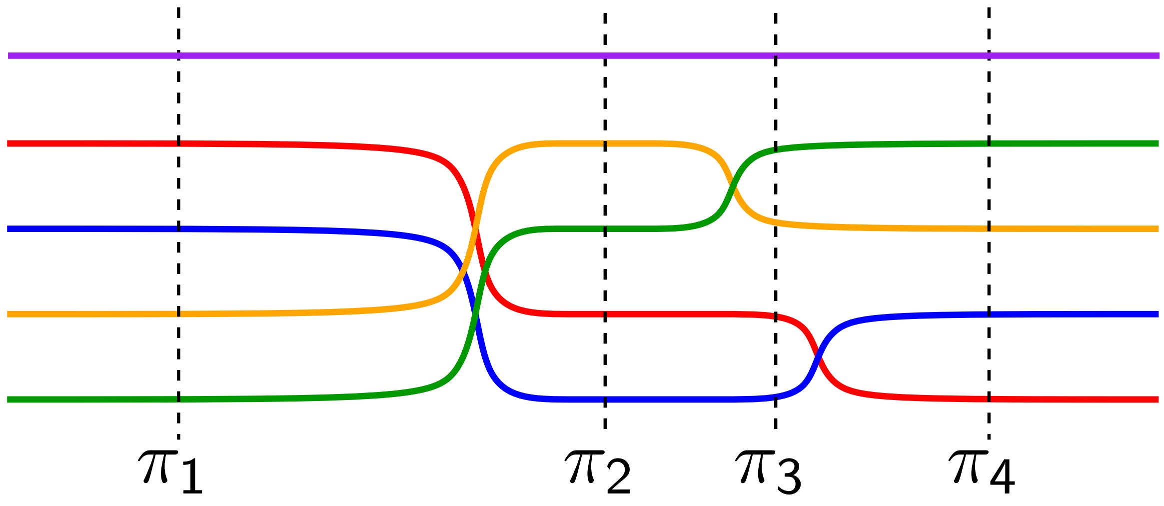

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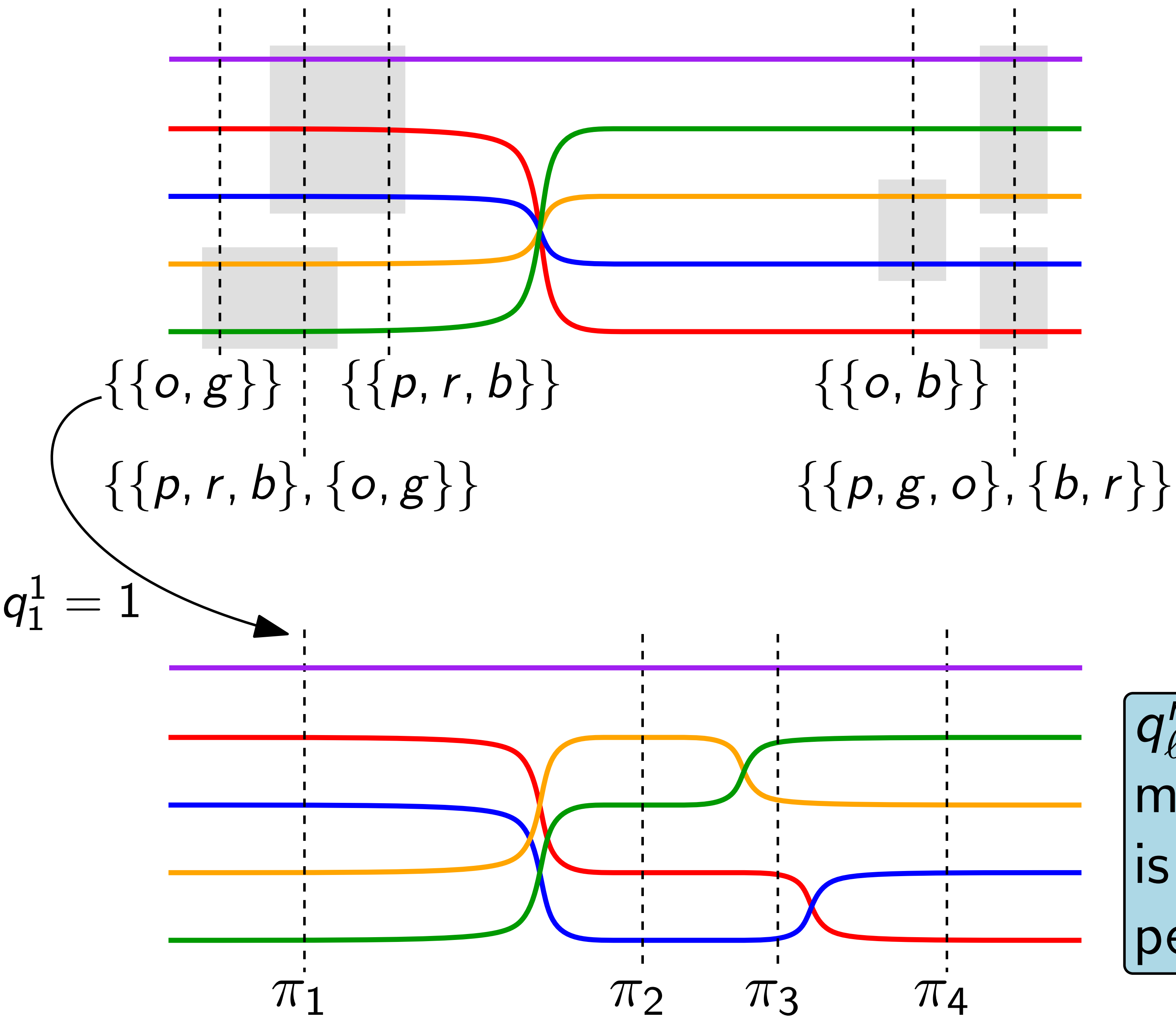

$q_\ell^r = 1 \Leftrightarrow$ meeting group ℓ is assigned to permutation π_r

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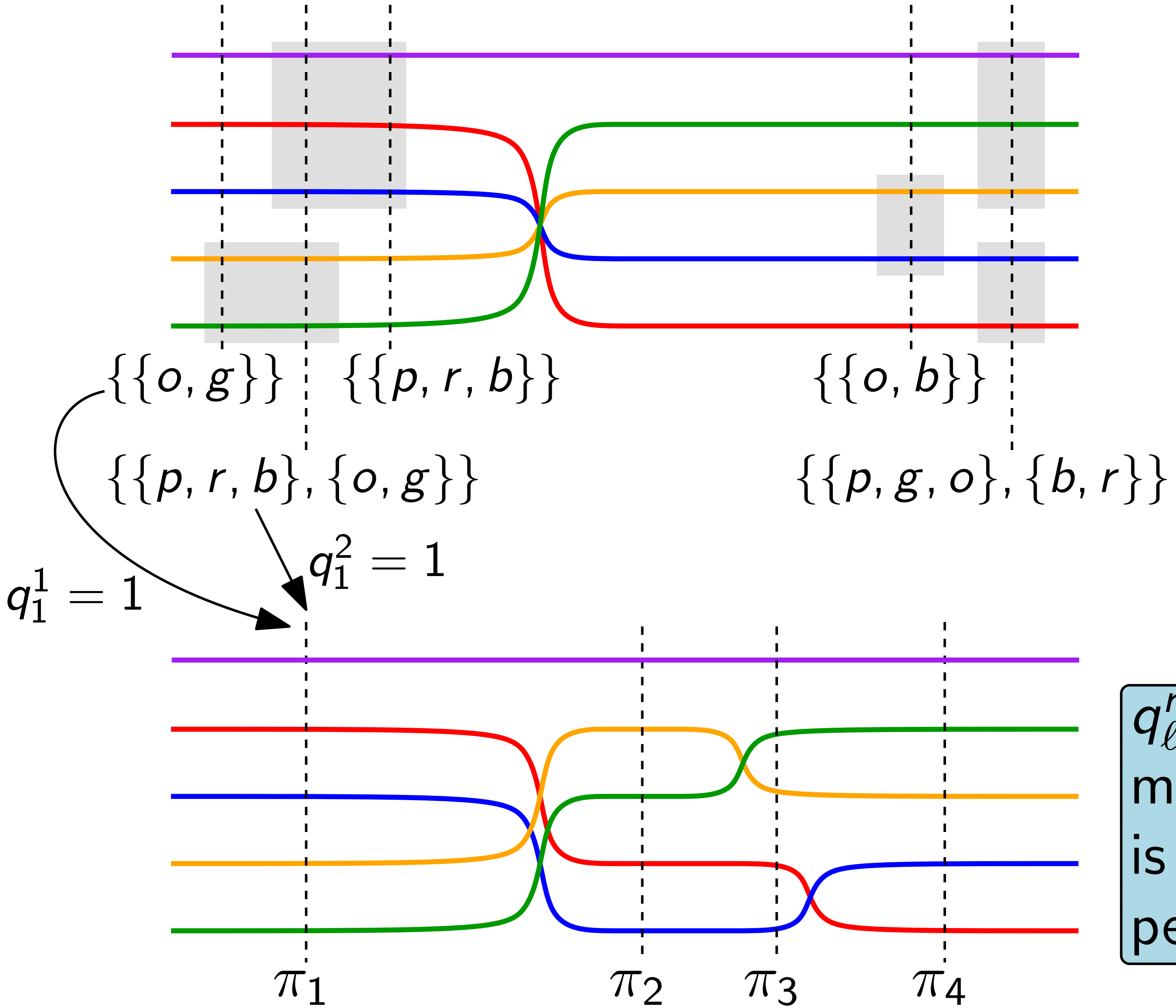

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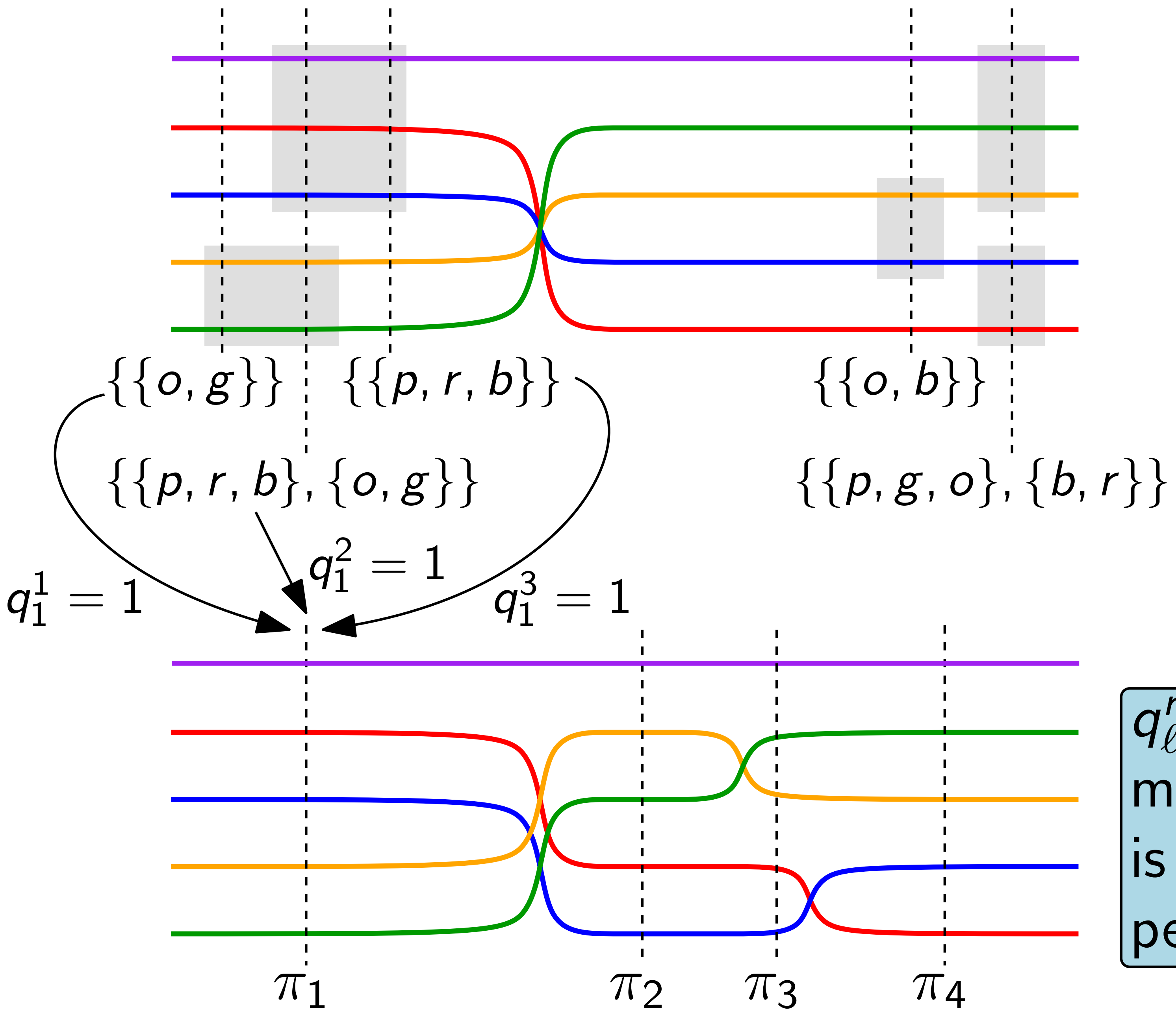

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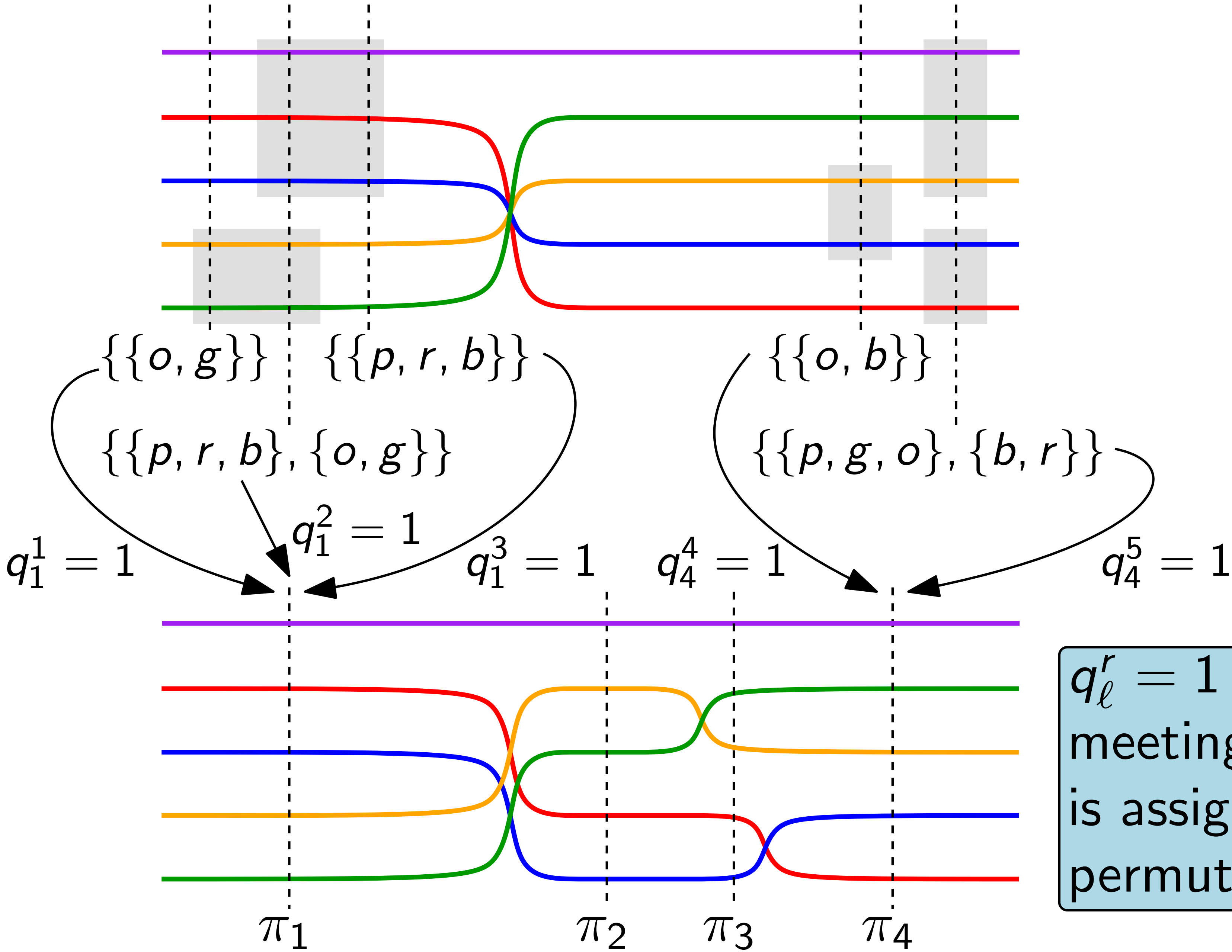

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- Map every meeting group exactly once:
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