Anisotropic Radial Layout for Visualizing Centrality and Structure in Graphs

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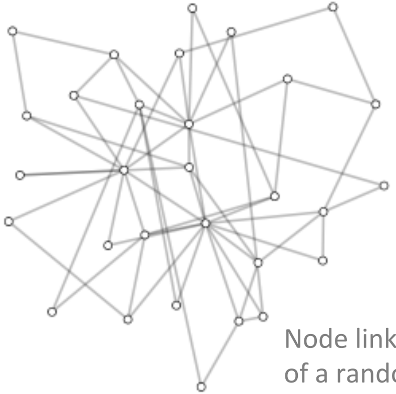
Node-link Diagram

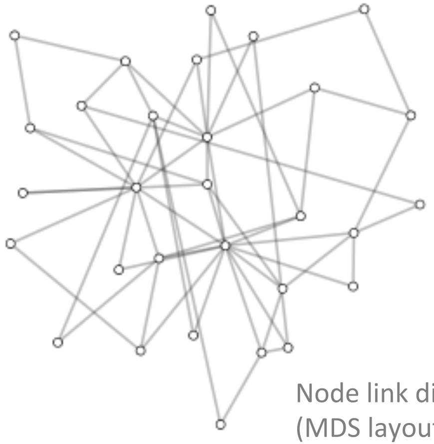
- Goals
  - Improve aesthetics
  - Reduce visual clutter
  - *Convey features of interest*
    - ***Structural features based on internode distances***
    - ***Importance of nodes***



Node link diagram
of a random graph

Visualizing Structure

- Preserve internode distances in the drawing
- Method:
  - Dimensionality reduction techniques
    - **Multidimensional Scaling (MDS)**
    - t-SNE (stochastic neighbor embedding)



Node link diagram (MDS layout)

Multidimensional Scaling (MDS)

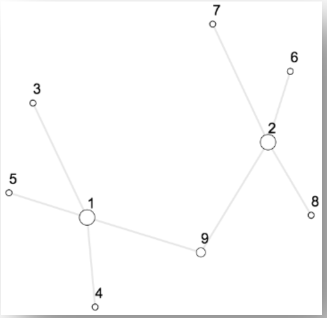
- Conveys similarity between objects
- Minimizes energy function (stress)

$$\sigma(X) = \sum_{u,v} w_{uv}\left(d_{uv} - ||\bar{x}_u - \bar{x}_v||_2\right)^2$$

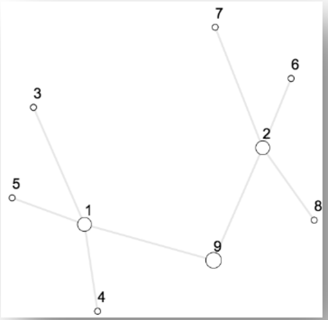
$X \in \mathbb{R}^{n\times 2}$
$d_{uv} :=$ ideal distance between nodes u and v
$w_{uv} :=$ weight on edge between u and v
$\bar{x}_u \in \mathbb{R}^2 :=$ position of x

Importance of Nodes

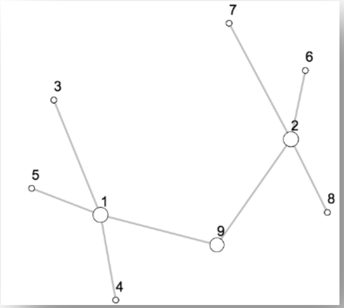
- Using graph structure
  - Node centrality
    - *Degree centrality:* number of neighbors
    - *Closeness centrality:* reciprocal of sum of distances
    - ***Betweenness centrality***: number of shortest paths passing through the node
    - Many more …



Degree centrality

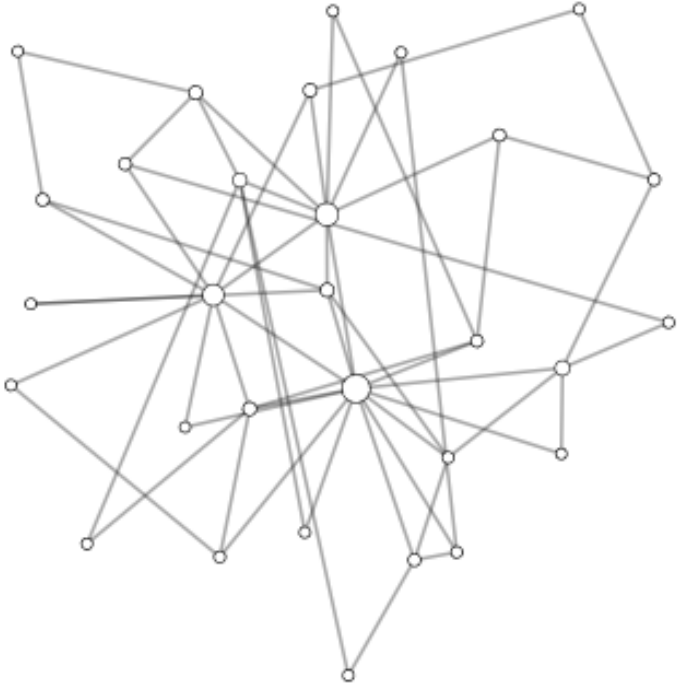


Closeness centrality



Betweenness centrality

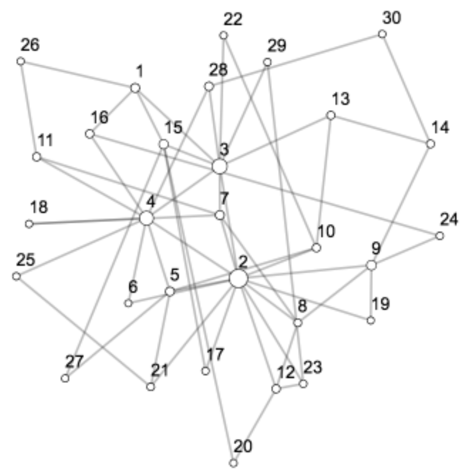
Visualizing Centrality

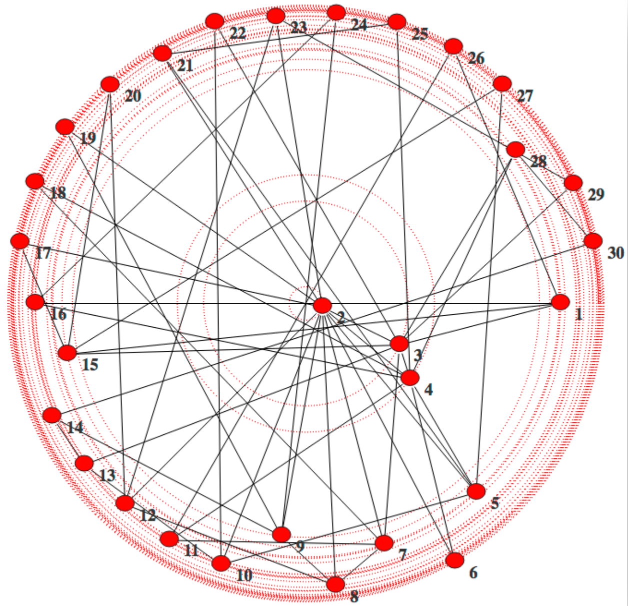


- Node size encodes centrality
- Conflict between position and size channels

Visualizing Centrality

- Radial layout [Yee 2001]



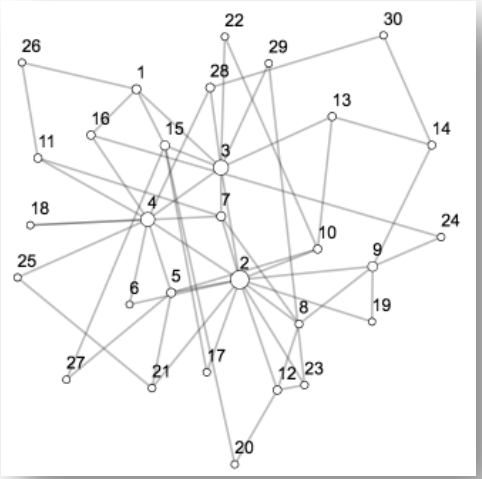


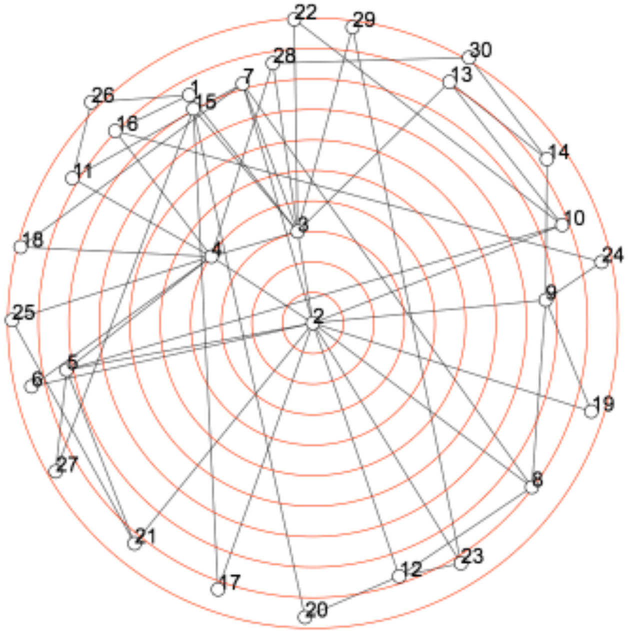
Distance from center encodes betweenness centrality.

* Figure generated using SocNetV.

Visualizing Centrality and Structure

- (Approximate) Distance preserving radial layout [Brandes 2009, Baingana 2014]

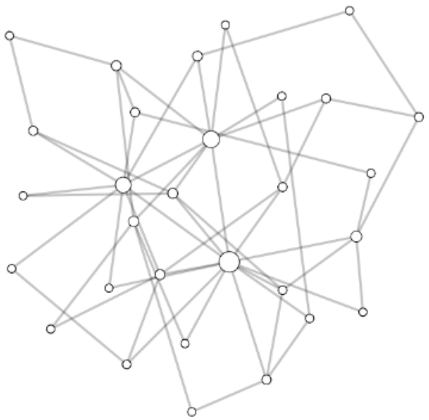




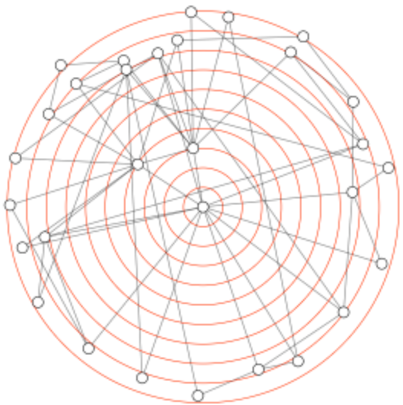
$$\text{Minimize:} \quad \underbrace{\sigma(X)}_{\text{MDS objective}} + \underbrace{\rho(X, \bar{c})}_{\text{Penalty}}$$

Objectives

- Convey centrality
- Avoid conflicting perceptual cues
- Reduce structural (distance) distortion



MDS layout preserves structure



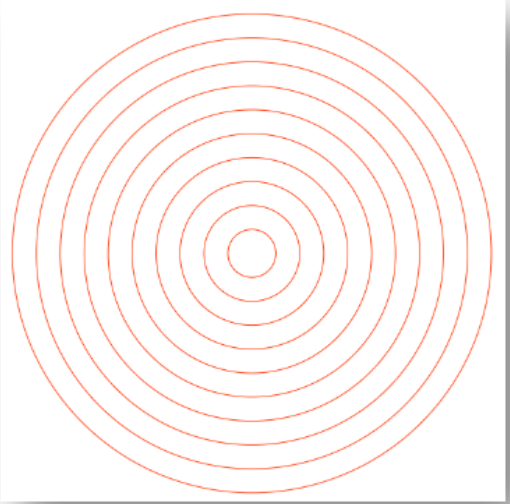
Radial layout highlights node centrality

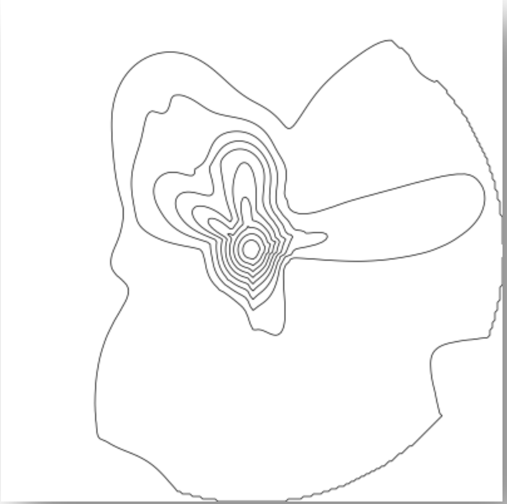
?

Highlight centrality and minimize distance distortion

Anisotropic Radial Layouts

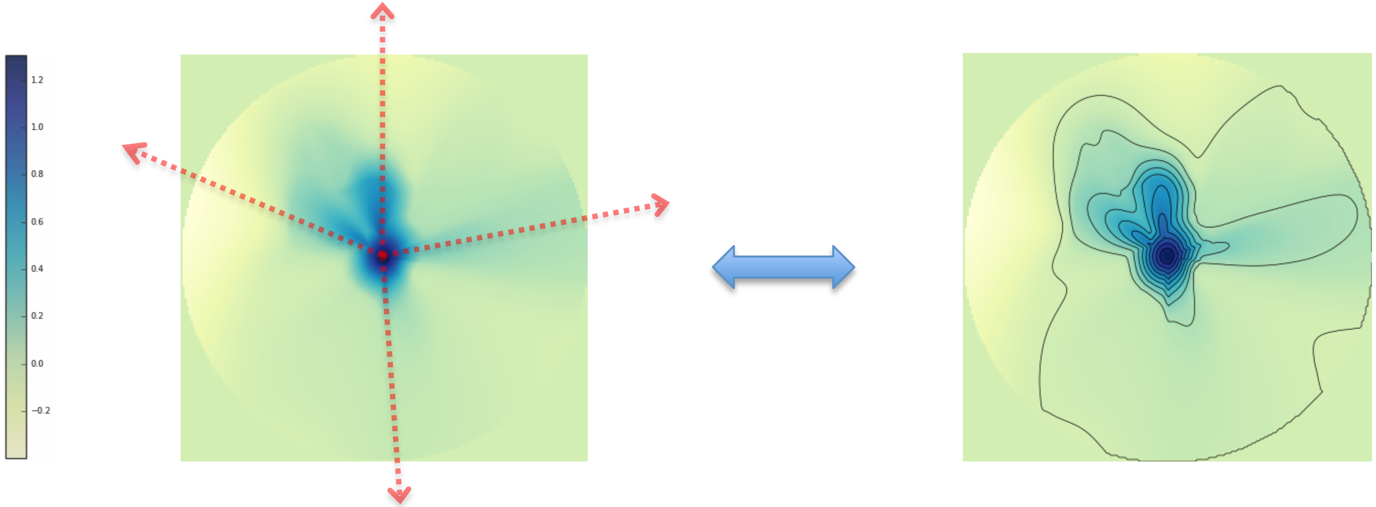
- Relax circular constraint
- Use star shaped curves





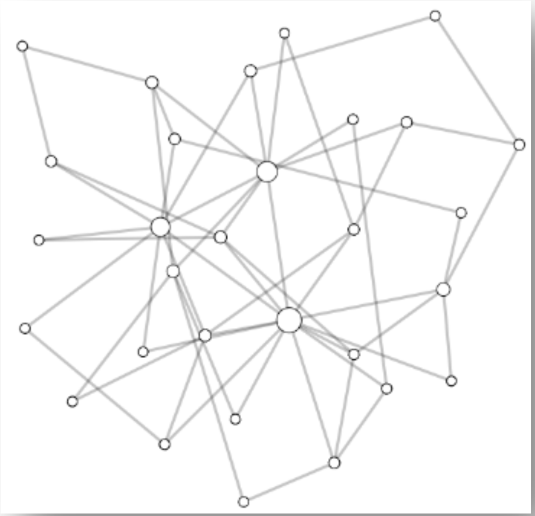
Radial Monotonicity

- Strictly decreasing along all center outward directions
- Guarantees star shaped contours



Step1: Initialization

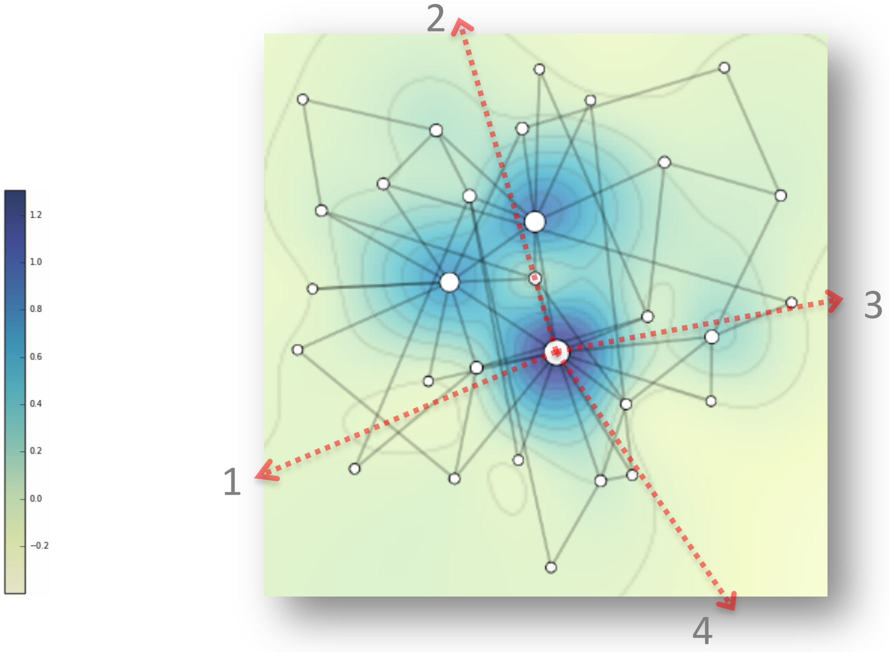
- Initialize positions using MDS.



Node sizes encode centrality.

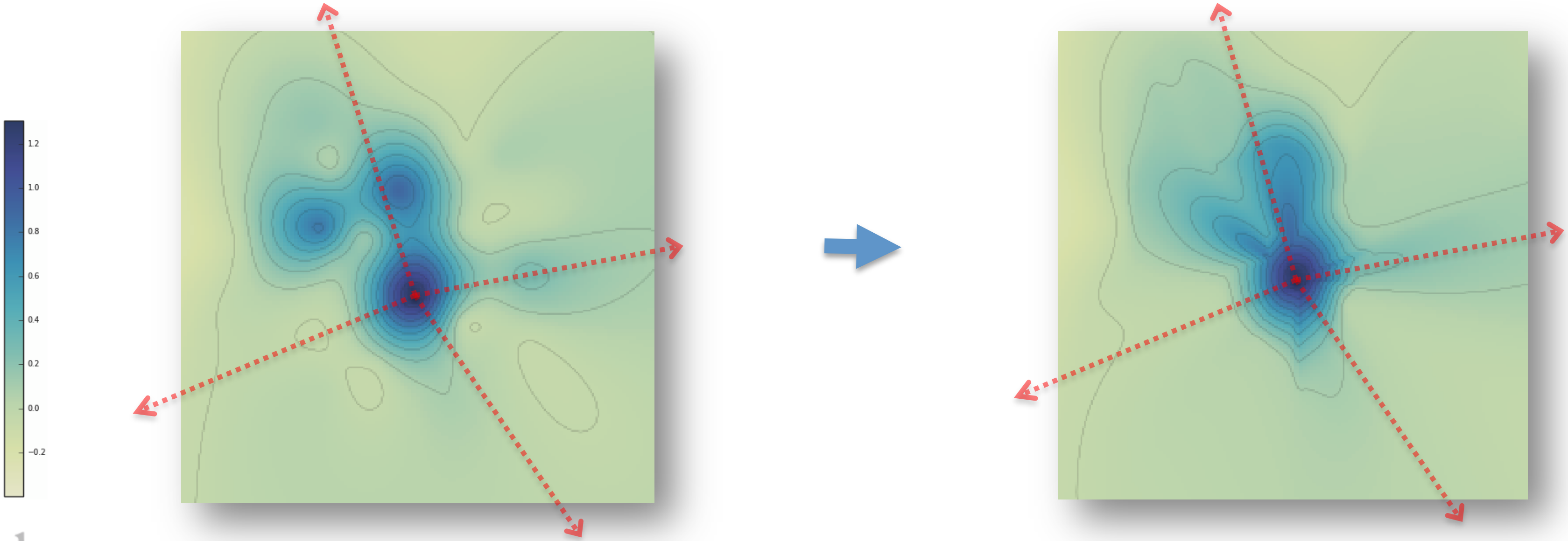
Step2: Interpolation

- Smooth interpolation of the centrality (e.g. using thin plate spline)



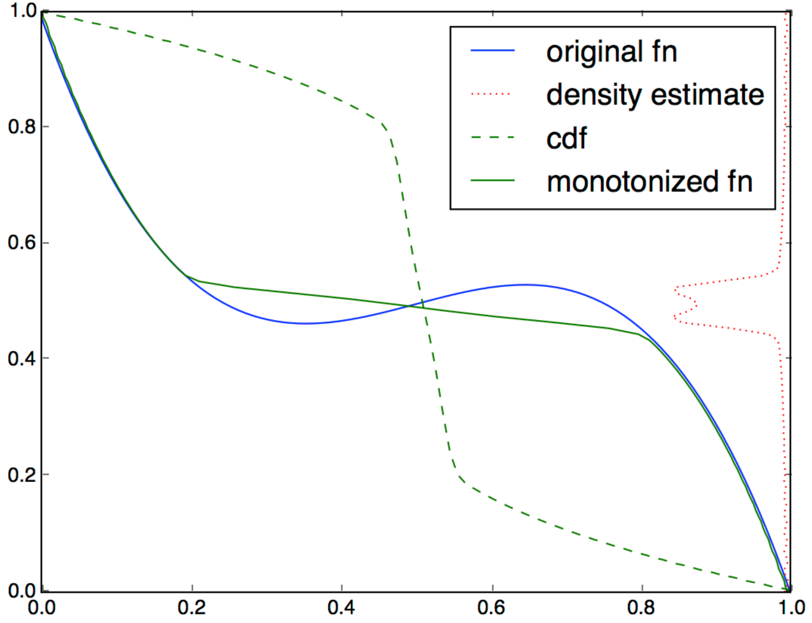
Step 3: Monotonization

- Monotonizing along radial axis



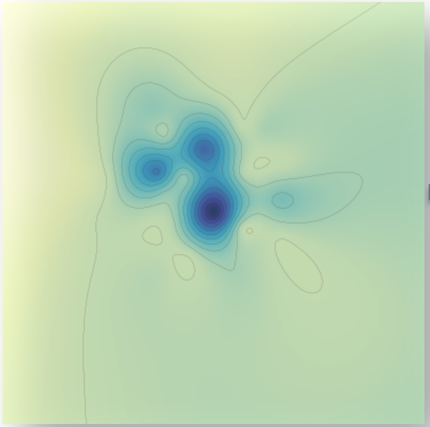
Step3: Monotonization

- Monotonizing a 1D function [Dette et al 2006]
- Steps
  1. Construct density estimate
  2. Compute cdf
  3. Invert cdf
- Output: smooth monotonic approximation

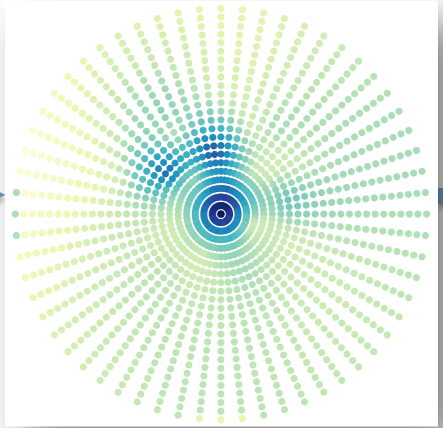


Step3: Monotonization

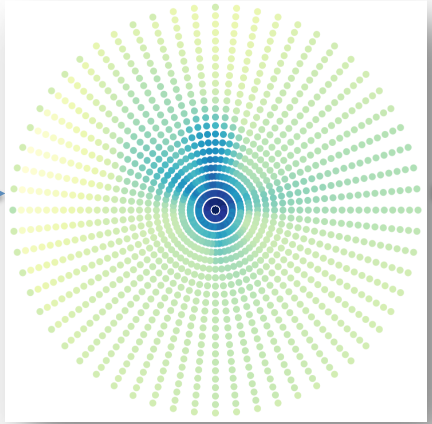
- Radially monotonizing a 2D field



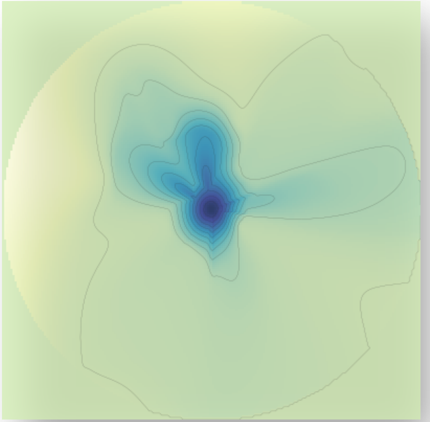
Interpolation field on a Cartesian grid



Resample on a regular **polar** grid



Monotonize along each angle



Interpolated and resampled **Monotonic field 'M'**

- If input field is smooth, independent 1D monotonization results in a smooth 2D field [Dette et al 2006]

Step 4: Optimization objective

- Combine MDS with centrality
- Add penalty for deviation of node centrality from associated field value

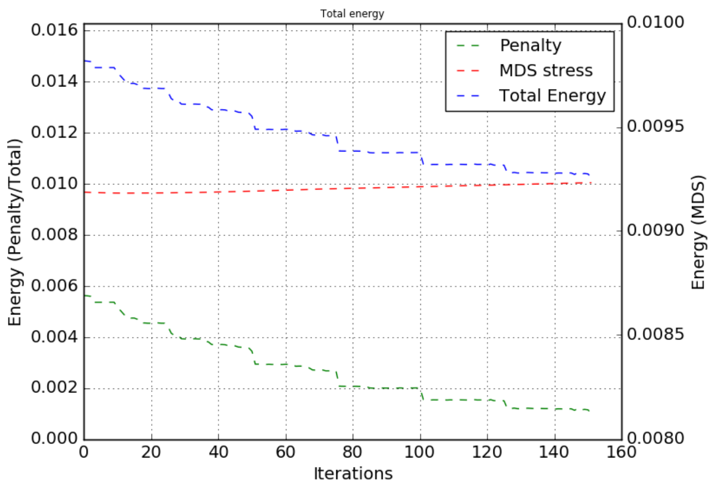
$$\gamma(X) = \underbrace{\sigma(X)}_{\text{MDS stress}} + w_\rho \underbrace{\left(M_{X,\bar{c}}(X) - \bar{c}\right)}_{\text{Penalty}}$$

$X \in \mathbb{R}^{n\times 2}$ $\bar{c} \in \mathbb{R}^n$ $M_{X,\bar{c}}(\cdot) :=$ Monotonic field

Step 5: Optimization

- Using gradient descent

$$\nabla\gamma(X) = \nabla\sigma(X) + w_\rho \times \underbrace{2\left(M_{X,\bar{c}}(X) - \bar{c}\right) \odot \nabla M_{X,\bar{c}}(X)}_{\text{Gradient of penalty}}$$



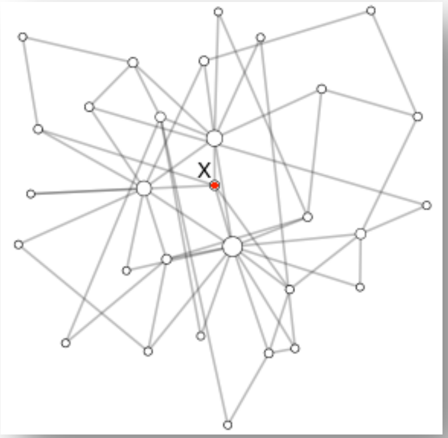
$X \in \mathbb{R}^{n\times 2}$

$\bar{c} \in \mathbb{R}^{n}$

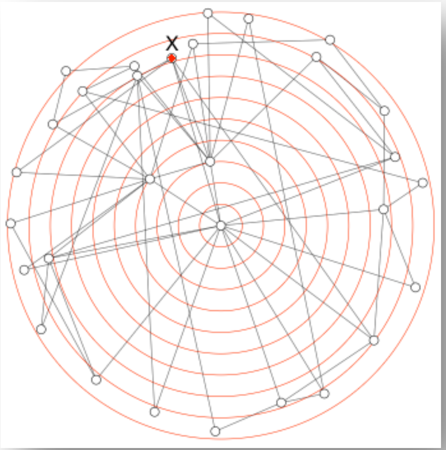
$M_{X,\bar{c}}(\cdot) :=$ Monotonic field

Results

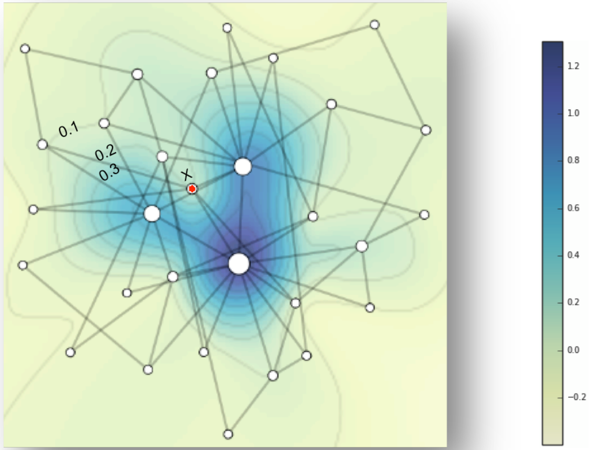
- Random graph generated using the Barabasi-Albert model.



MDS layout



Radial layout



Anisotropic radial layout

Results

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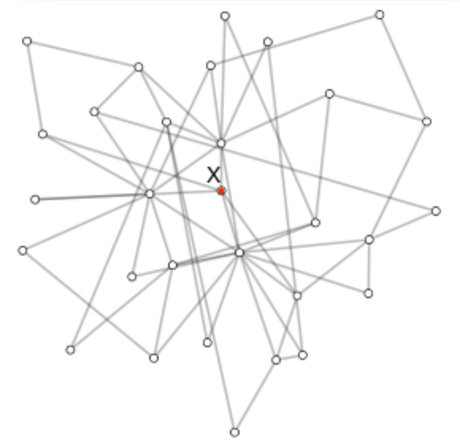
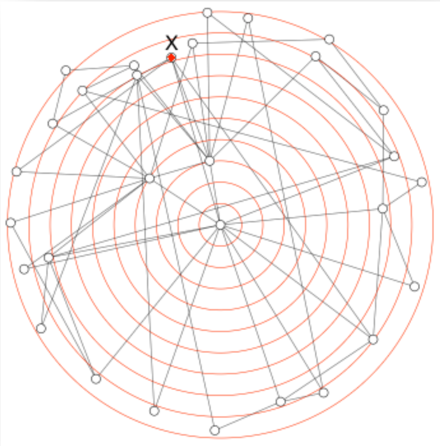
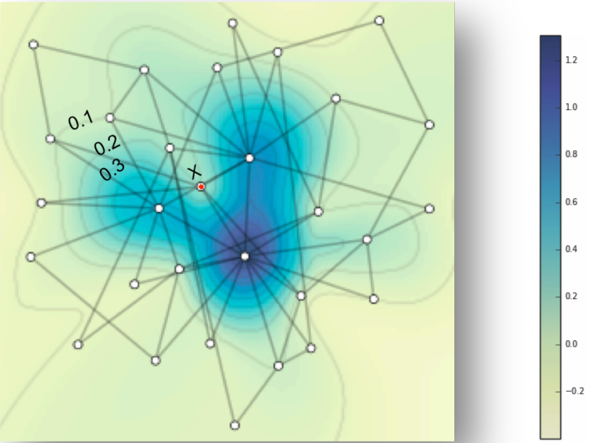


MDS layout



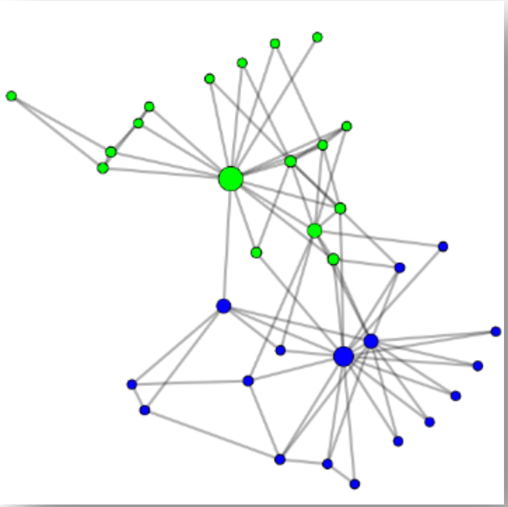
Radial layout



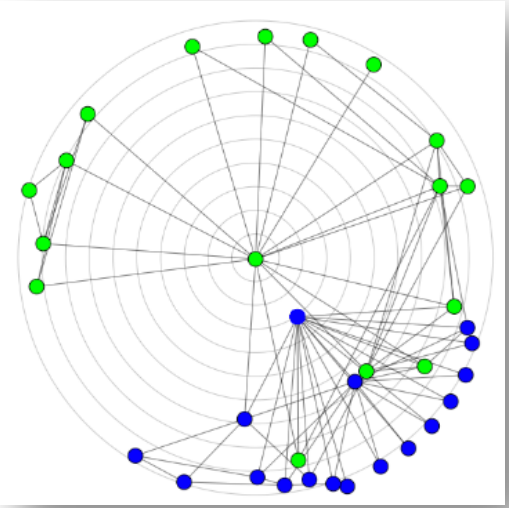
Anisotropic radial layout

Results

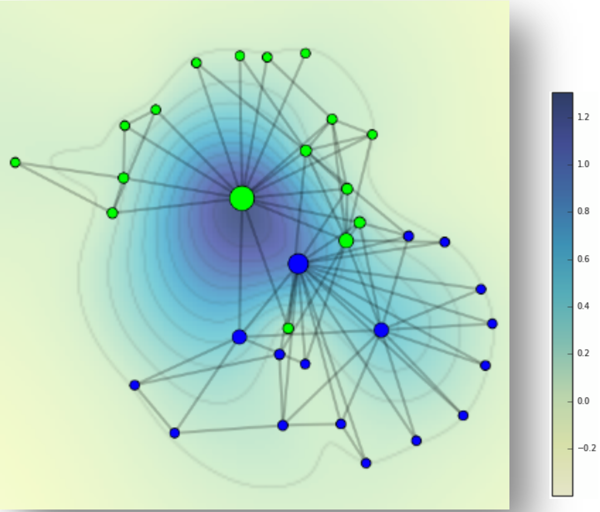
- Zachary's karate club network [Zachary 1977]



MDS layout



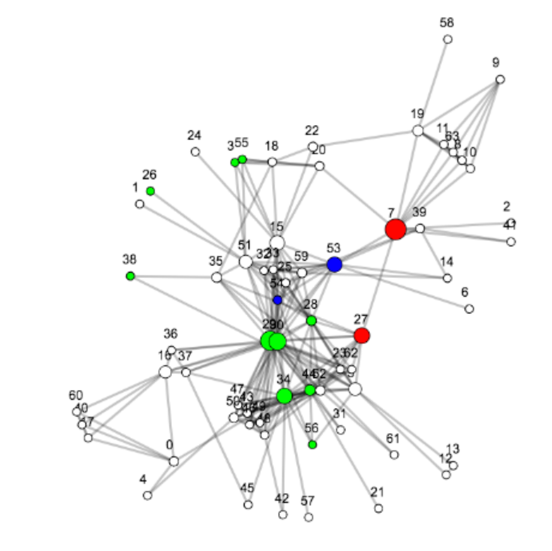
Radial layout



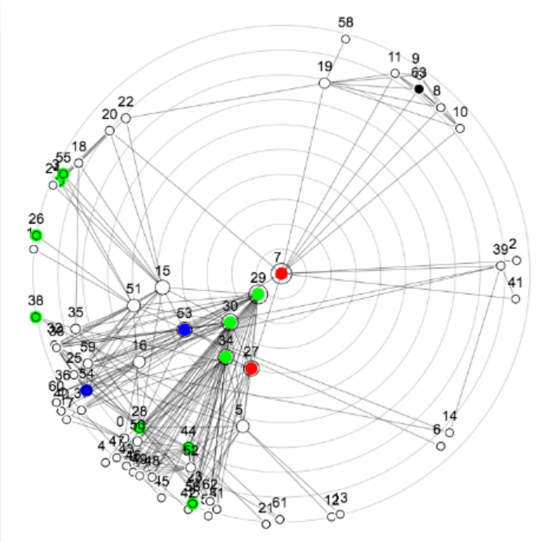
Anisotropic radial layout

Results

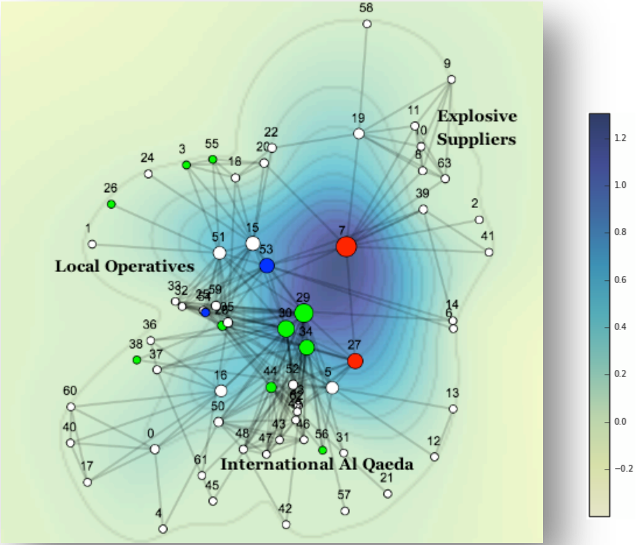
- Terrorist network from the Madrid train bombing incident in 2004 [Rodriguez 2005]



MDS layout



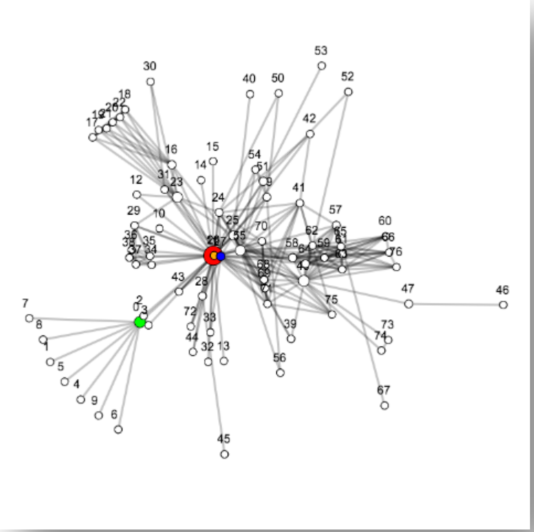
Radial layout



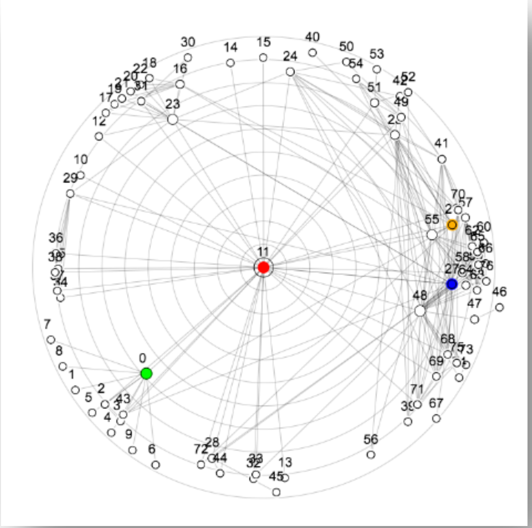
Anisotropic radial layout

Results

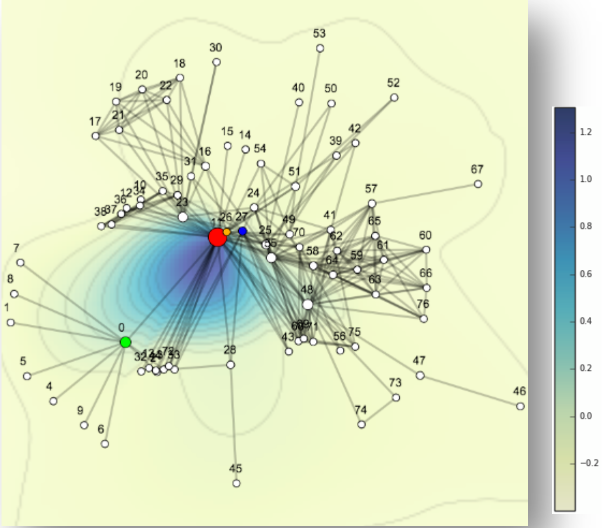
- Les Miserables character associations



MDS layout



Radial layout



Anisotropic radial layout

Conclusions

- Strategy for preserving centrality and structure
- Layout Algorithm
- Suitable for real world networks
- Future work
  - Automatic parameter estimation
  - Better optimization method
  - Experiment with large graph (higher node count)

Thanks

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