Colored Point-set Embeddings of Acyclic Graphs

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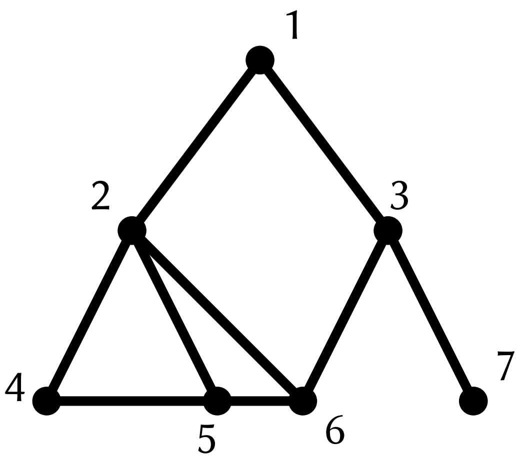
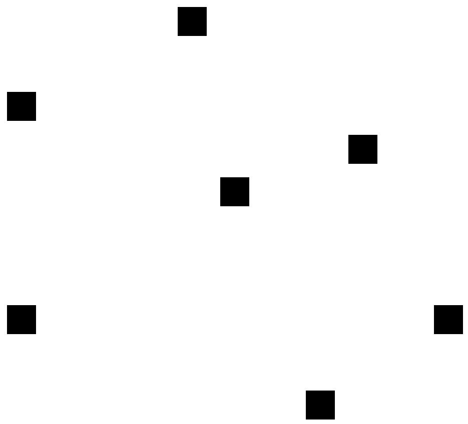
Point-set Embedding (PSE)

Given a planar graph G=(V, E) and a point set S ($|V|$=$|S|$)



G



S

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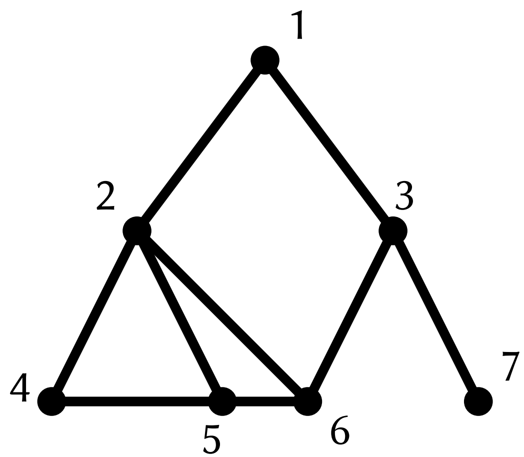
Given a planar graph G=(V, E) and a point set S ($|V|$=$|S|$)

A *Point-set embedding of G on S* is a planar drawing such that each vertex is represented by a point of S



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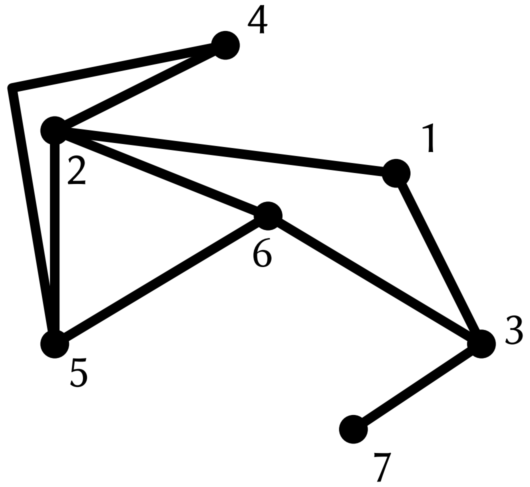

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Given a planar graph $G=(V,E)$ and a point set S ($|V|=|S|$)

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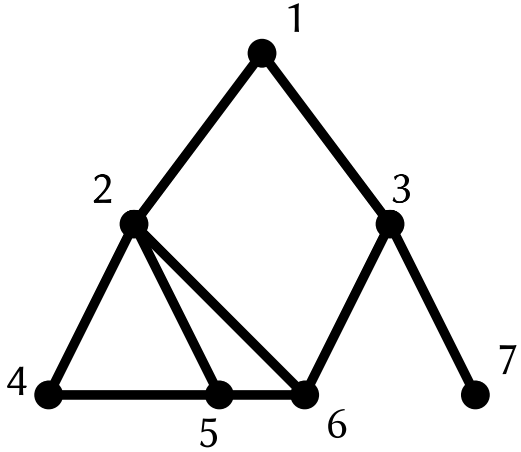
The *curve complexity (CC)* of a PSE is the maximum number of bends along any edge



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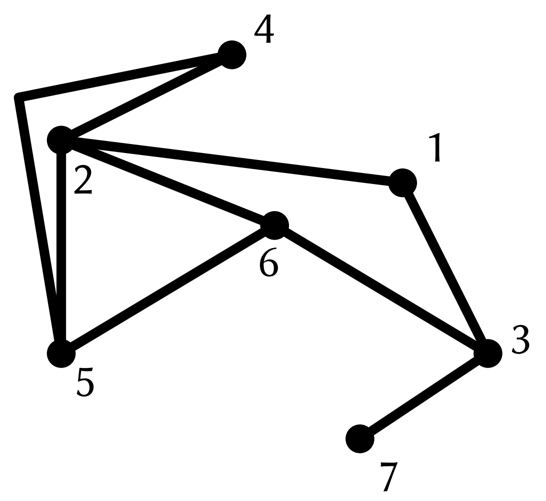

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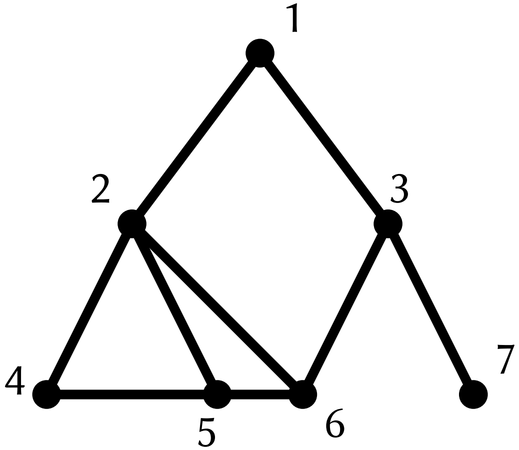

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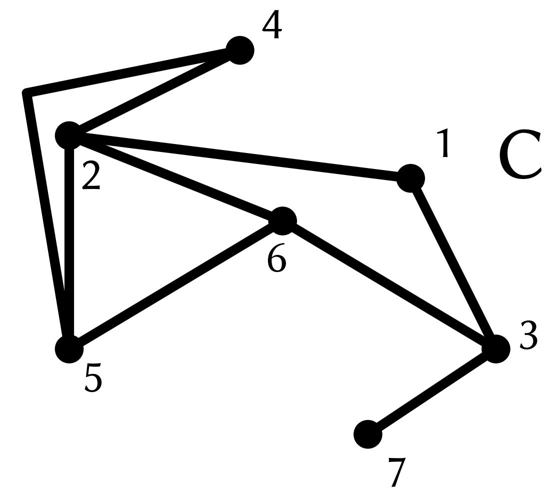

CC=1

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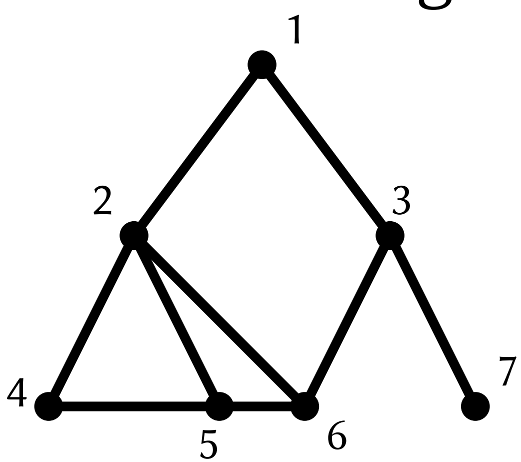

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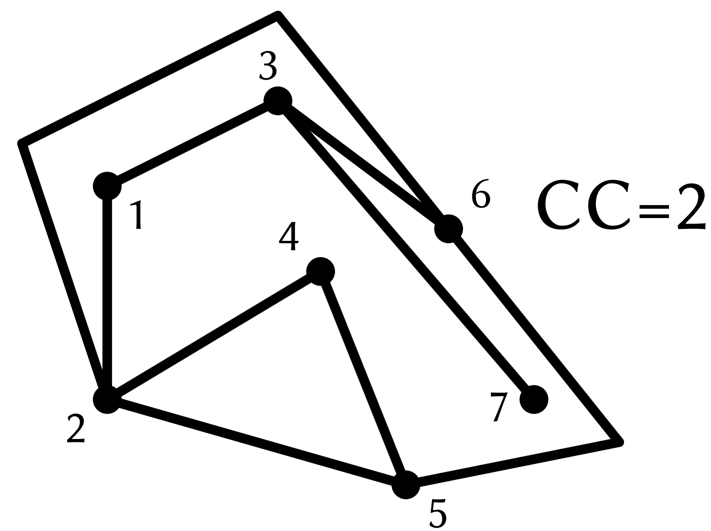

S

Colored PSE

In a *colored PSE* vertices and points are colored; a vertex can only be represented by a point of its color

Colors are used to describe the mapping between vertices and points. In particular:

- 1 color $\equiv$ No mapping
- n colors $\equiv$ Complete mapping
- $1 < k < n \equiv$ Partial mapping

PSE: Known Results (upper and lower bounds on CC)

	Paths	Caterp.	Trees	Outerpl.	Planar
1					
2					
3					
. . .	. . .	. . .	. . .	. . .	. . .
n					

[1] Badent, Di Giacomo, Liotta TCS 2008 [2] Bose CGTA 2002 [3] Di Giacomo et al. JGAA 2008
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	Paths	Caterp.	Trees	Outerpl.	Planar
1				0 [2]	
				0 [trivial]	
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3					
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2					
3					
...	...	...	...	...	...
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		Paths	Caterp.	Trees	Outerpl.	Planar
1	upper	0	0	0	0 [2]	2 [6]
	lower	0 [trivial]	0 [trivial]	0 [trivial]	0 [trivial]	2 [6]
2	upper					
	lower					
3	upper					
	lower					
...		...	...	...	...	...
n	upper					$O(n)$ [7]
	lower	$\Omega(n)$ [7]				

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2					
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...	...	...	...	...	...
n	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$ [7]
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2	1 [4]	2 [4]		5 [3]	
	1 [5]				
3					
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
n	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$ [7]
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	lower	1 [5]	1	1	1	
3	upper					
	lower					
…		…	…	…	…	…
n	upper	$O(n)$	$O(n)$	$O(n)$	$O(n)$	$O(n)$ [7]
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	lower	1 [5]	1	1	1	$\Omega(n)$ [1]
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$\cdots$		$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
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	lower	1 [5]	1	1	1	$\Omega(n)$ [1]
3	upper					
	lower				$\Omega(\sqrt[3]{n})$ [3]	
$\cdots$		$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
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Our results

- There exists a 3-colored forest of 3 stars F_n and a point set S_n such that in any PSE of F_n on S_n there are $\Omega(n^{\frac{2}{3}})$ edges with $\Omega(n^{\frac{1}{3}})$ bends.

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$\Rightarrow$ There exists a 3-colored caterpillar C_n and a point set S_n such that in any PSE of C_n on S_n there are $\Omega(n^{\frac{2}{3}})$ edges with $\Omega(n^{\frac{1}{3}})$ bends.

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- Every 3-colored path admits a PSE with CC ≤ 5 onto any 3-colored point set.

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• Every 4-colored path such that the vertices of two colors precede all the vertices of the other two colors admits a PSE with CC ≤ 5 onto any 4-colored point set.

3-Colored PSE of stars

PSE of stars in the real world

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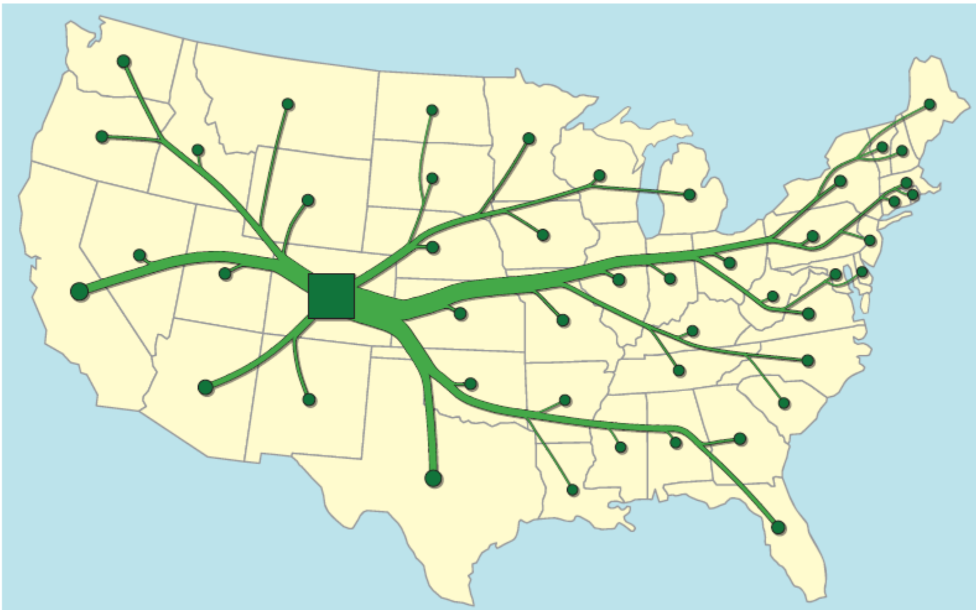
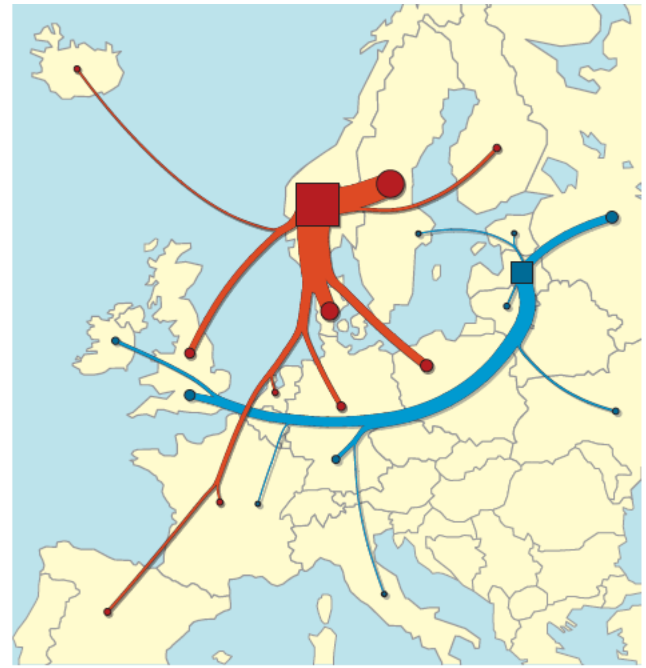
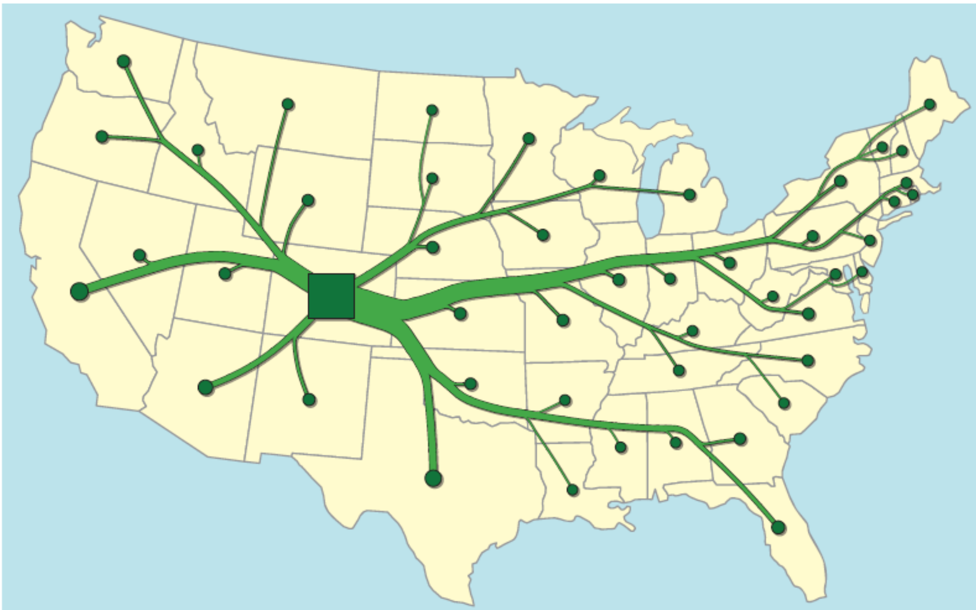


Verbeek, Buchin, Speckmann IEEE TVCG 2011



Flow maps

PSE of stars in the real world



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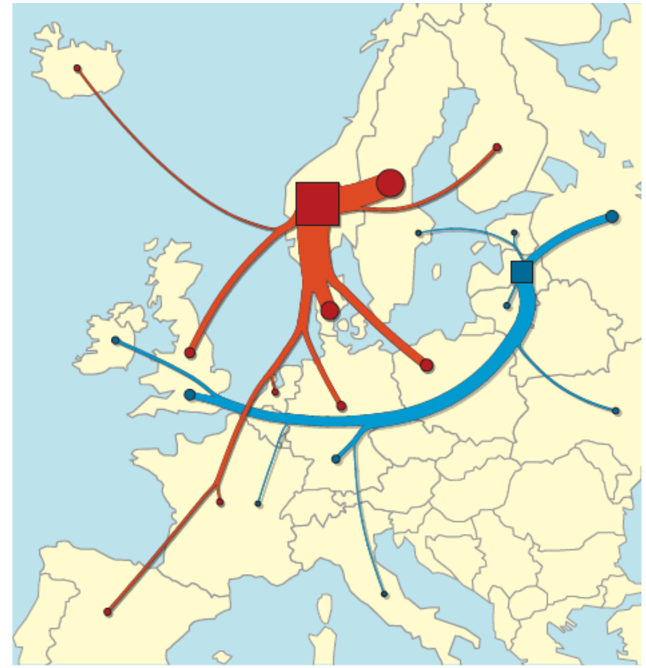

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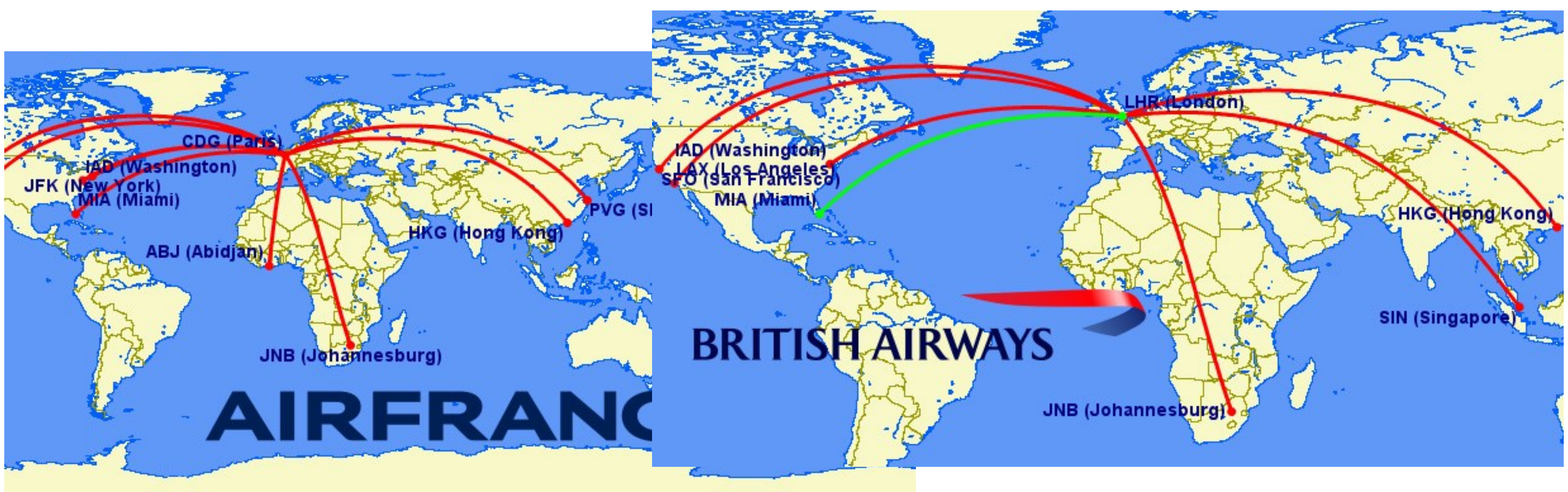

Route maps

http://weekendblitz.com/airbus-a380-current-routes-operators/

3-colored PSE of stars

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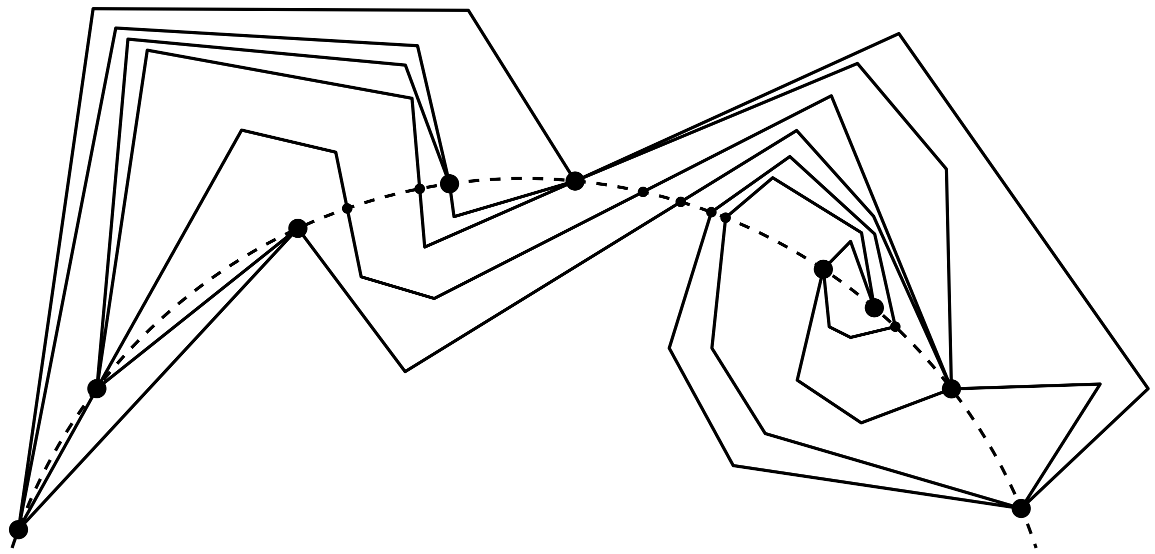
What I will show you:

- There exists a 3-colored forest of 3 stars F_n and a point set S_n such that in any PSE of F_n on S_n there is at least one edge with $\Omega(n^{\frac{1}{3}})$ bends.

A useful result

Theorem 1 *If a graph G has a PSE on a semiconvex point set S s.t. each crosses $CH(S)$ at most b times, then G admits a PSE on S with $CC \leq 2b + 1$.*



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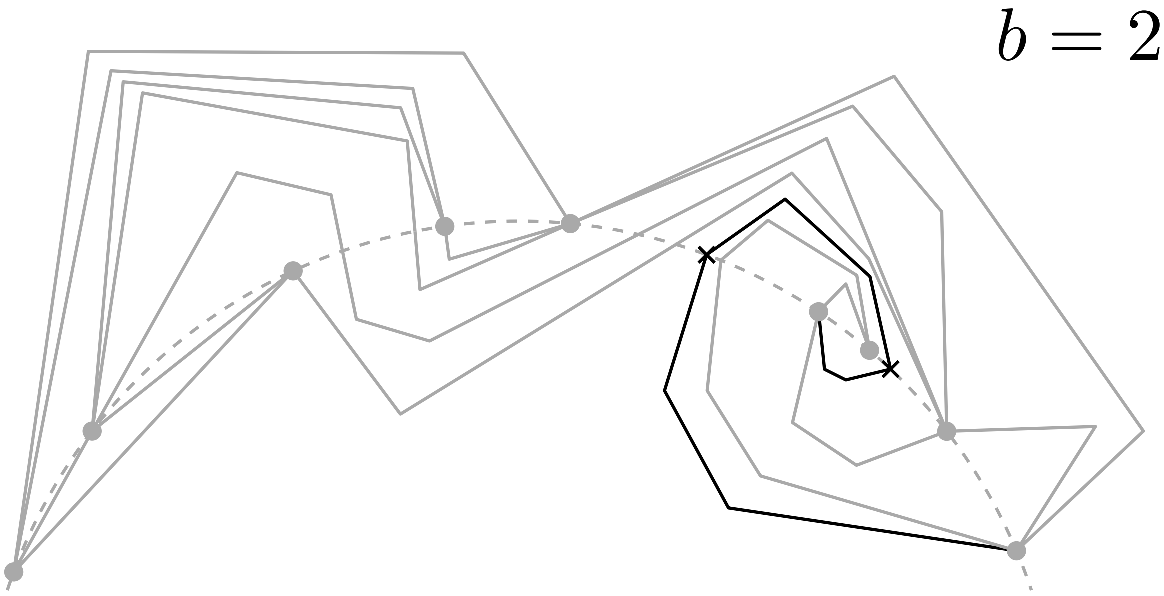

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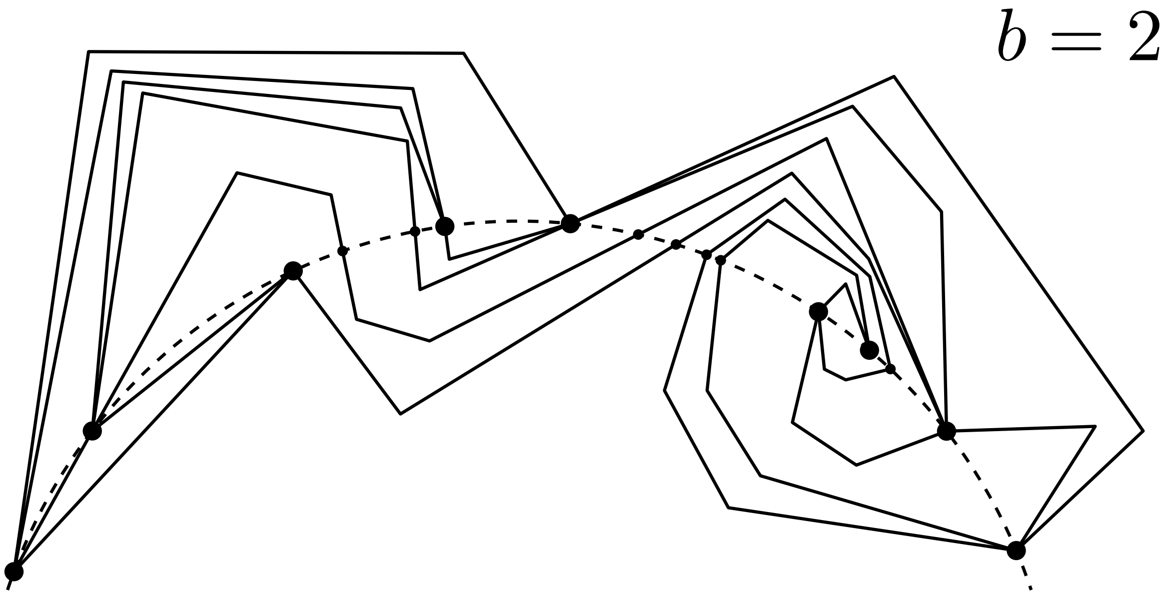

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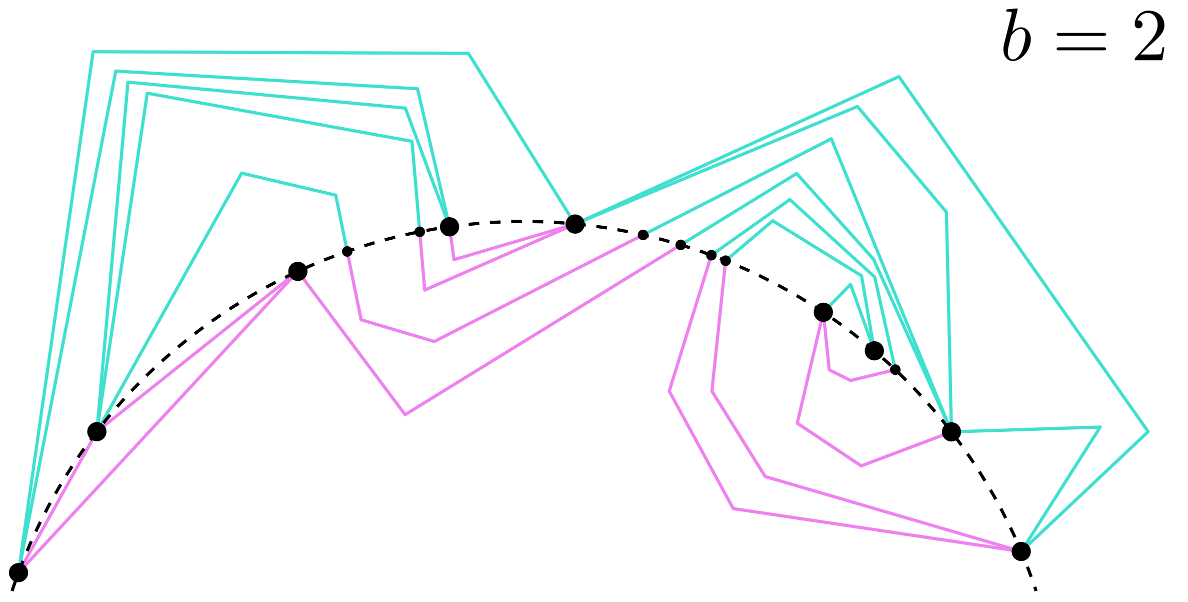

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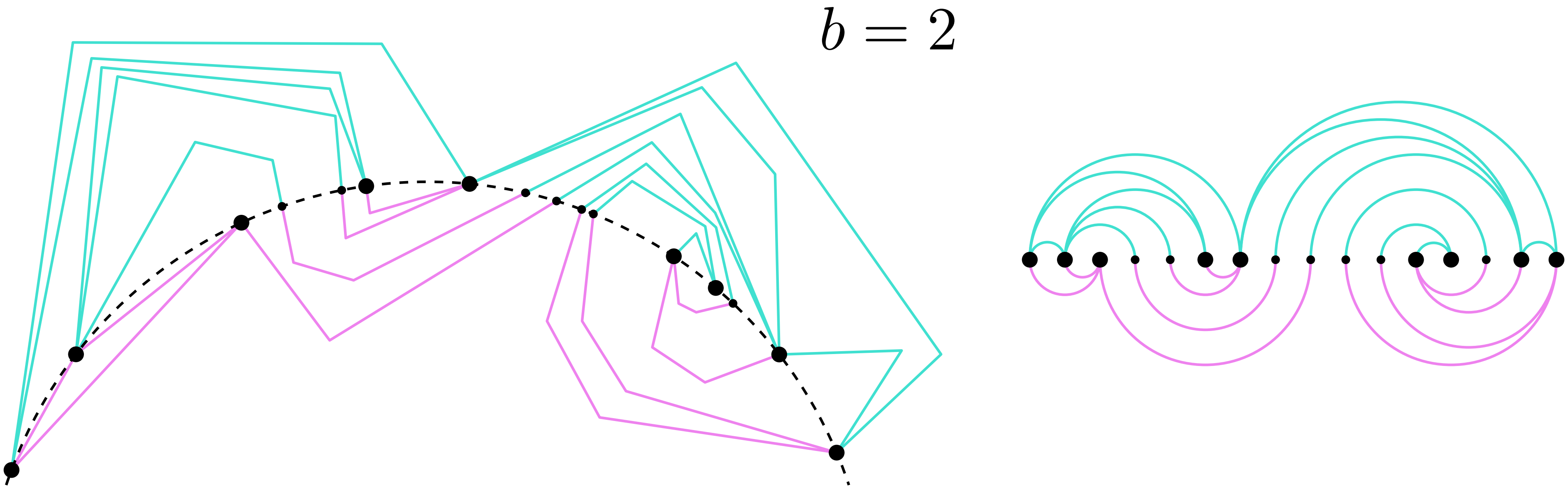

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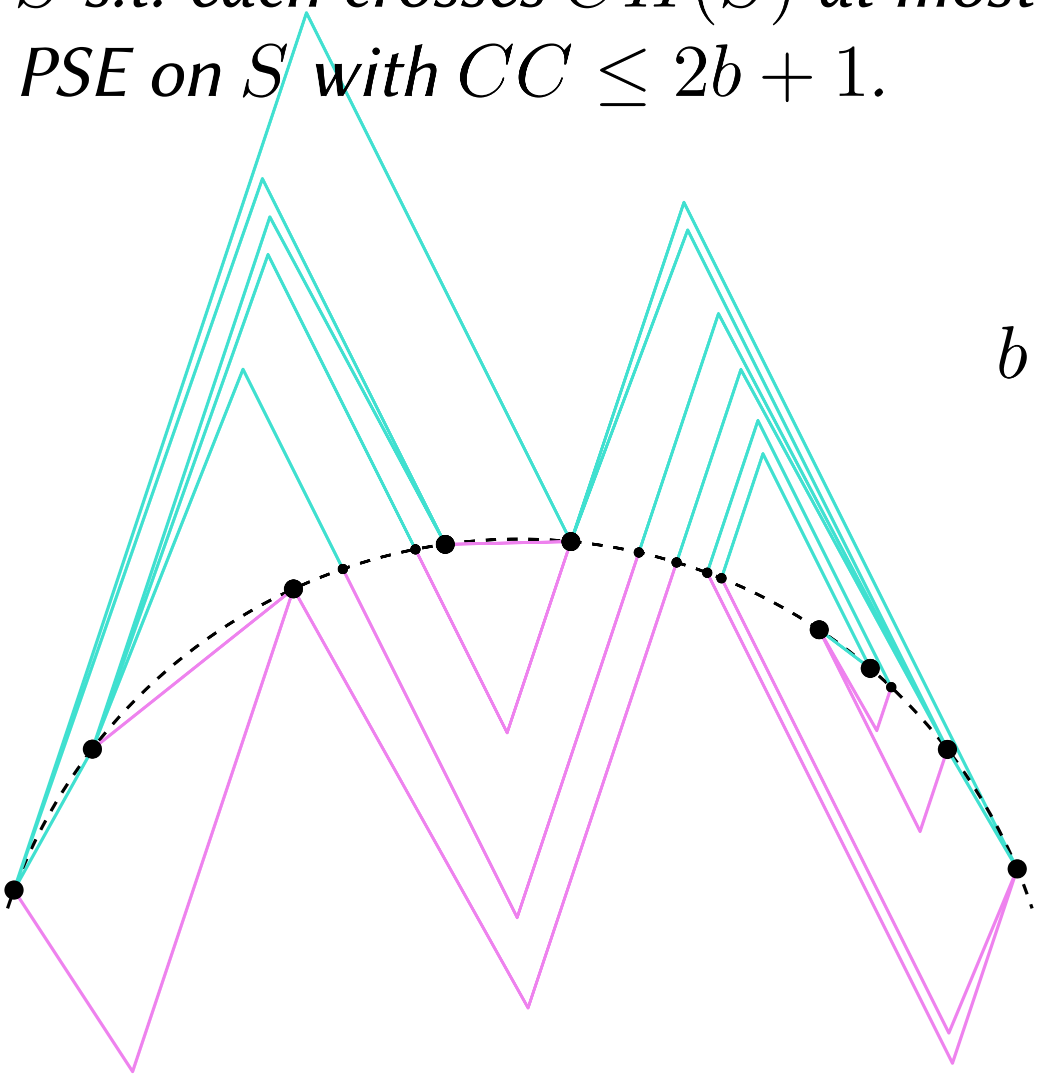
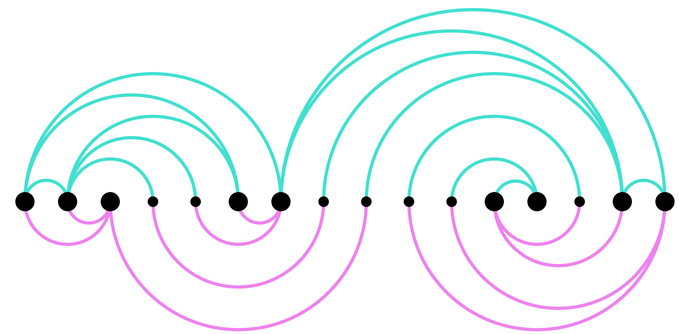


$b = 2$



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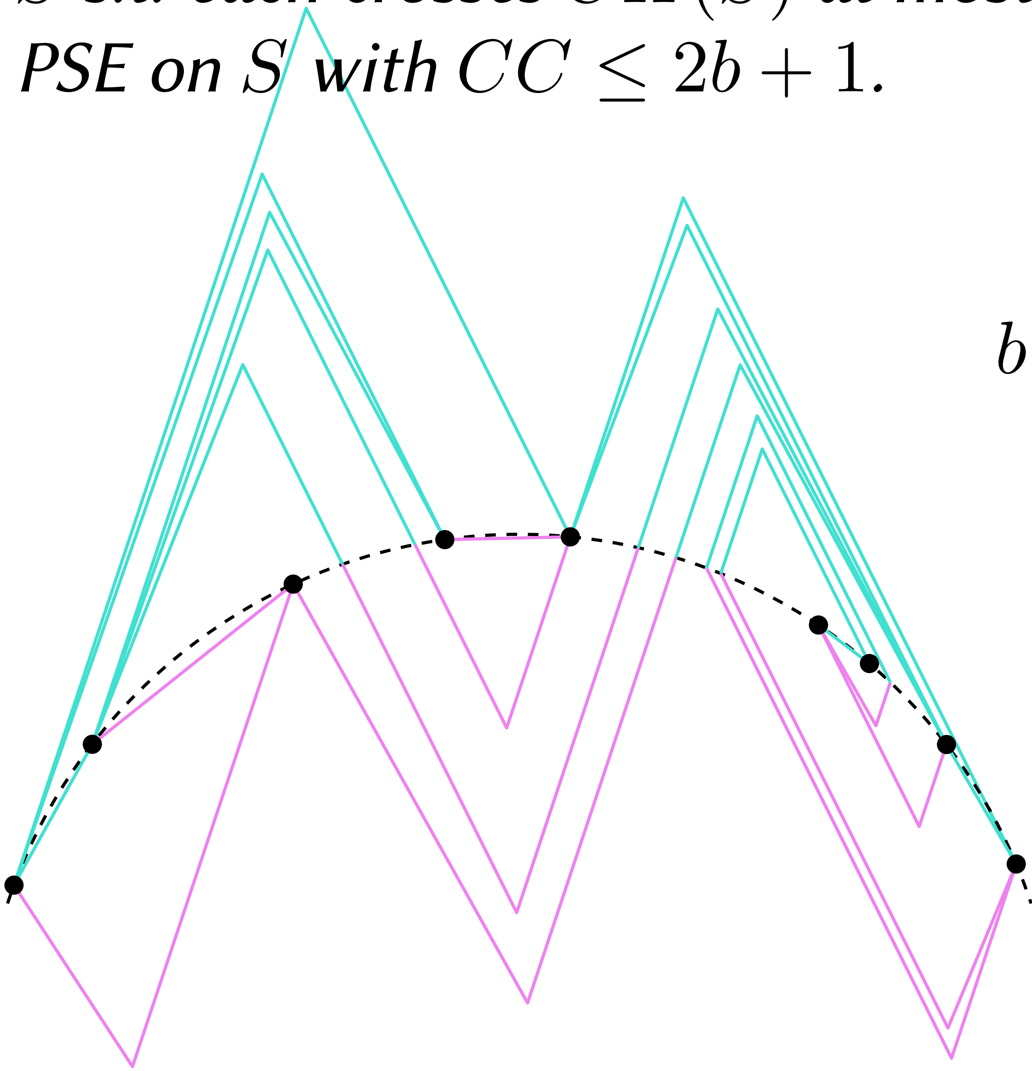

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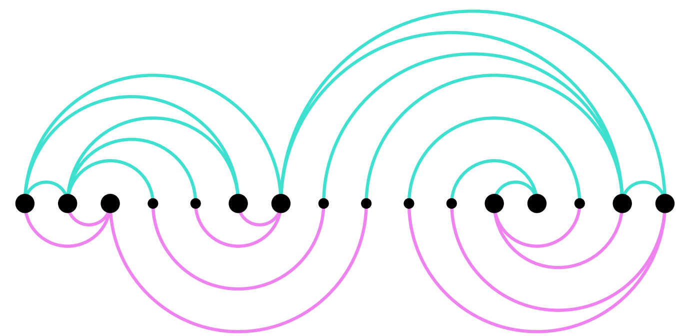

A useful result

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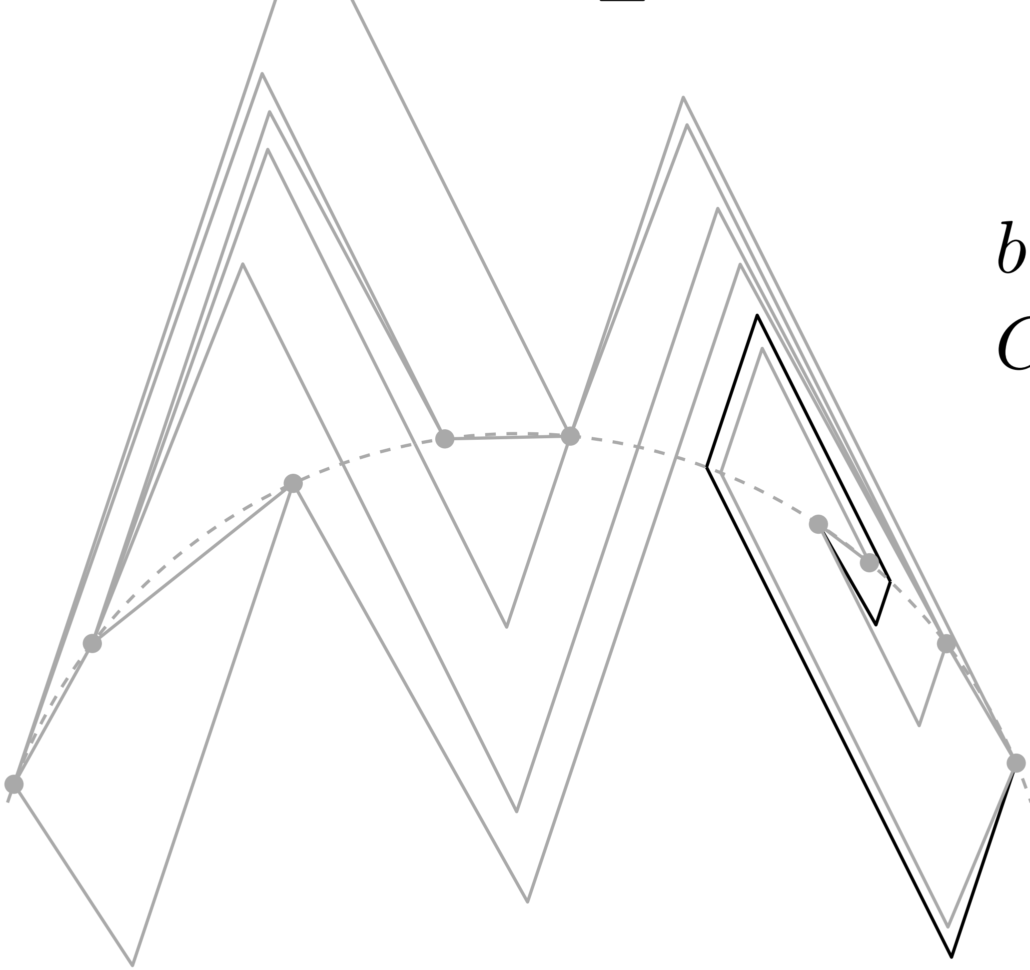
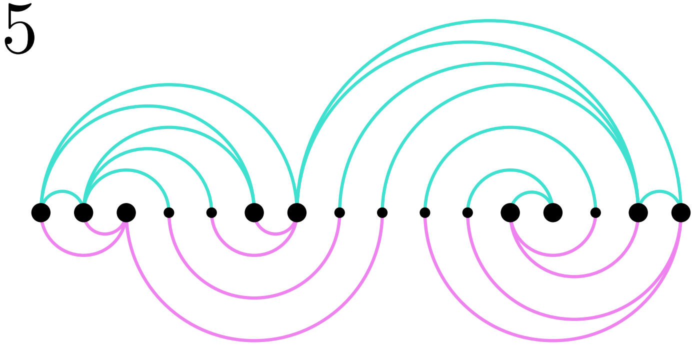


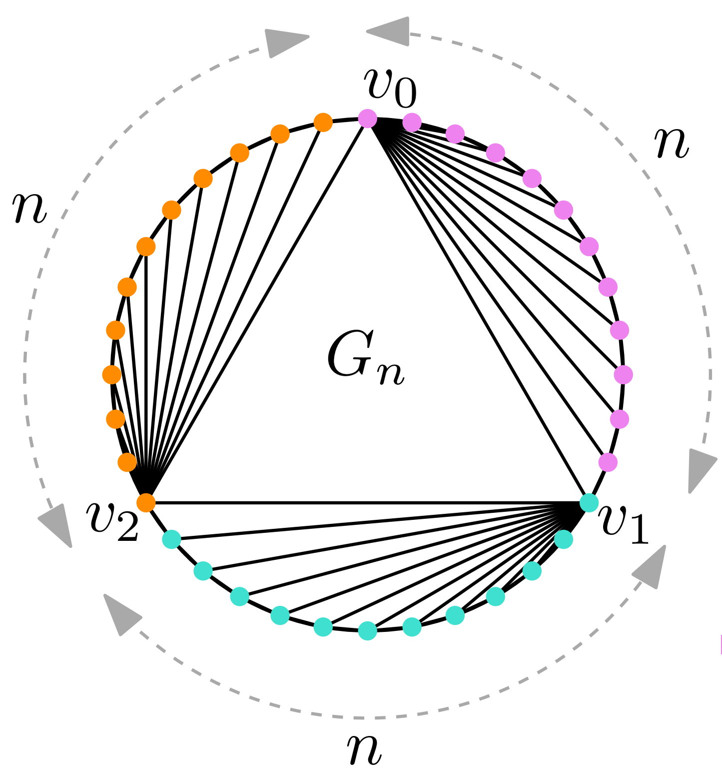
$b = 2$

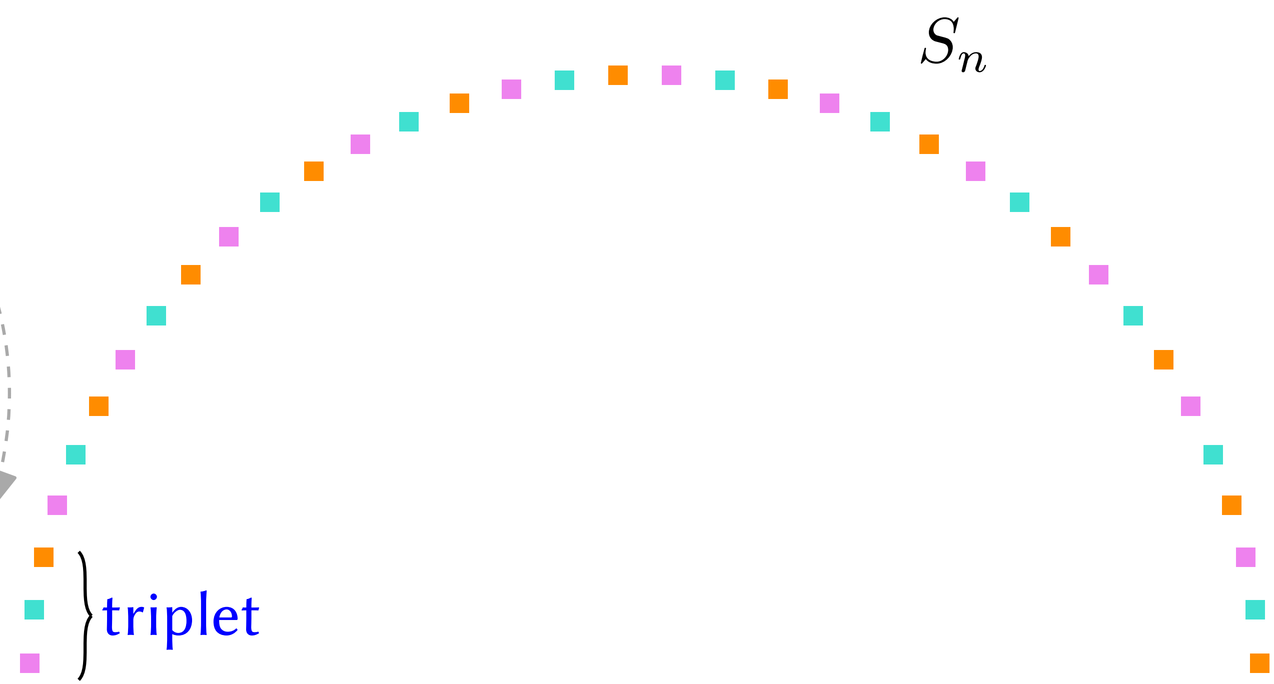
$CC = 5$



Another useful result

Theorem 2 [3] *For every $h > 0$ there exists a 3-colored biconnected outerplanar graph G_n, with $n \geq 79h^3$, and a 3-colored set of points S_n s.t. in every PSE of G_n on S_n there is at least one edge with more than h bends.*



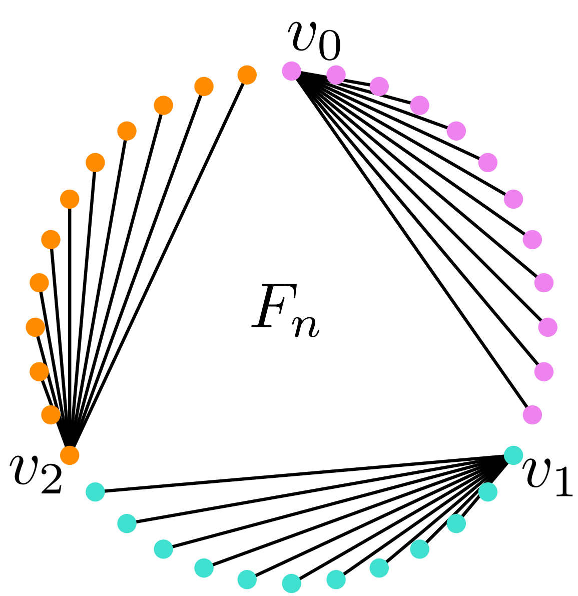


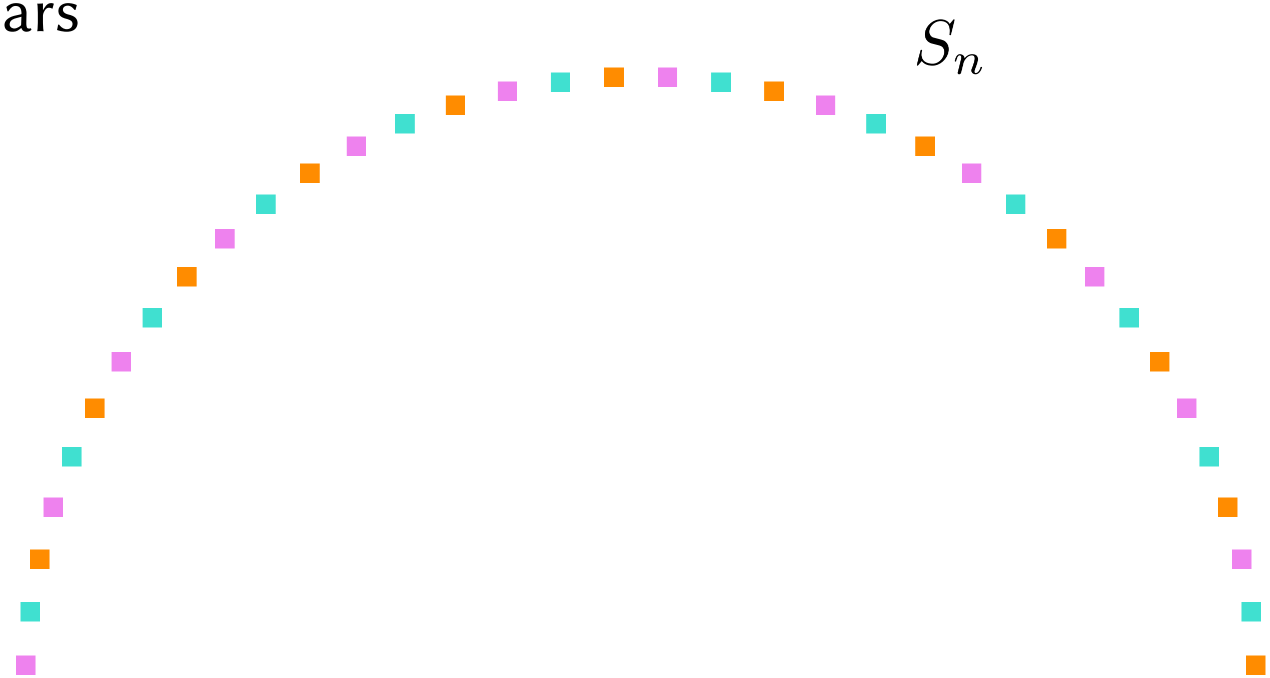
[3] Di Giacomo et al. JGAA 2008

Another useful result

Theorem 2 [3] *For every* $h > 0$ *there exists a* 3*-colored biconnected outerplanar graph* G_n*, with* $n \geq 79h^3$*, and a* 3*-colored set of points* S_n *s.t. in every PSE of* G_n *on* S_n *there is at least one edge with more than* h *bends.*

A 3-colored forest of 3 stars





[3] Di Giacomo et al. JGAA 2008

Another useful result

Theorem 2 [3] *For every $h > 0$ there exists a 3-colored biconnected outerplanar graph G_n, with $n \geq 79h^3$, and a 3-colored set of points S_n s.t. in every PSE of G_n on S_n there is at least one edge with more than h bends.*

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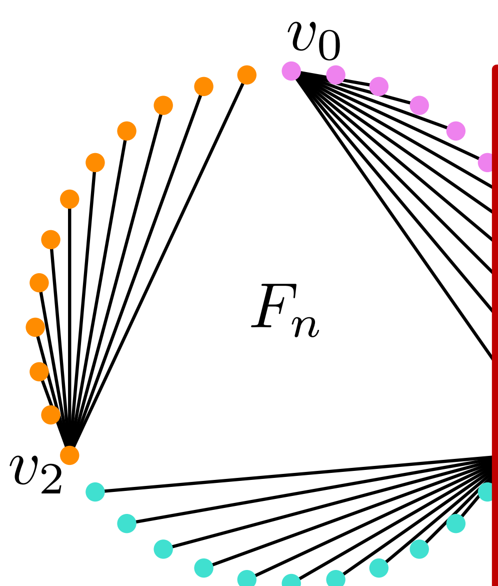
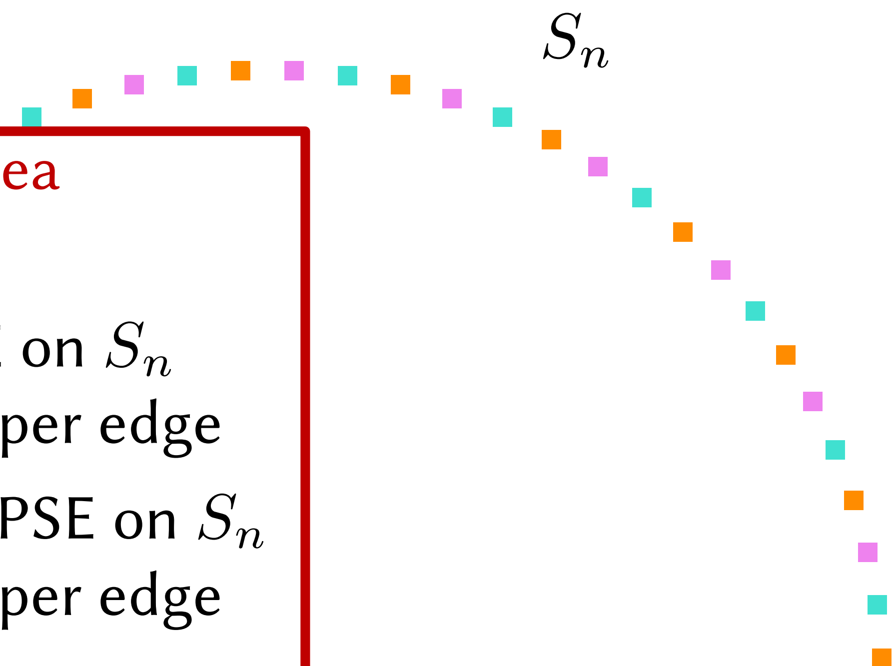


High-level idea

We prove that

if F_n admits a PSE on S_n with $o(n^{\frac{1}{3}})$ bends per edge

then G_n admits a PSE on S_n with $o(n^{\frac{1}{3}})$ bends per edge



[3] Di Giacomo et al. JGAA 2008

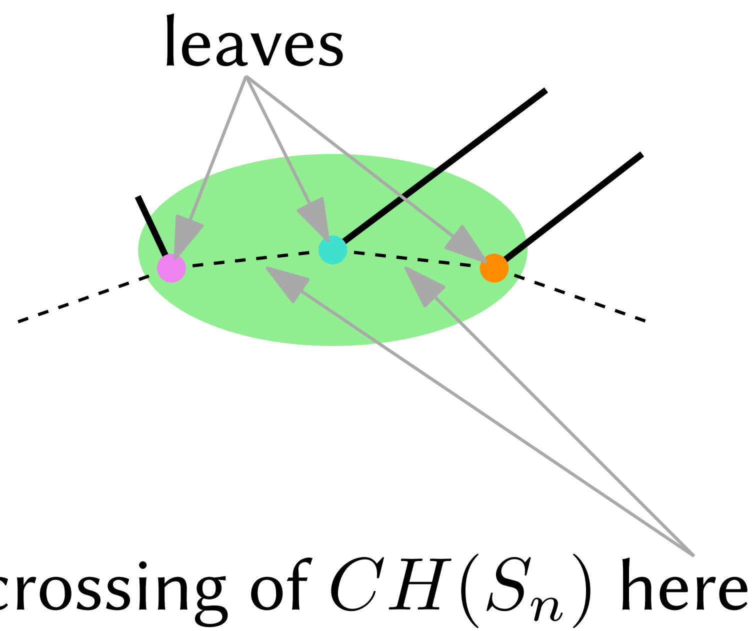
From a PSE of F_n to a PSE of G_n

Lemma 1 *If F_n has a PSE on S_n s.t.*

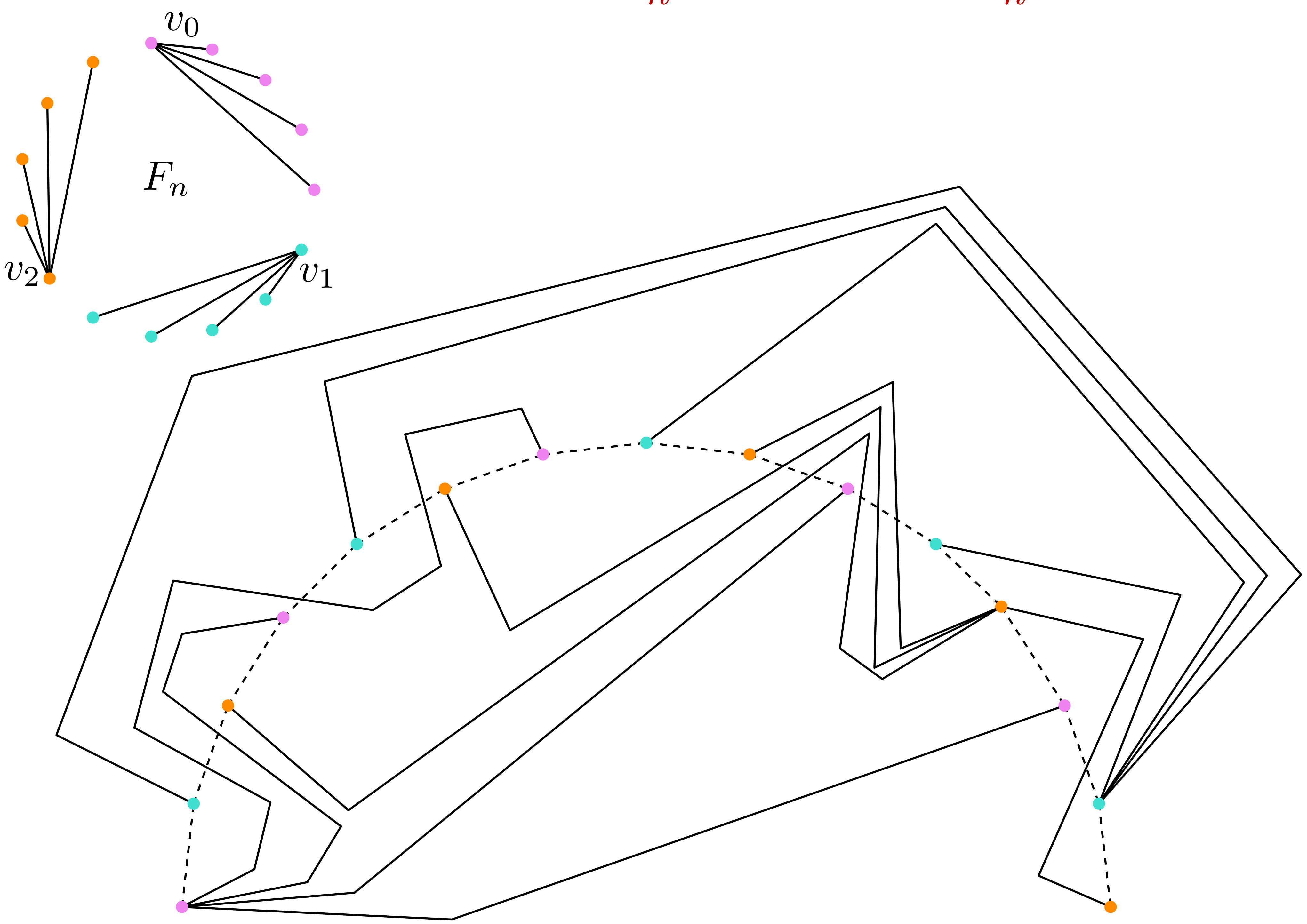
1. *each edge crosses $CH(S_n)$ at most b times;*
2. *there exists an* uncrossed triplet

then G_n has a PSE on S_n such that each edge crosses $CH(S_n)$ at most $3b + 2$ times.

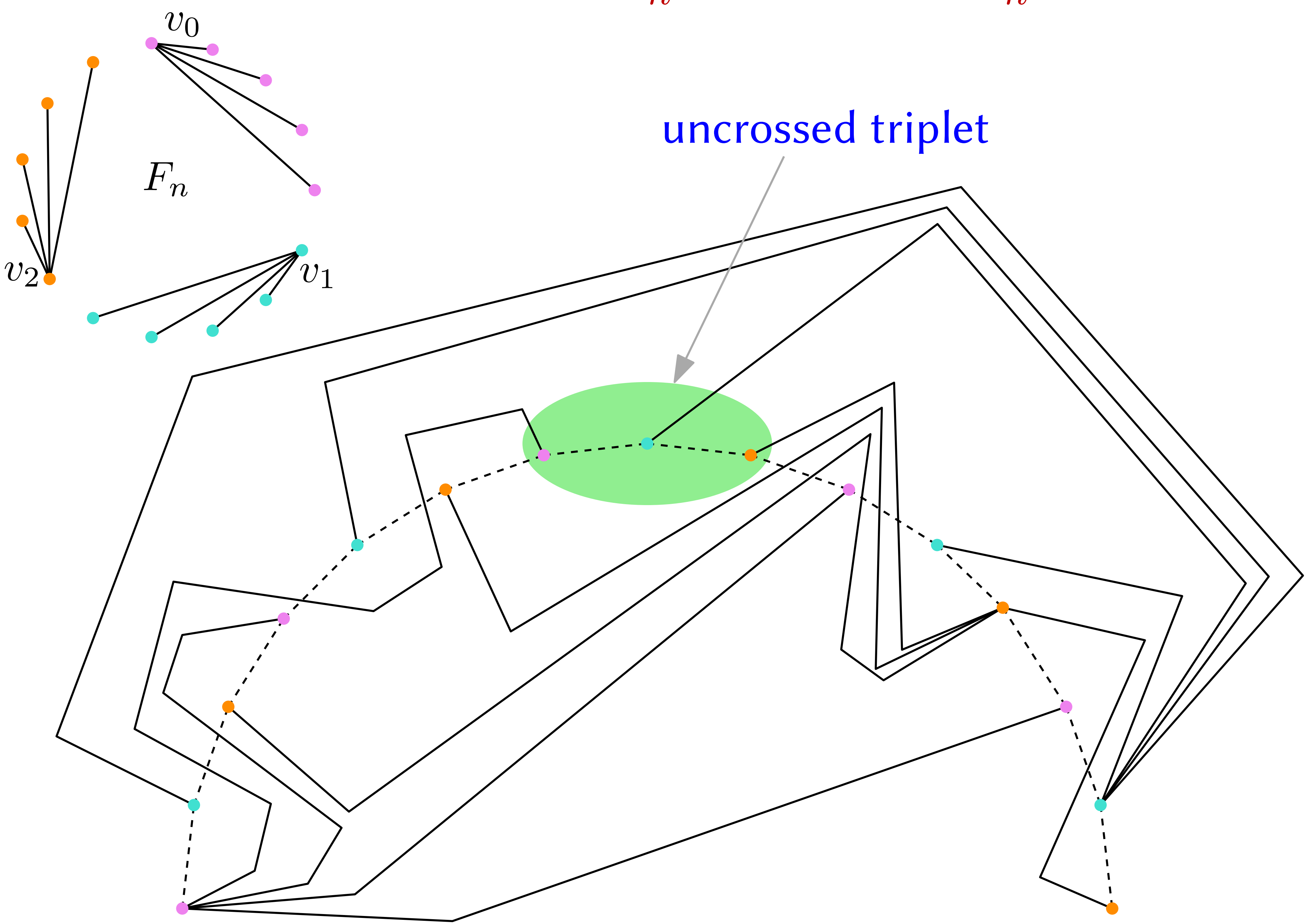
uncrossed triplet



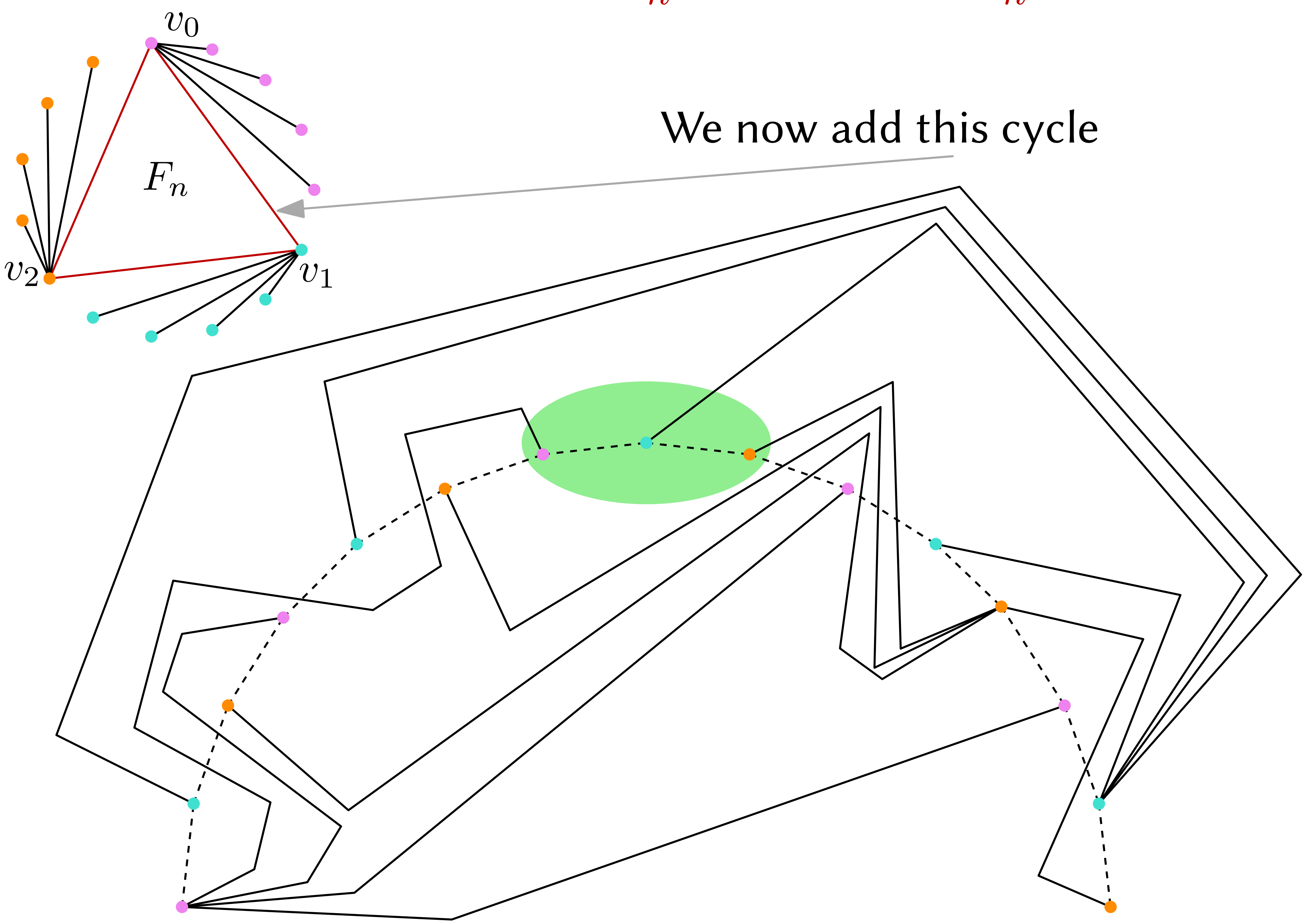
From a PSE of F_n to a PSE of G_n



From a PSE of F_n to a PSE of G_n



From a PSE of F_n to a PSE of G_n



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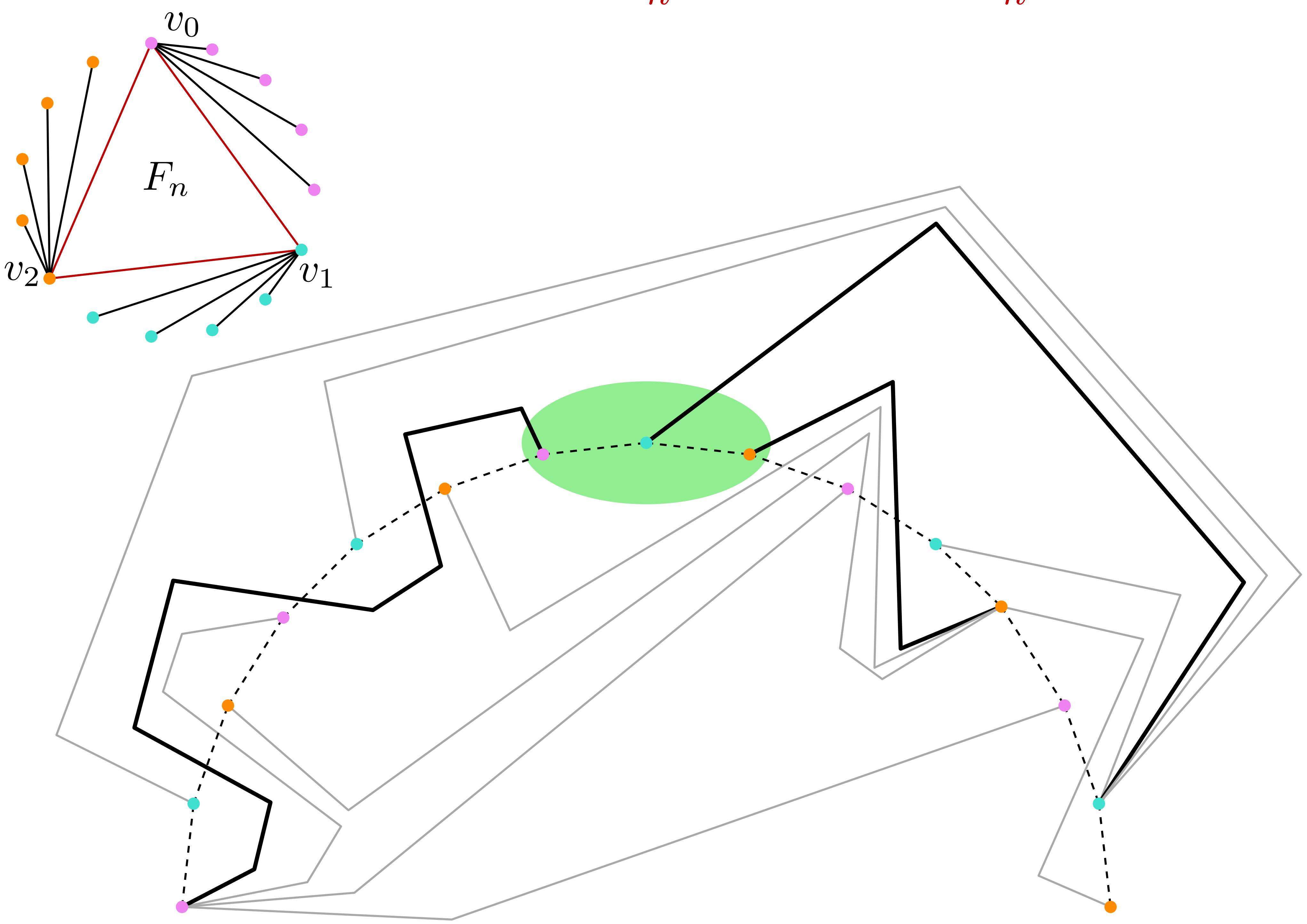
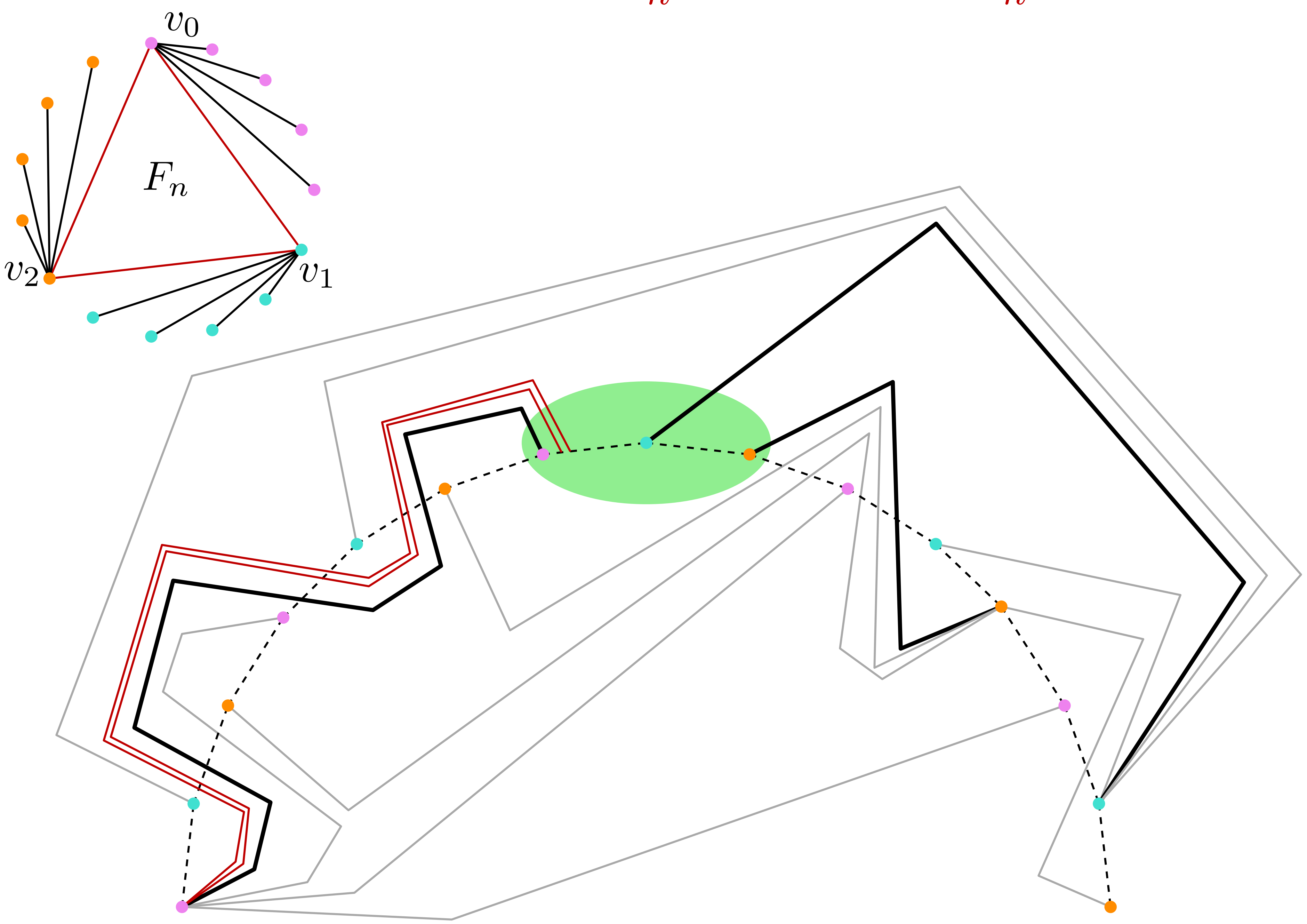
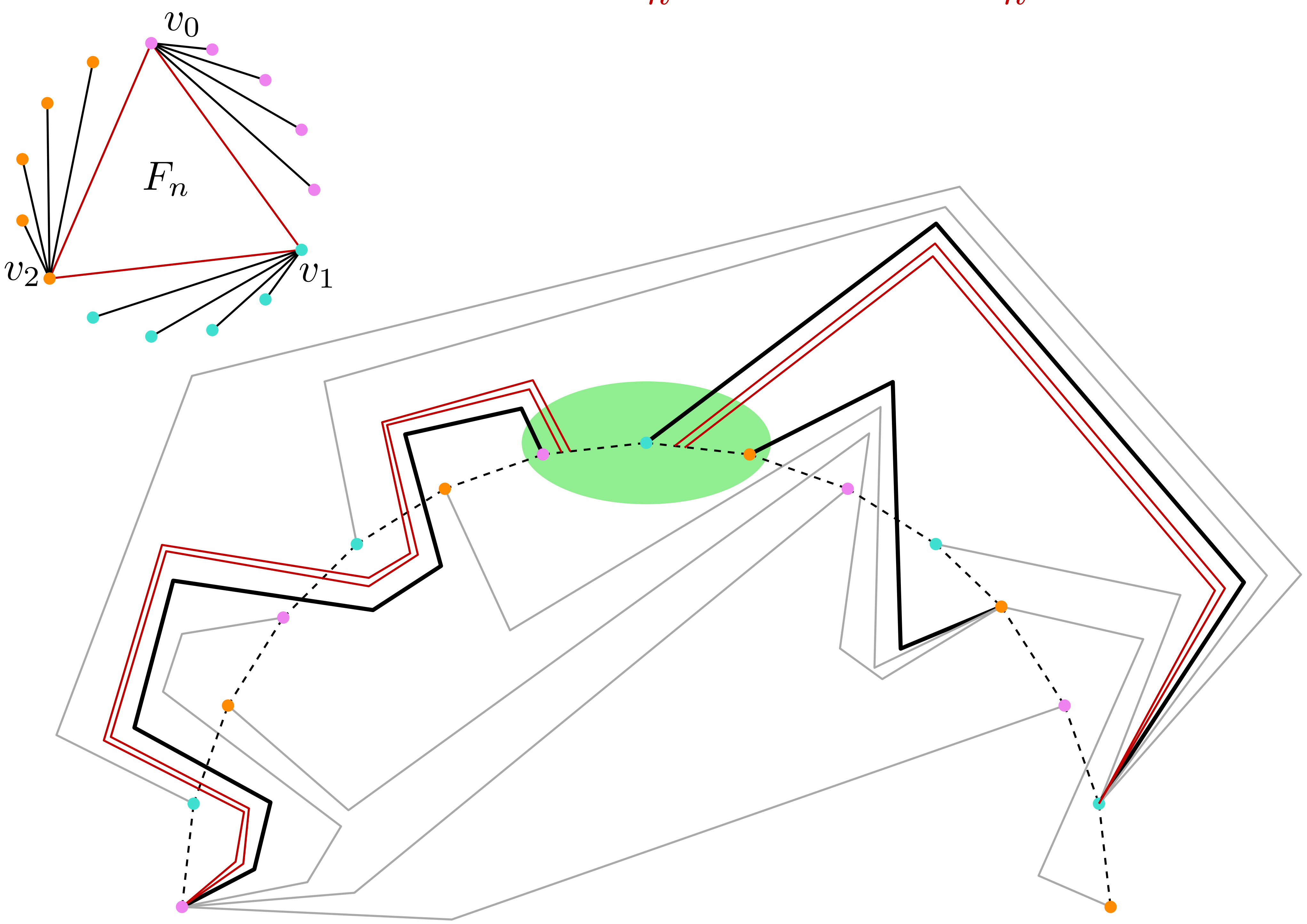


From a PSE of F_n to a PSE of G_n



From a PSE of F_n to a PSE of G_n



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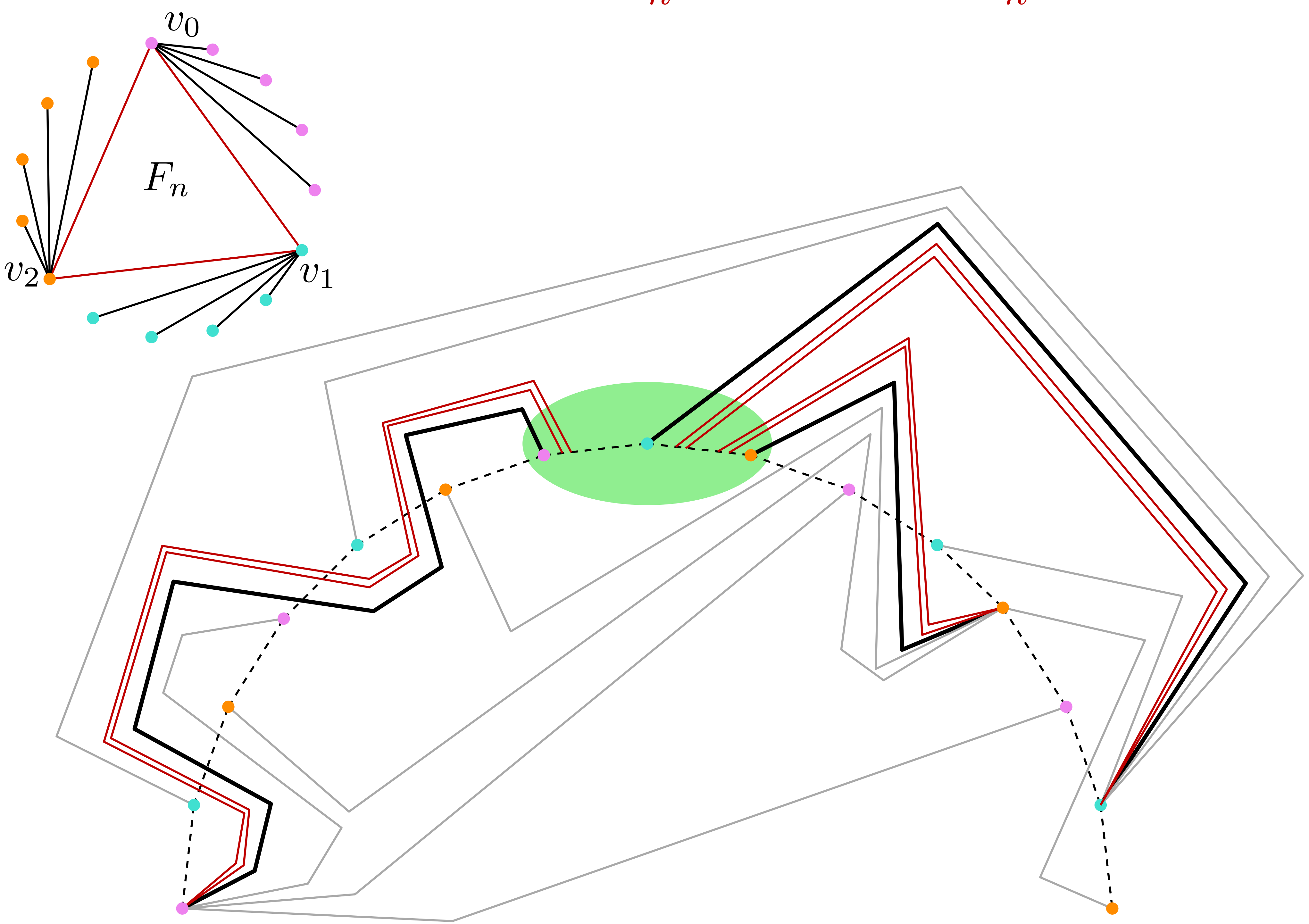
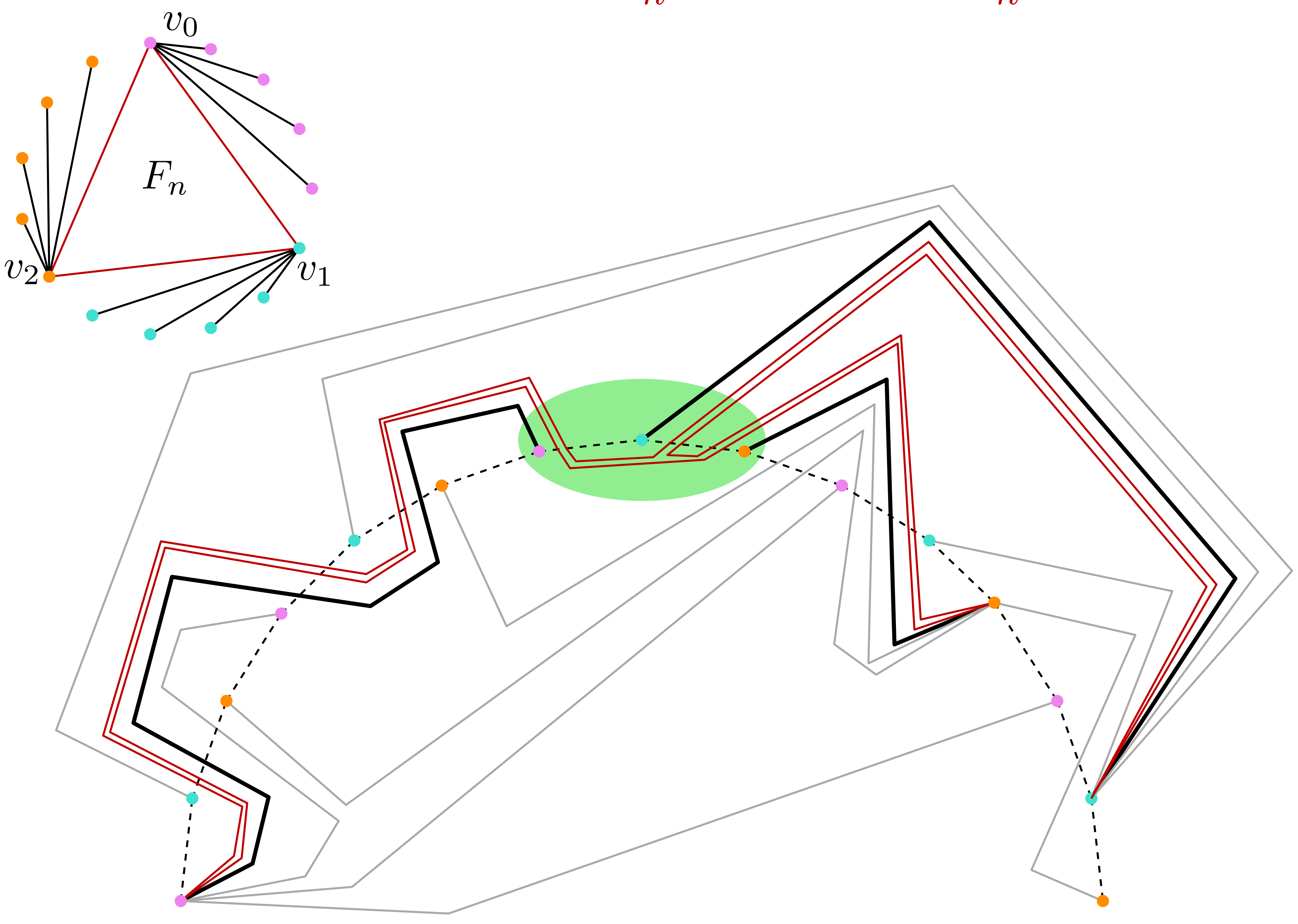
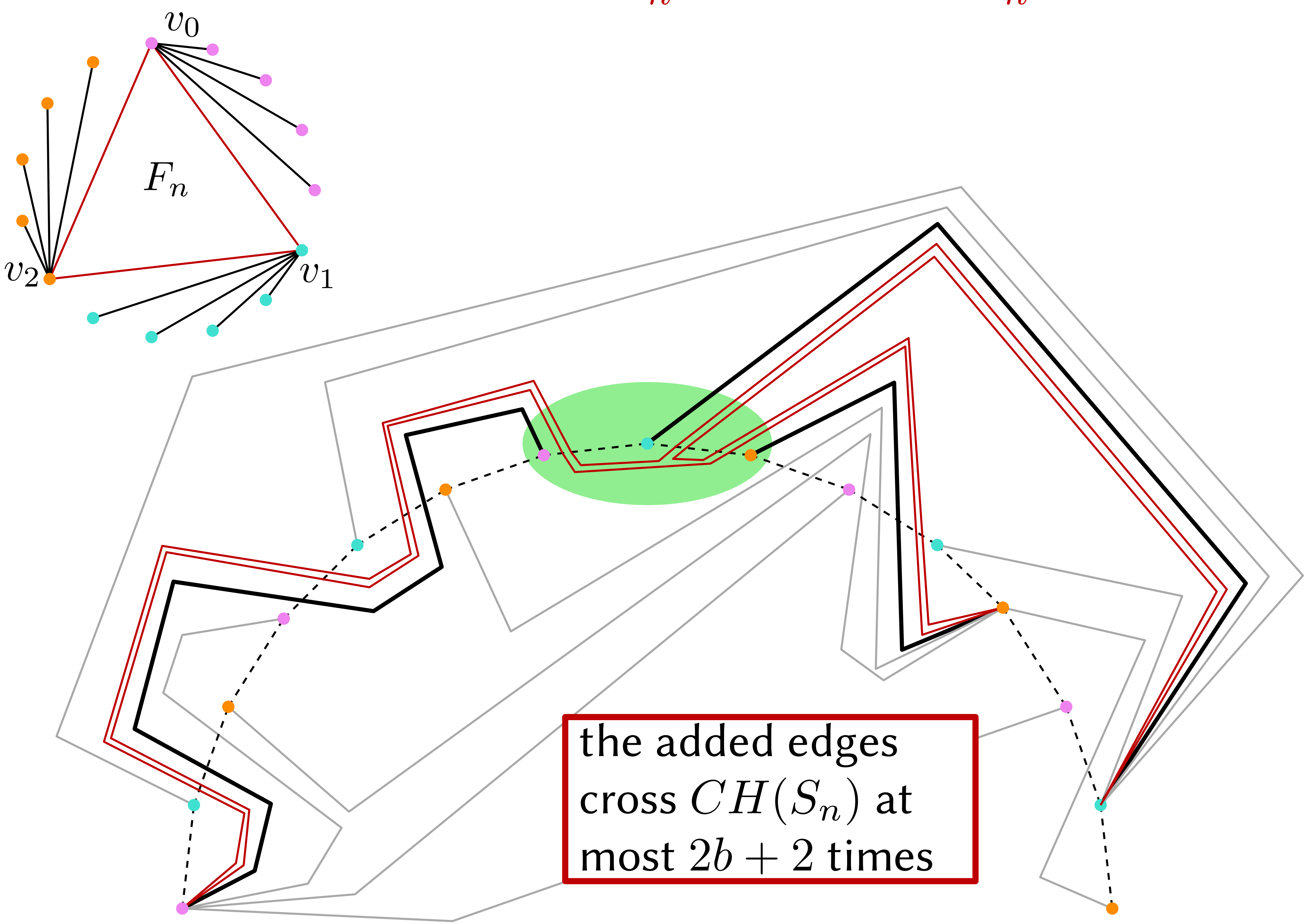


From a PSE of F_n to a PSE of G_n



From a PSE of F_n to a PSE of G_n



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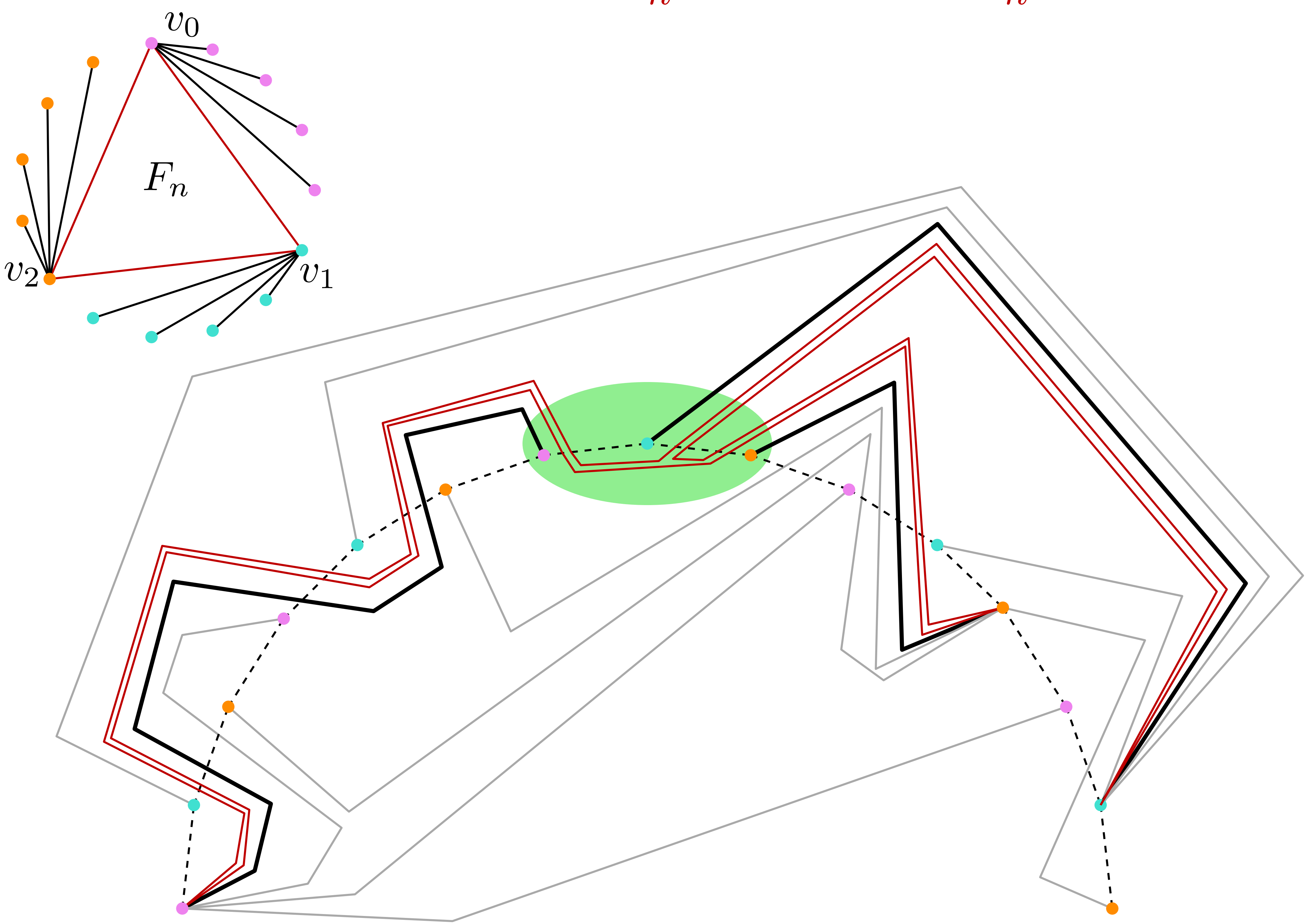
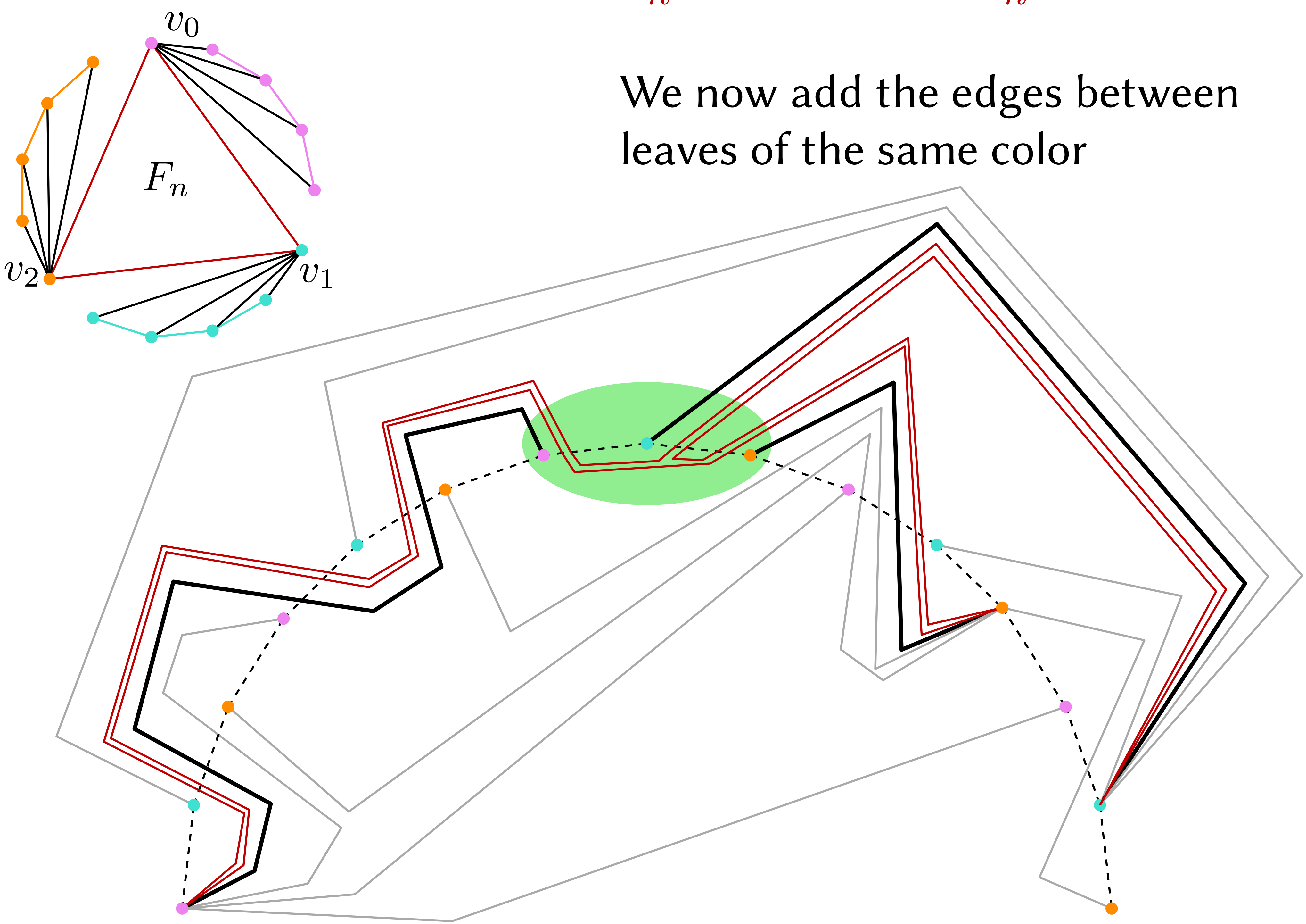
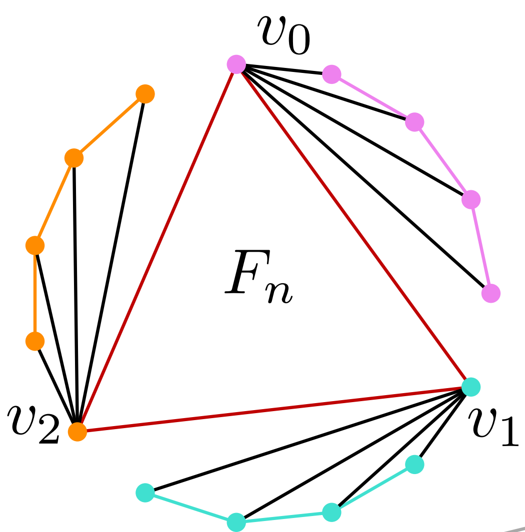


From a PSE of F_n to a PSE of G_n

We now add the edges between leaves of the same color



From a PSE of F_n to a PSE of G_n



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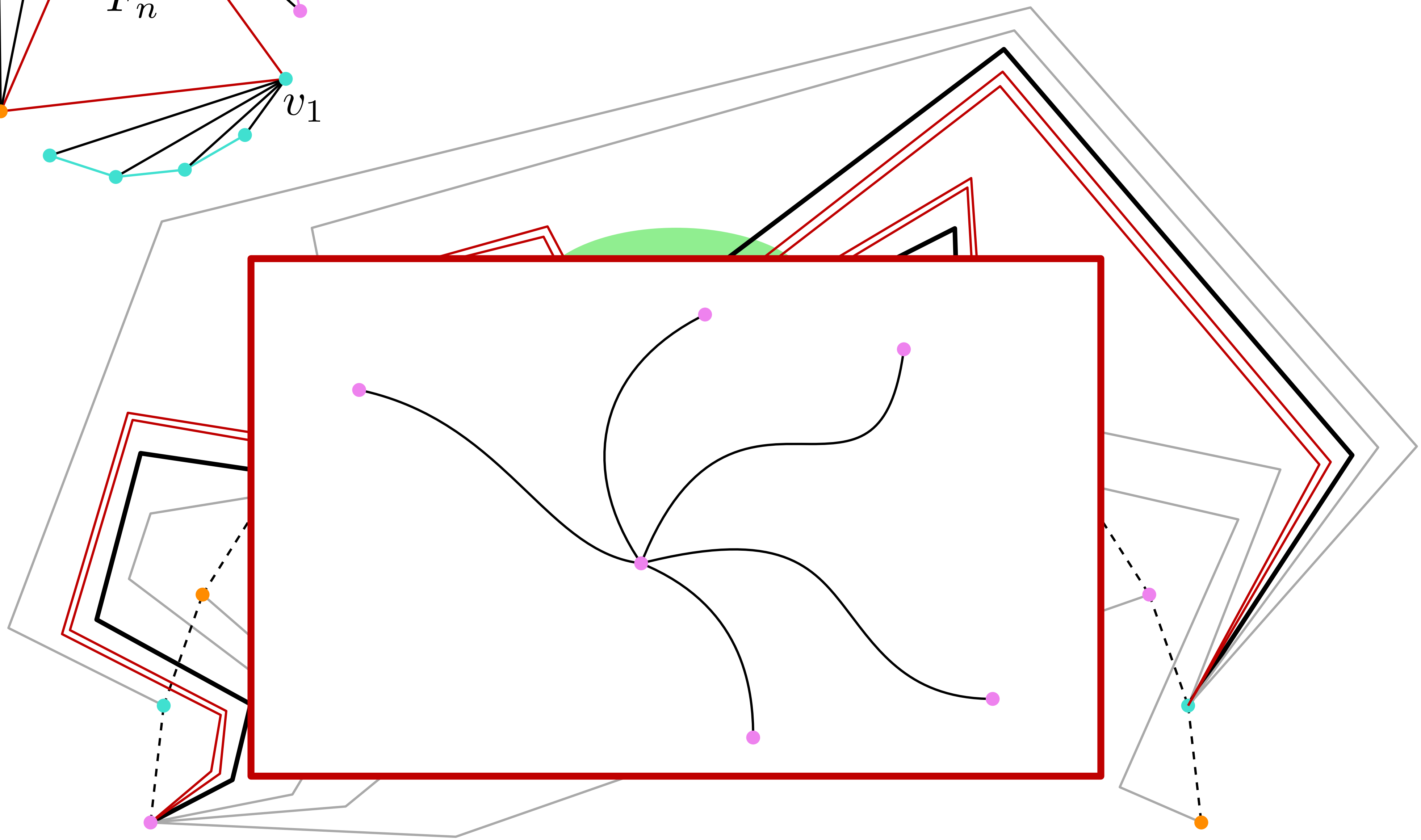
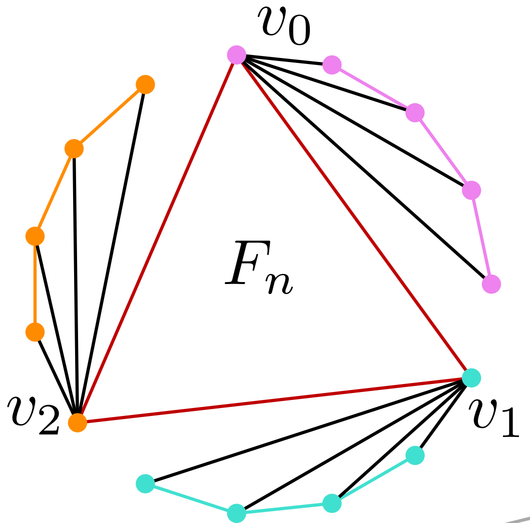


From a PSE of F_n to a PSE of G_n



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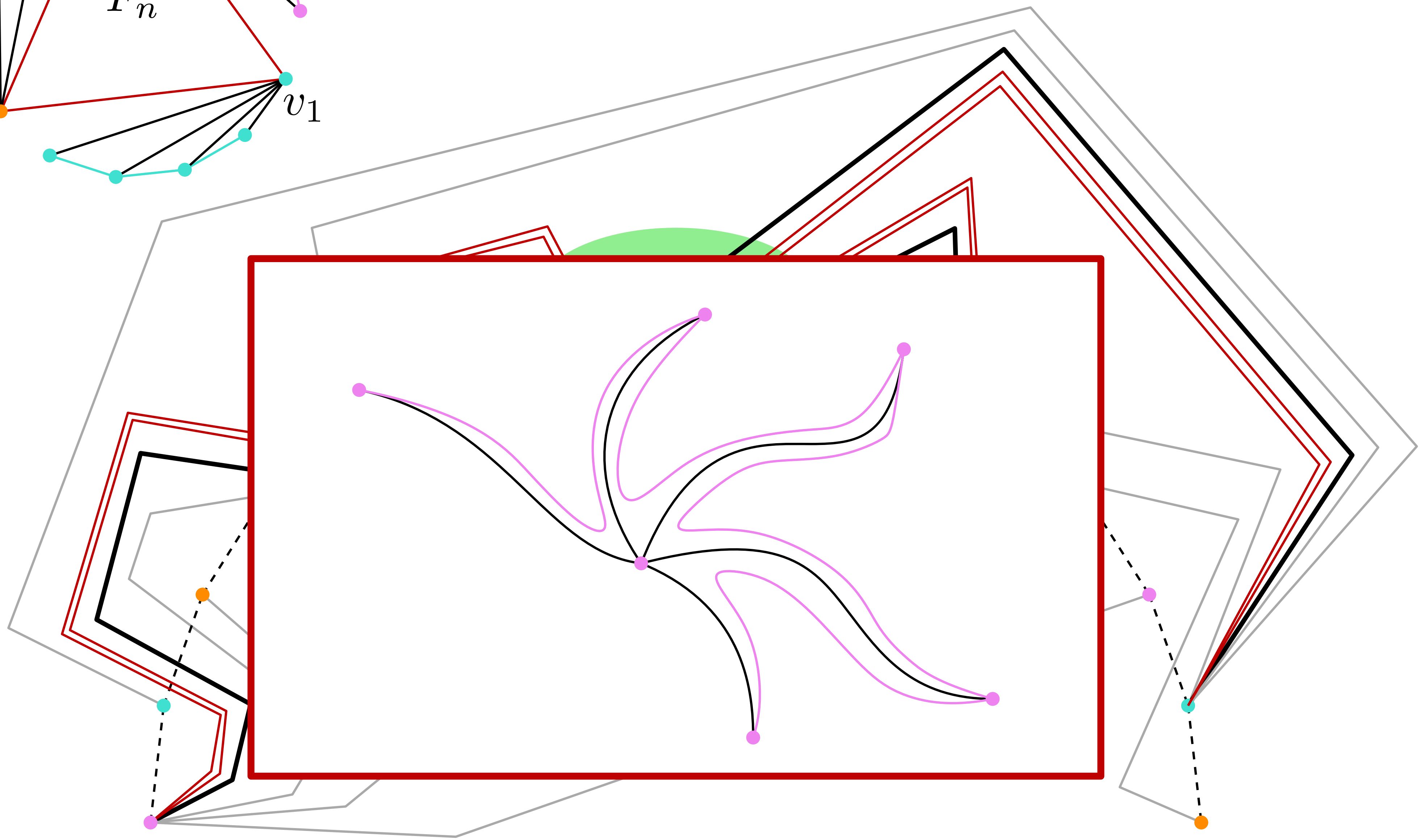
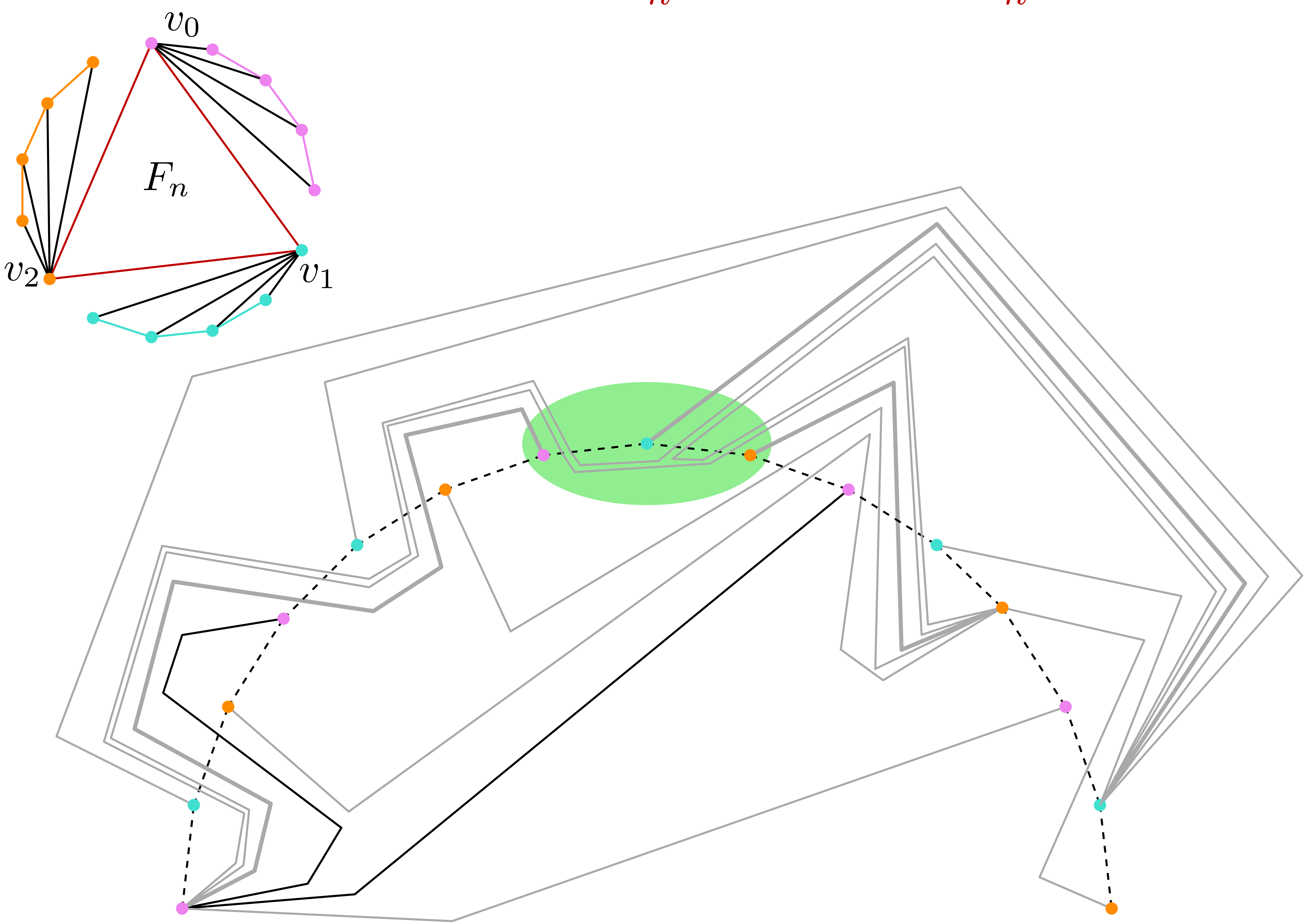


From a PSE of F_n to a PSE of G_n



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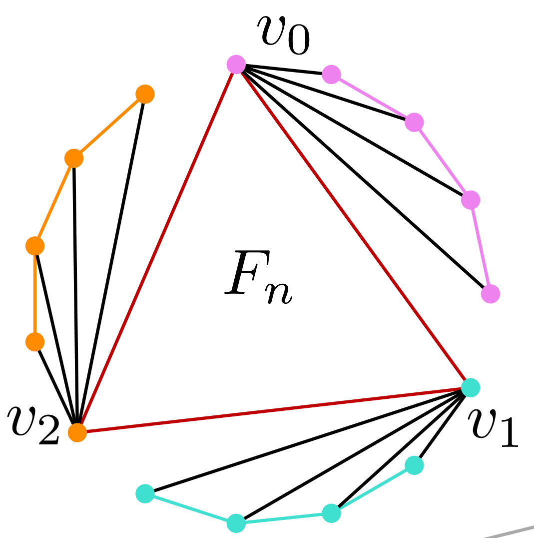

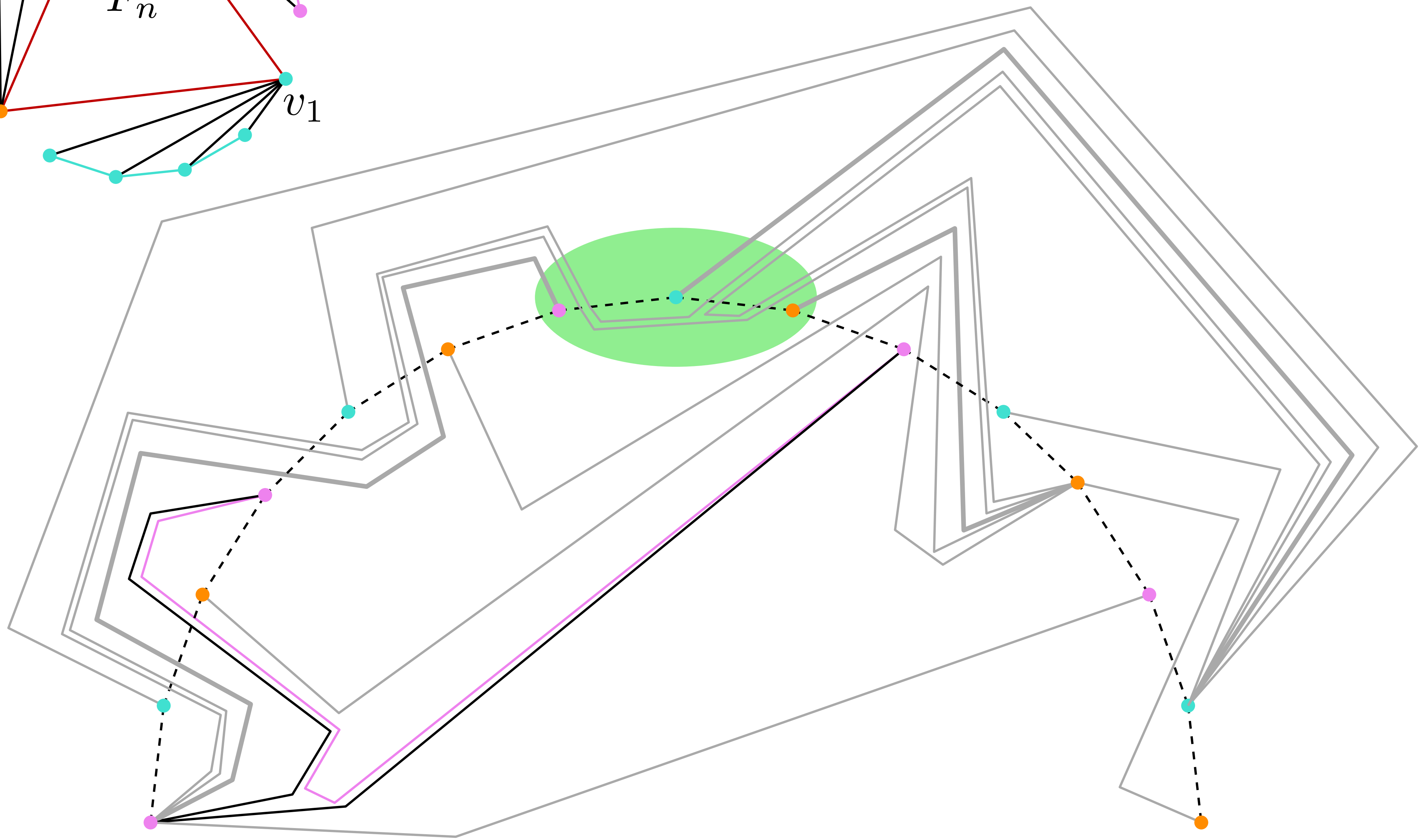

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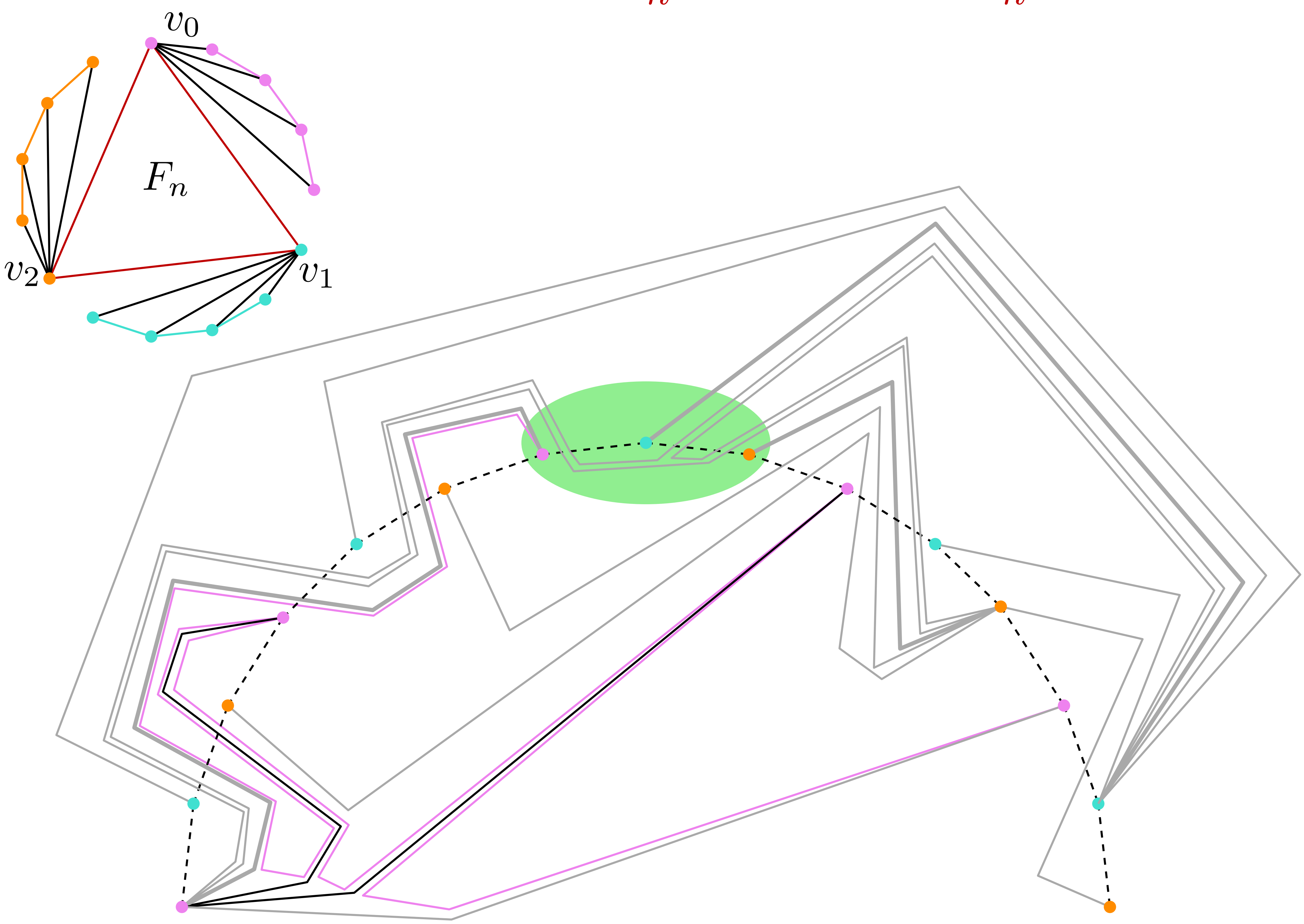

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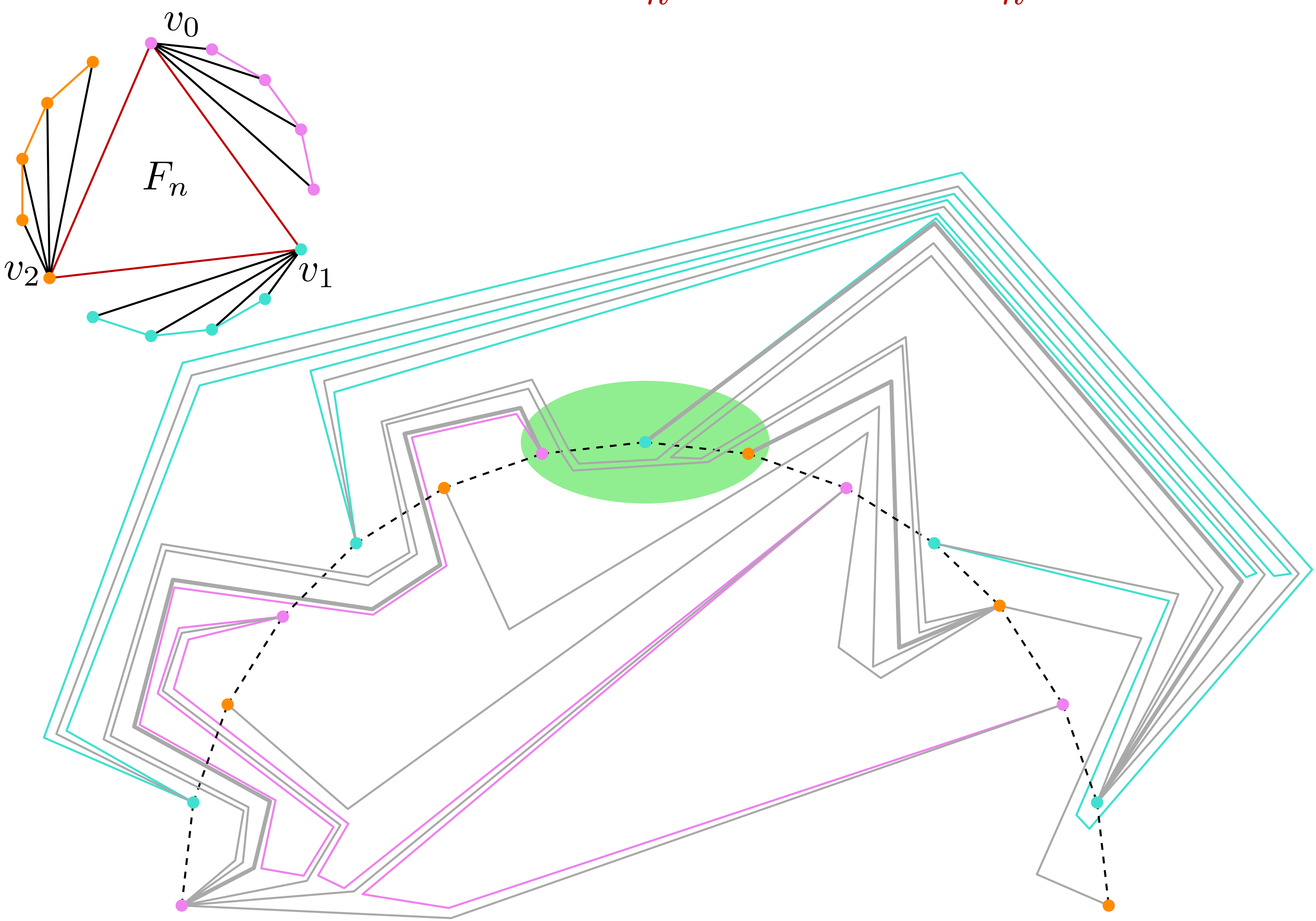

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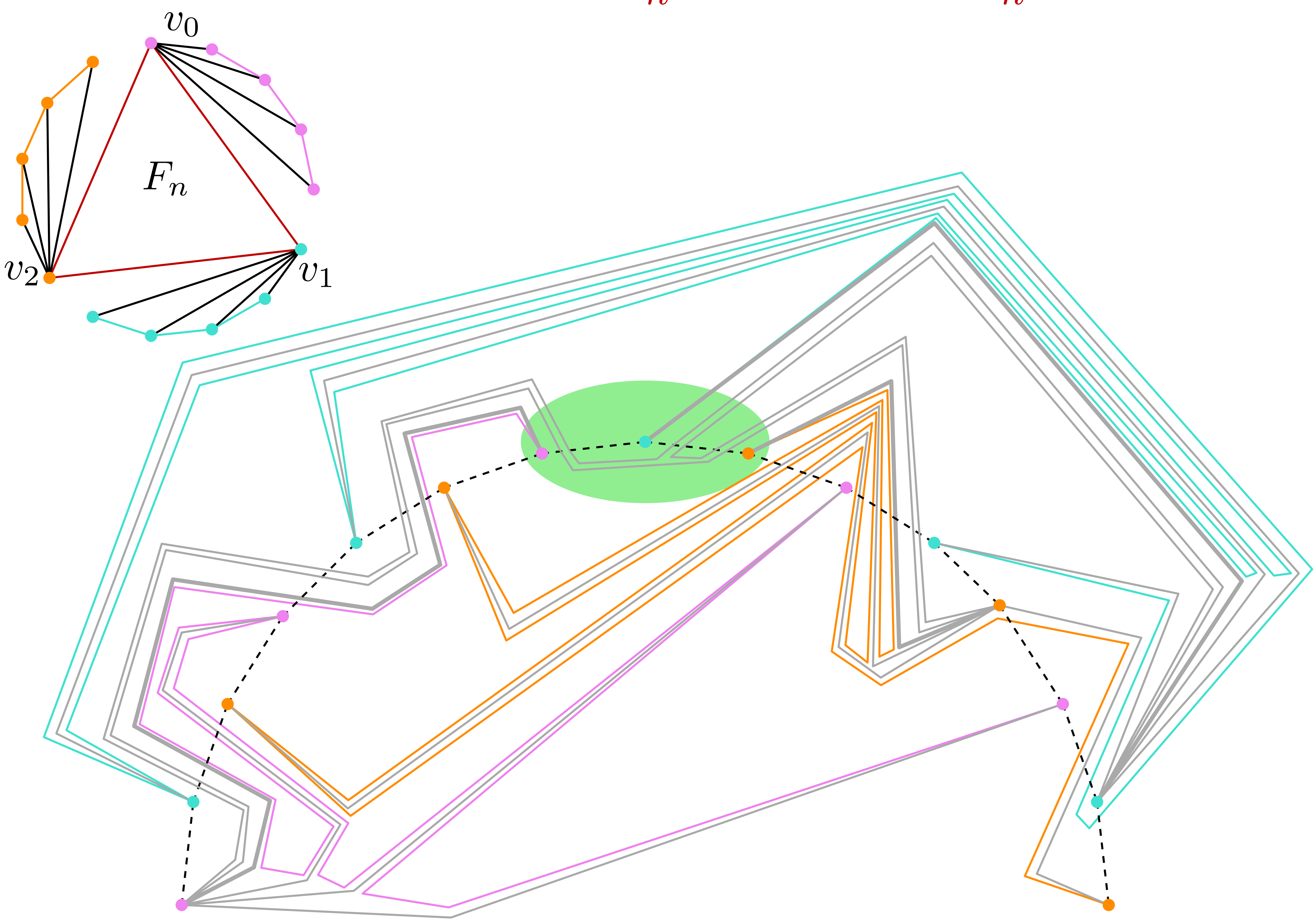

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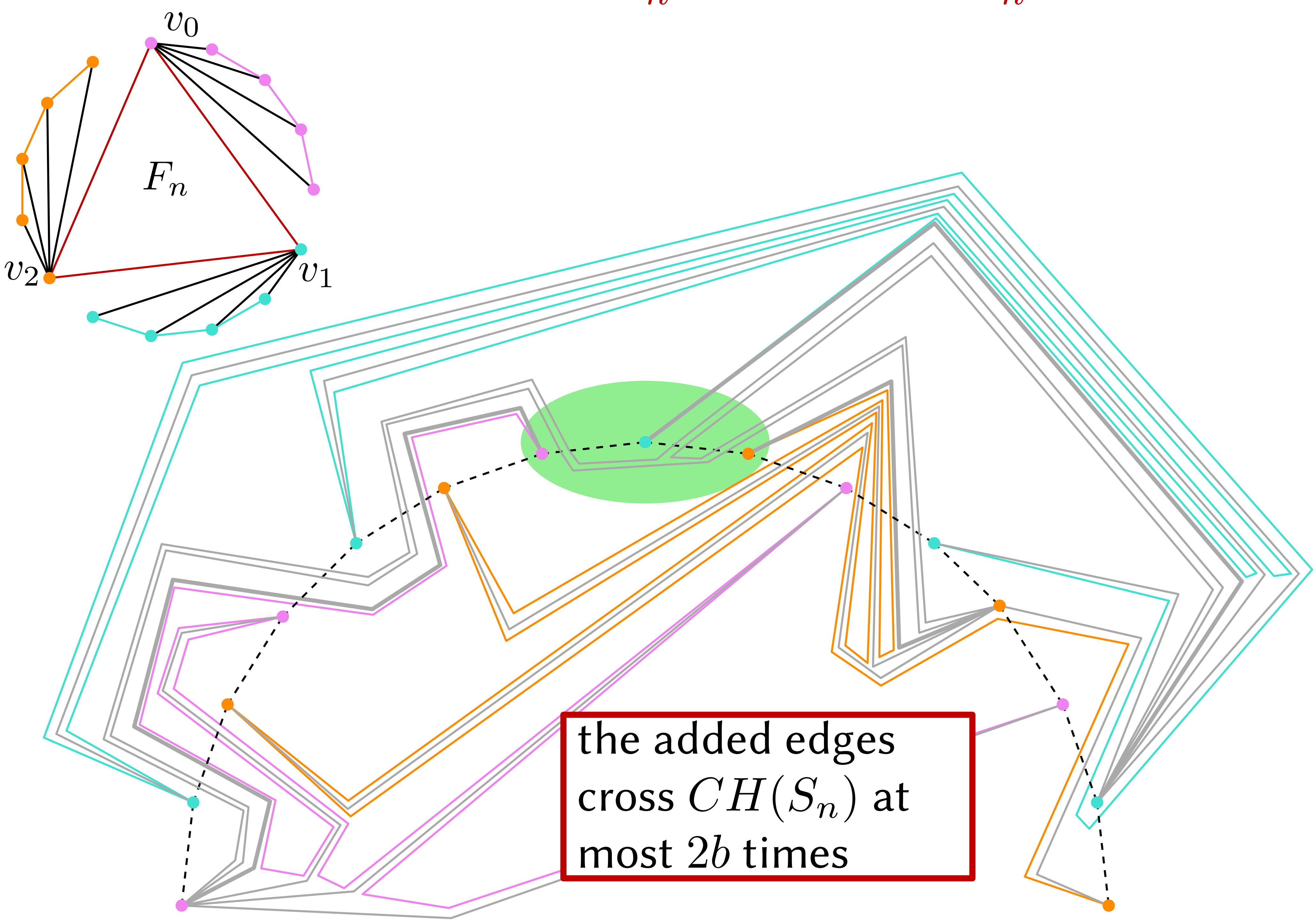
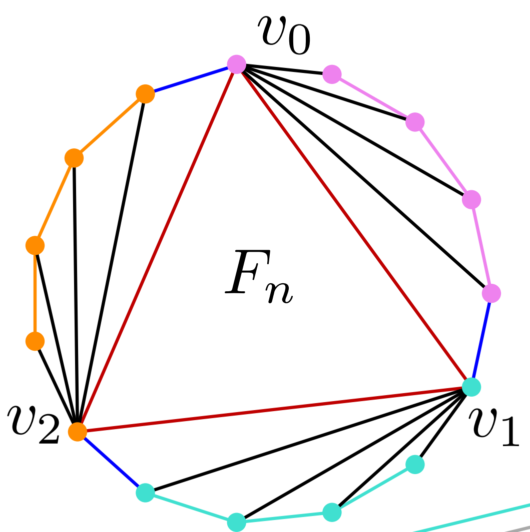
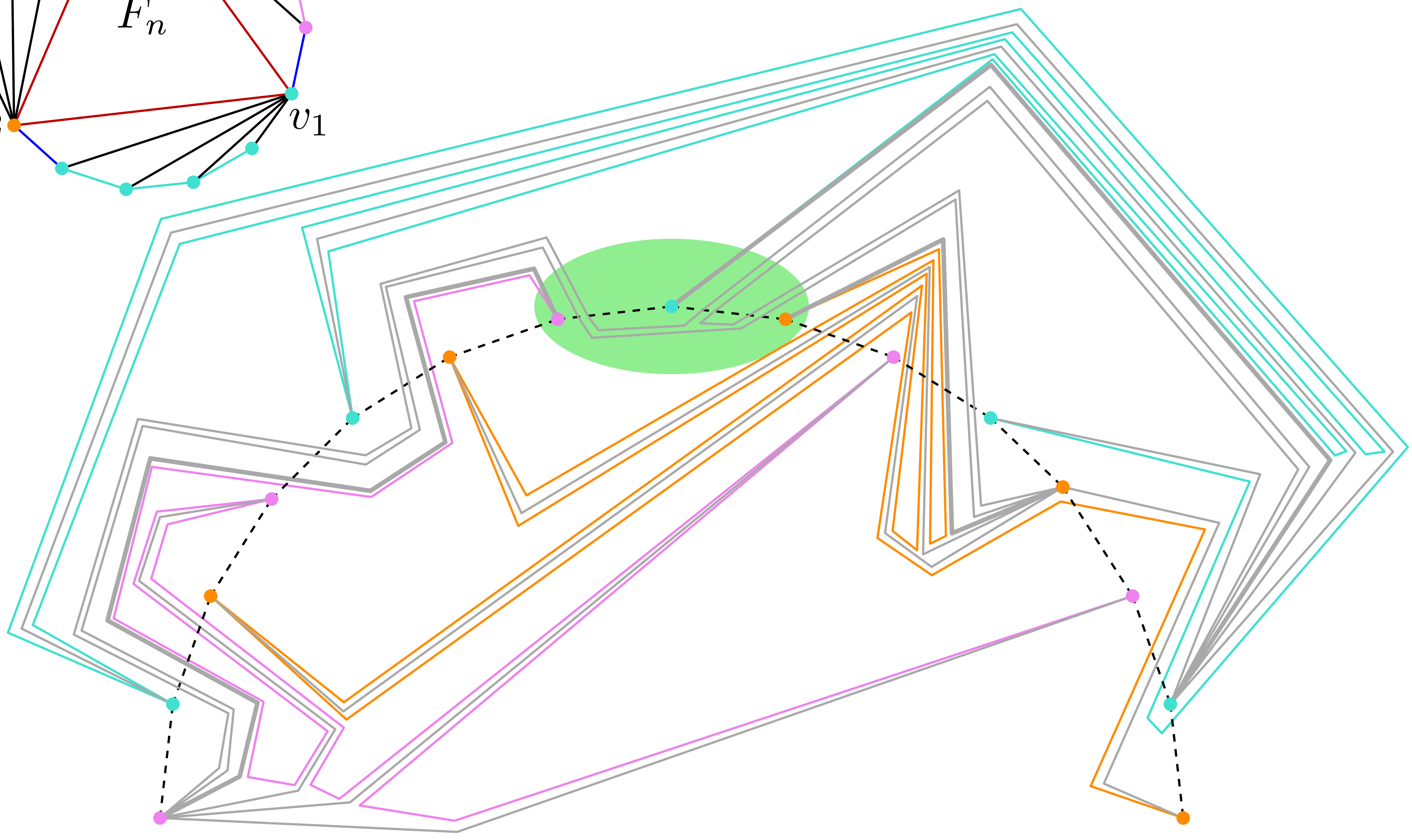


the added edges cross $CH(S_n)$ at most $2b$ times

From a PSE of F_n to a PSE of G_n



We now add the last three edges



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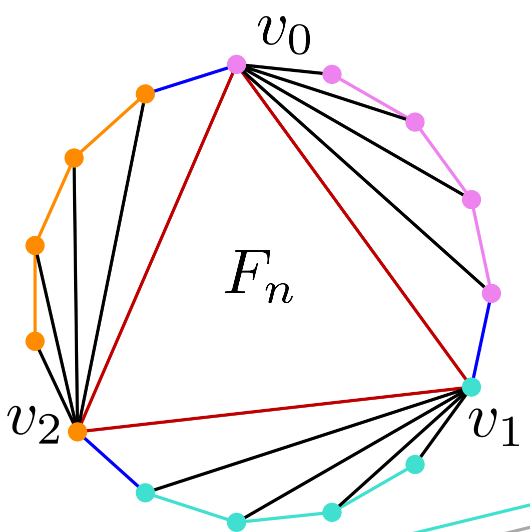

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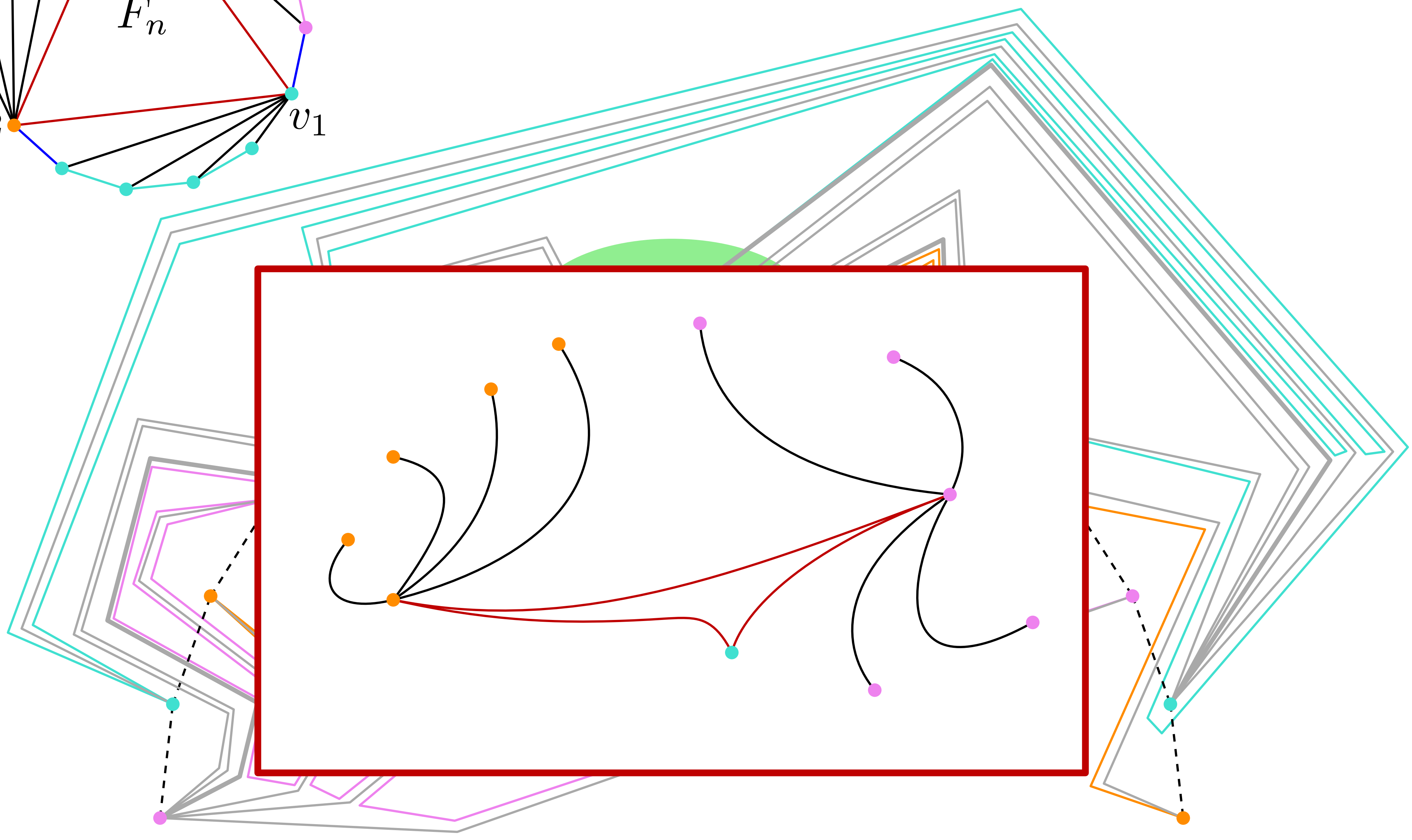

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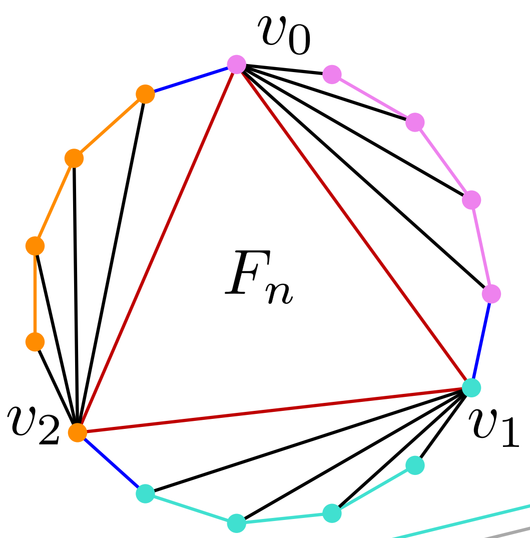

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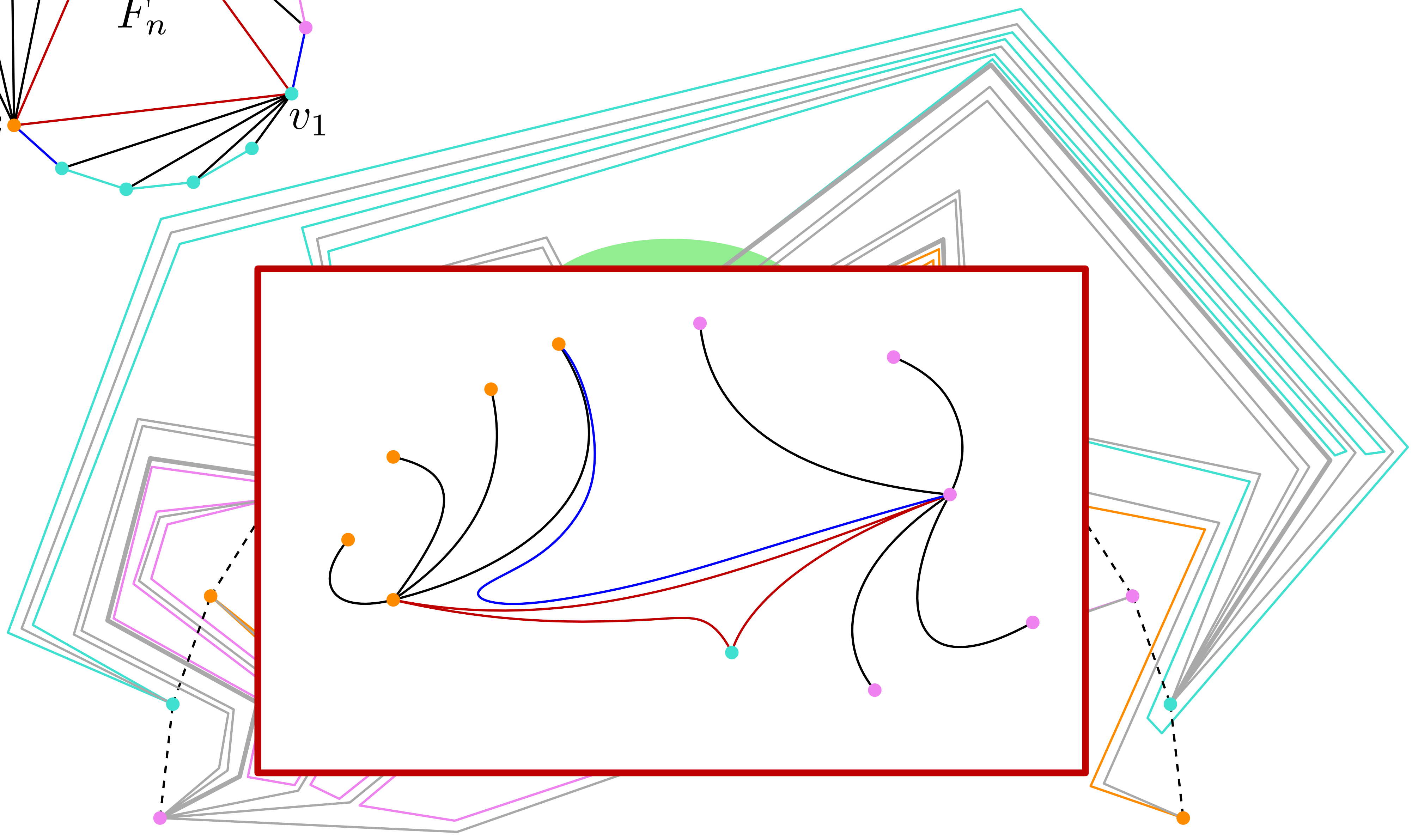

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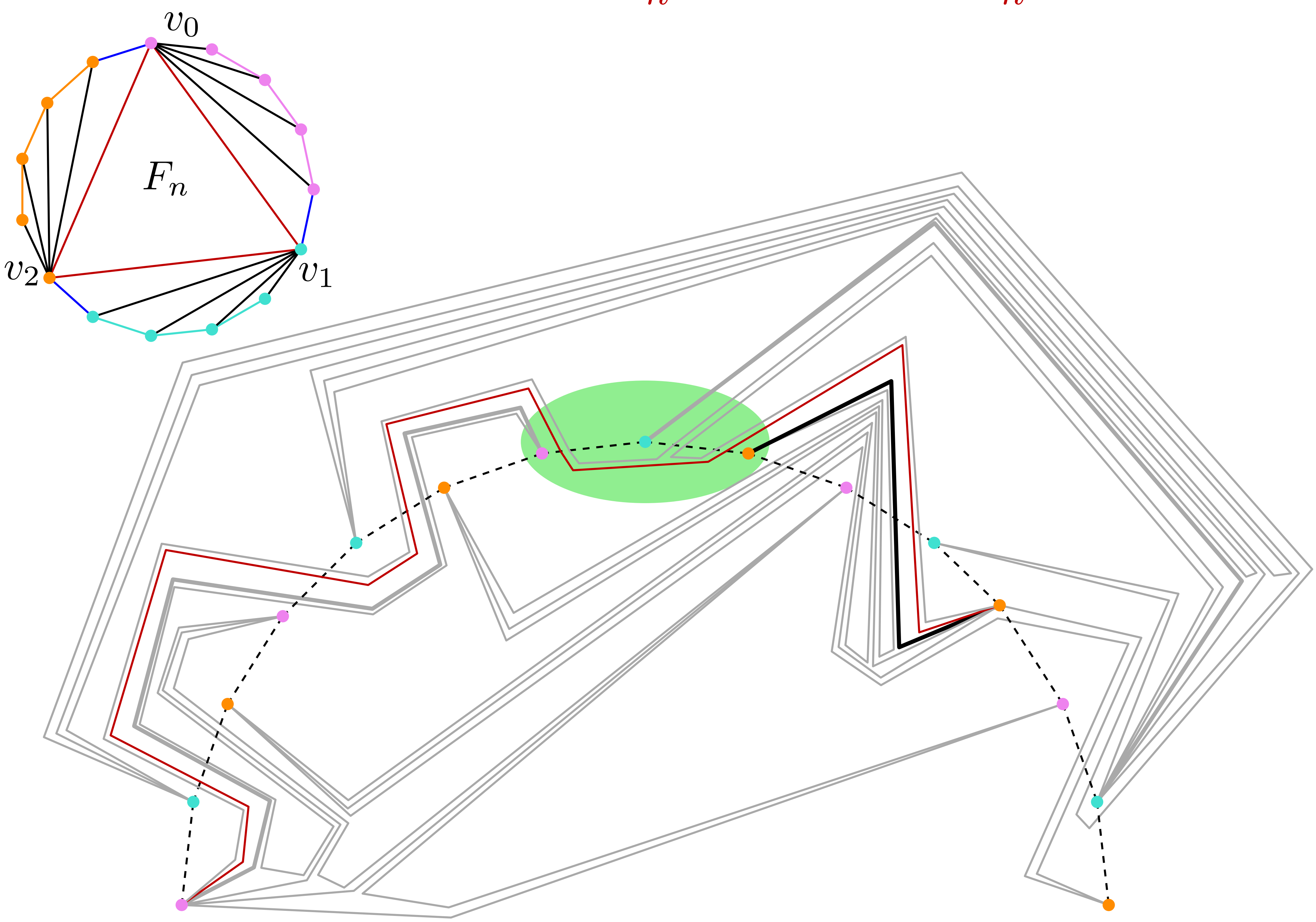

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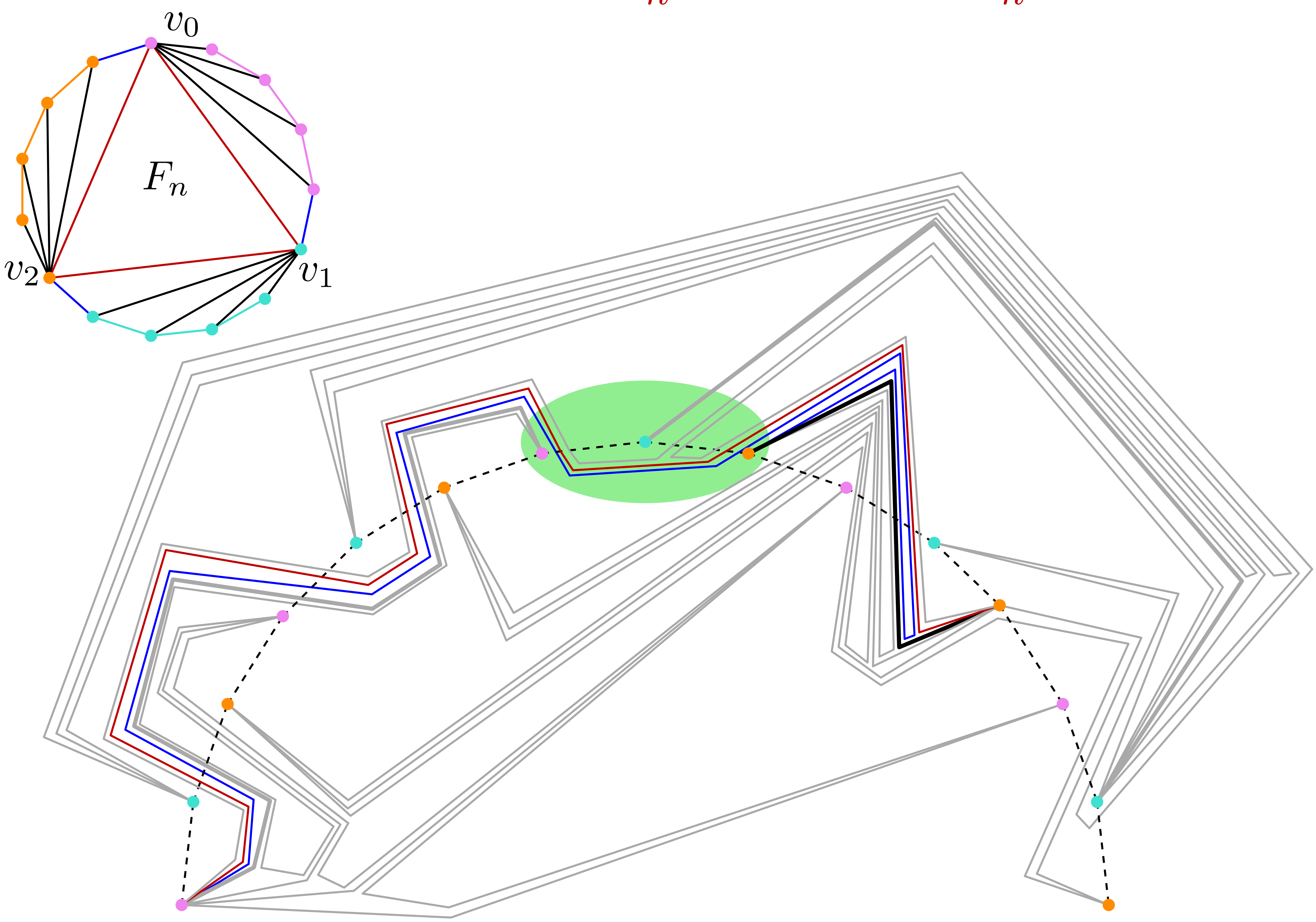

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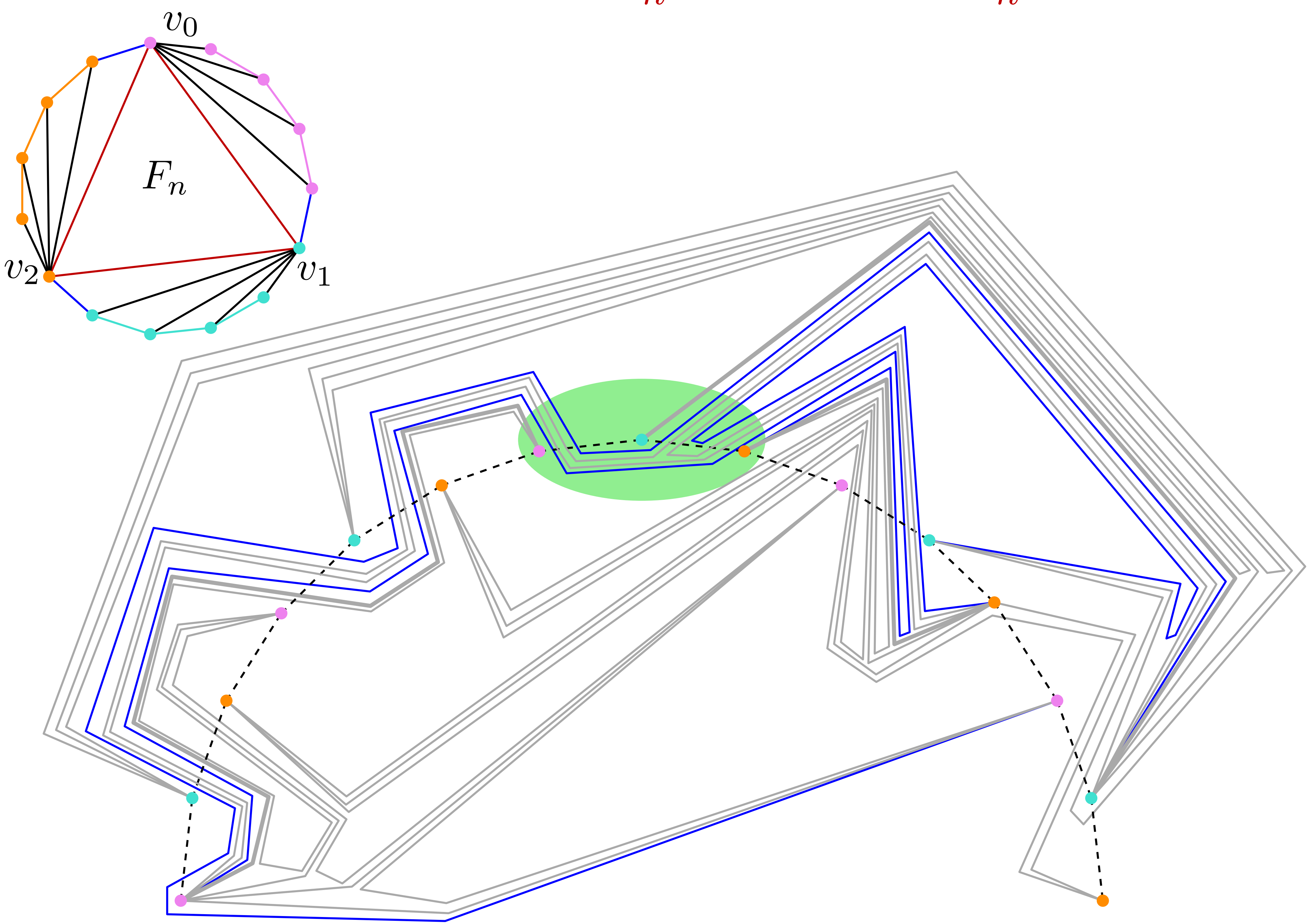

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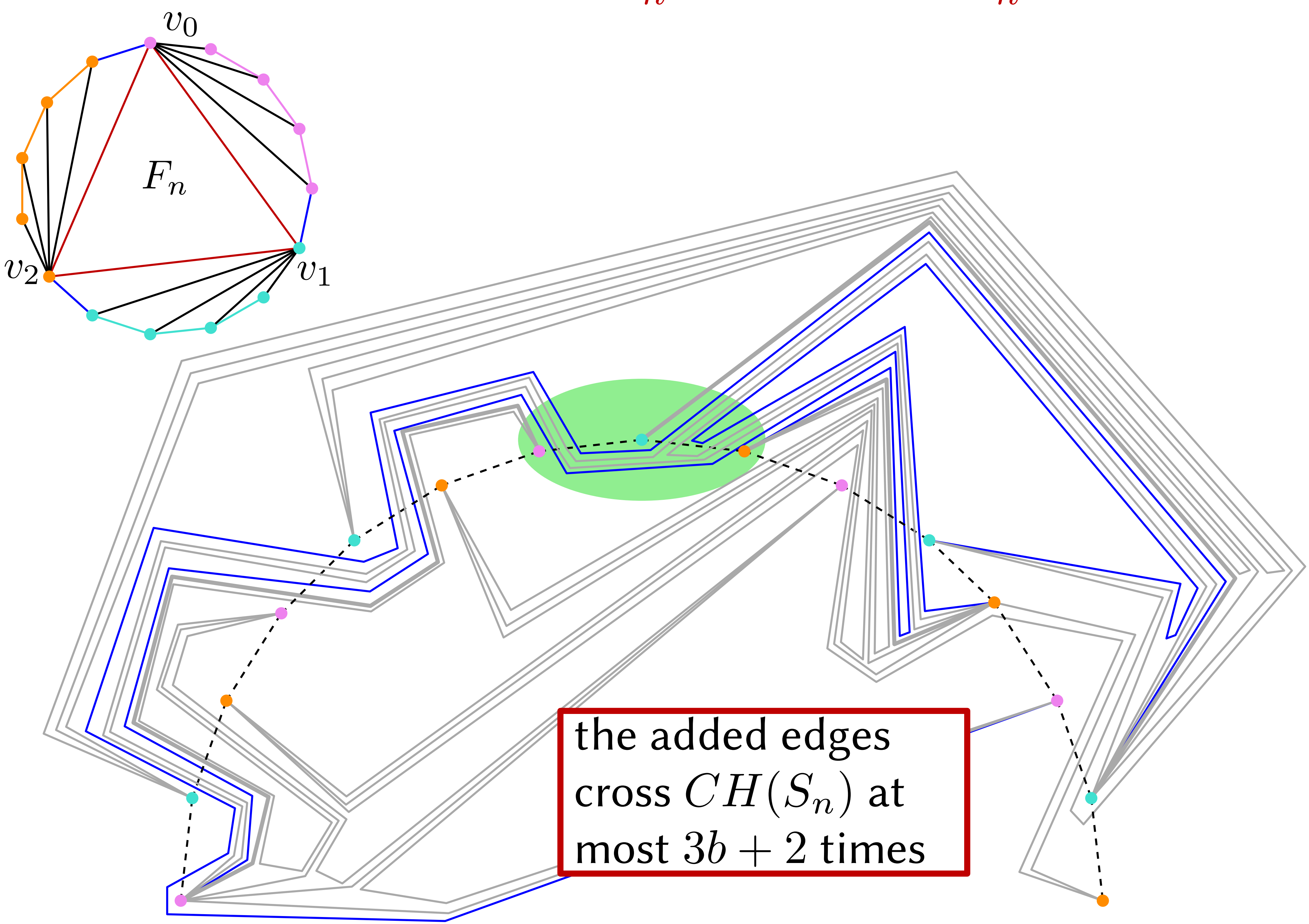

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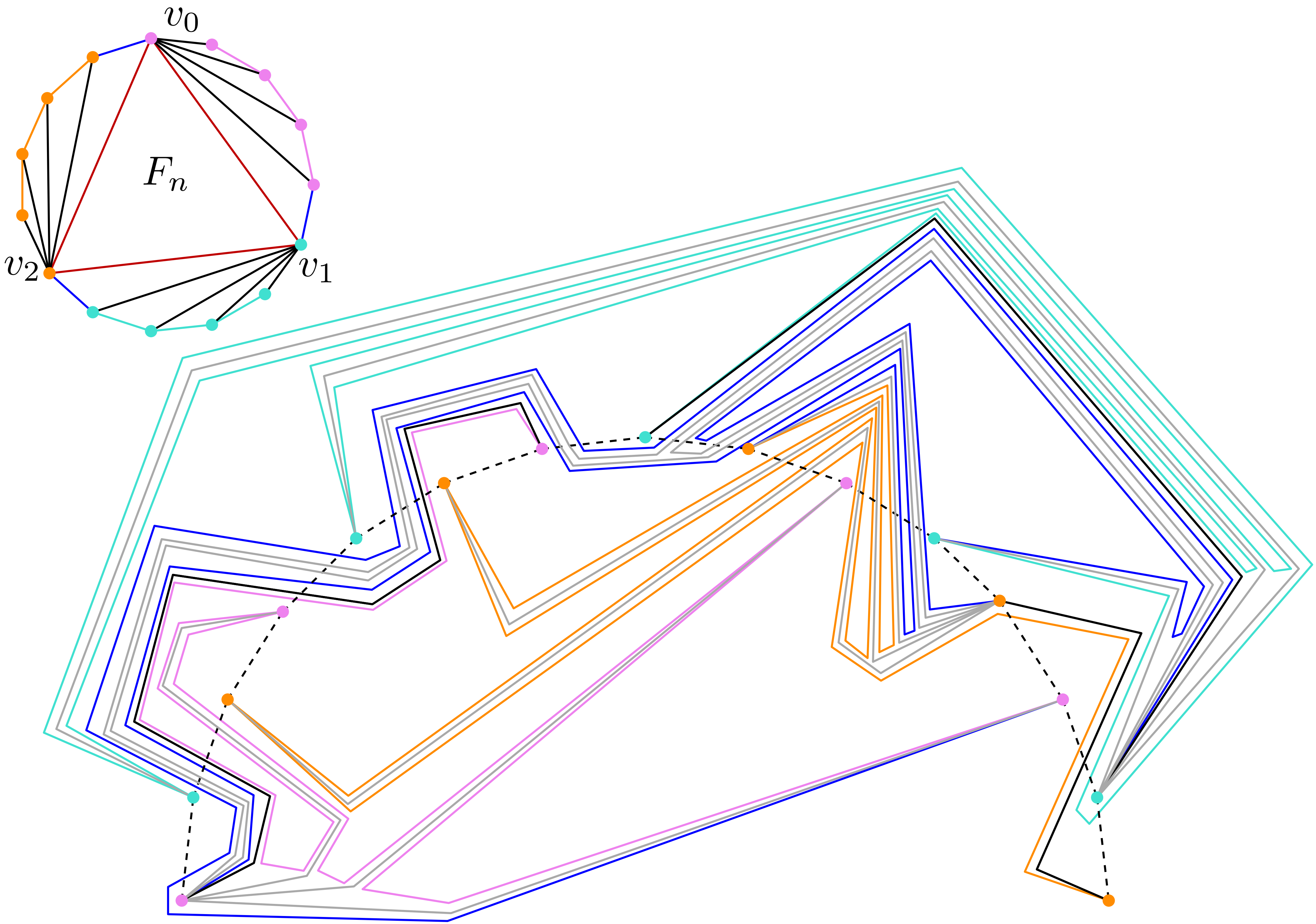

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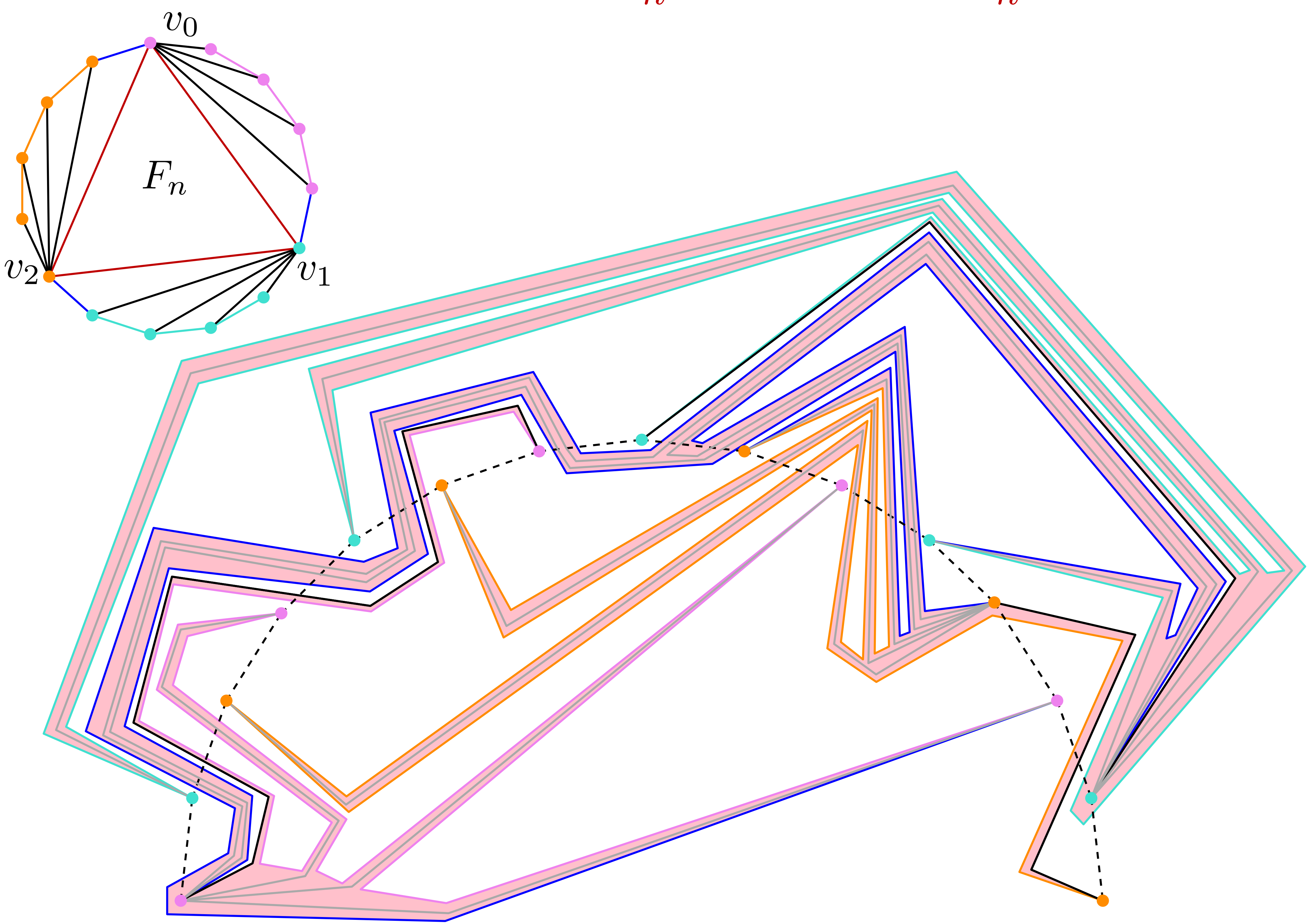

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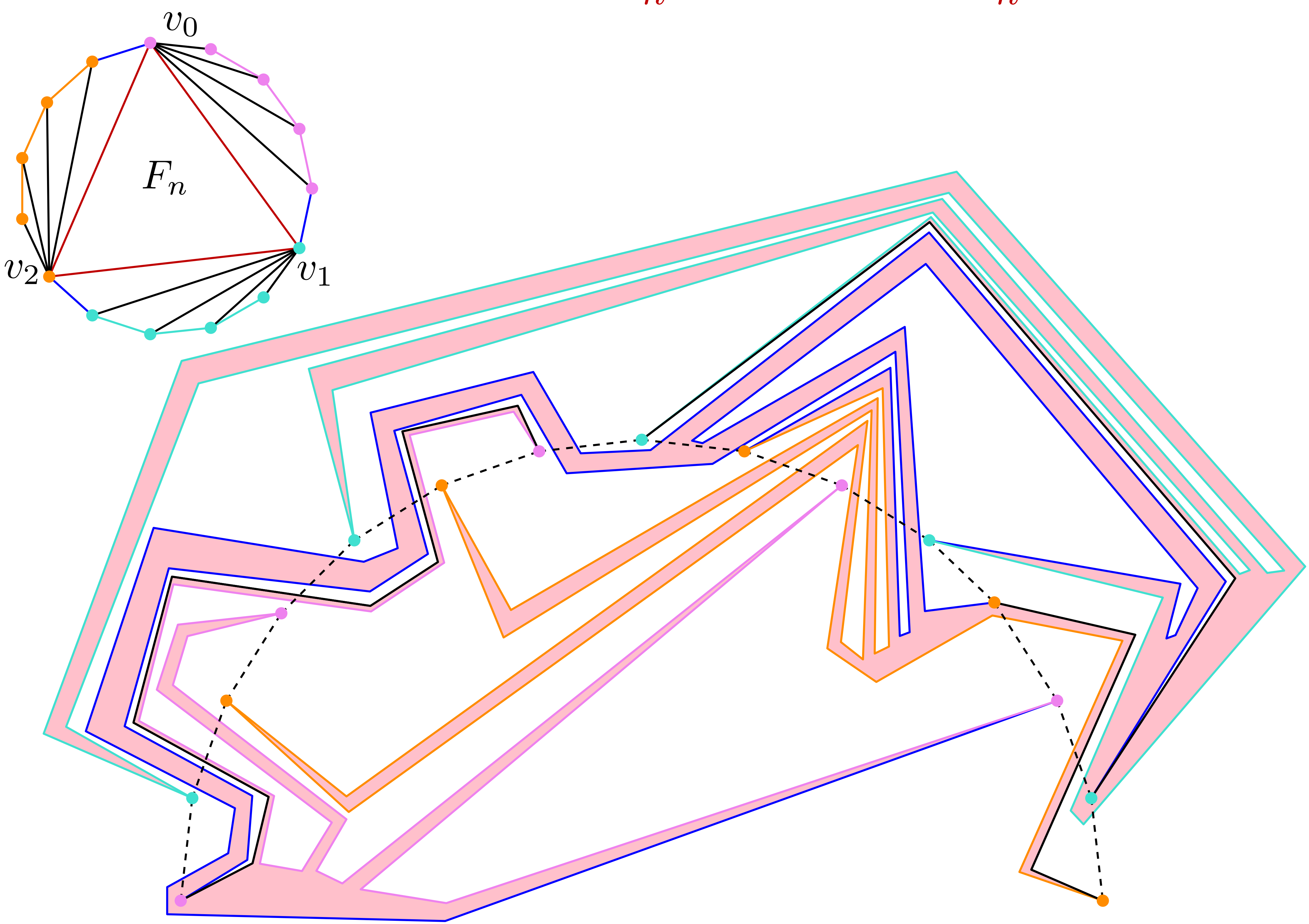

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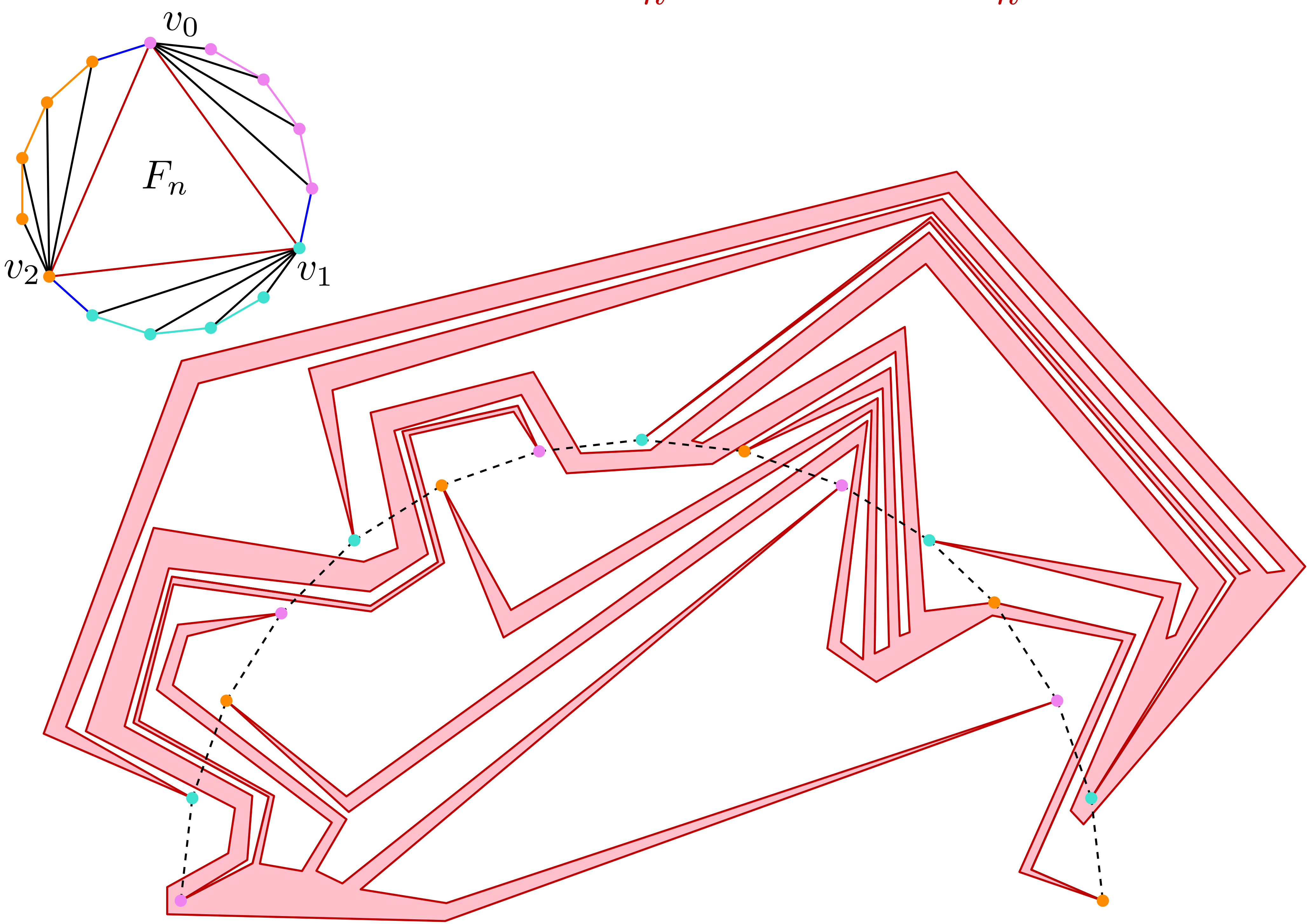


Putting all together

F_n admits a PSE with $CC \leq b$

$\Longrightarrow$

F_n admits a PSE with $2b + 1$ crossings of $CH(S_n)$

Lm. 1 $\Downarrow$

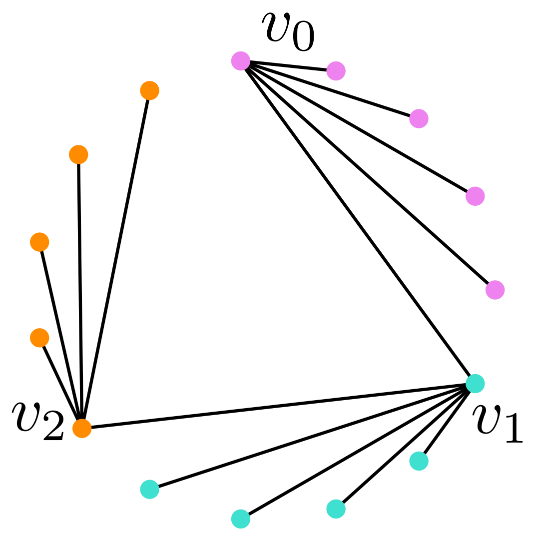
G_n admits a PSE with $6b + 2$ crossings of $CH(S_n)$

$\Longleftarrow$ Th. 1

G_n admits a PSE with $CC \leq 12b + 5$

Comments

Since there exists a caterpillar that is a supergraph of F_n for every n, $\Omega(n^{\frac{1}{3}})$ bends may be necessary also for 3-colored caterpillars



3-Colored PSE of paths

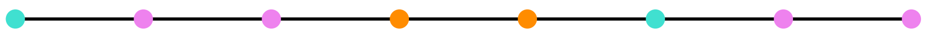
3-colored PSE of paths

Our result:

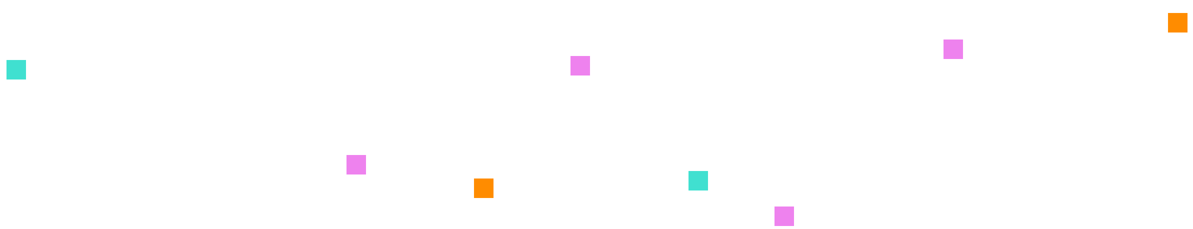
- Every 3-colored path admits a PSE with CC ≤ 5 onto any 3-colored point set.

Proof approach

Path

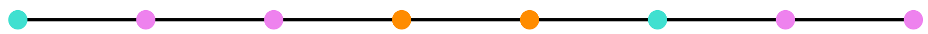


Point set

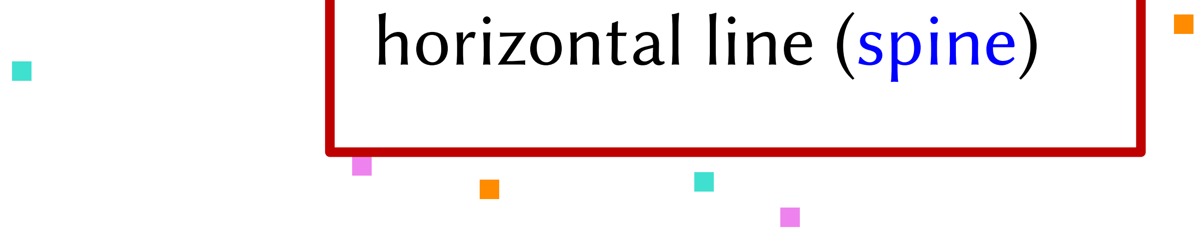


Proof approach

Path



Point set



Project the points on a horizontal line (spine)

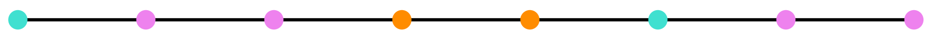
Proof approach

Path

Sequence of colors

Proof approach

Path

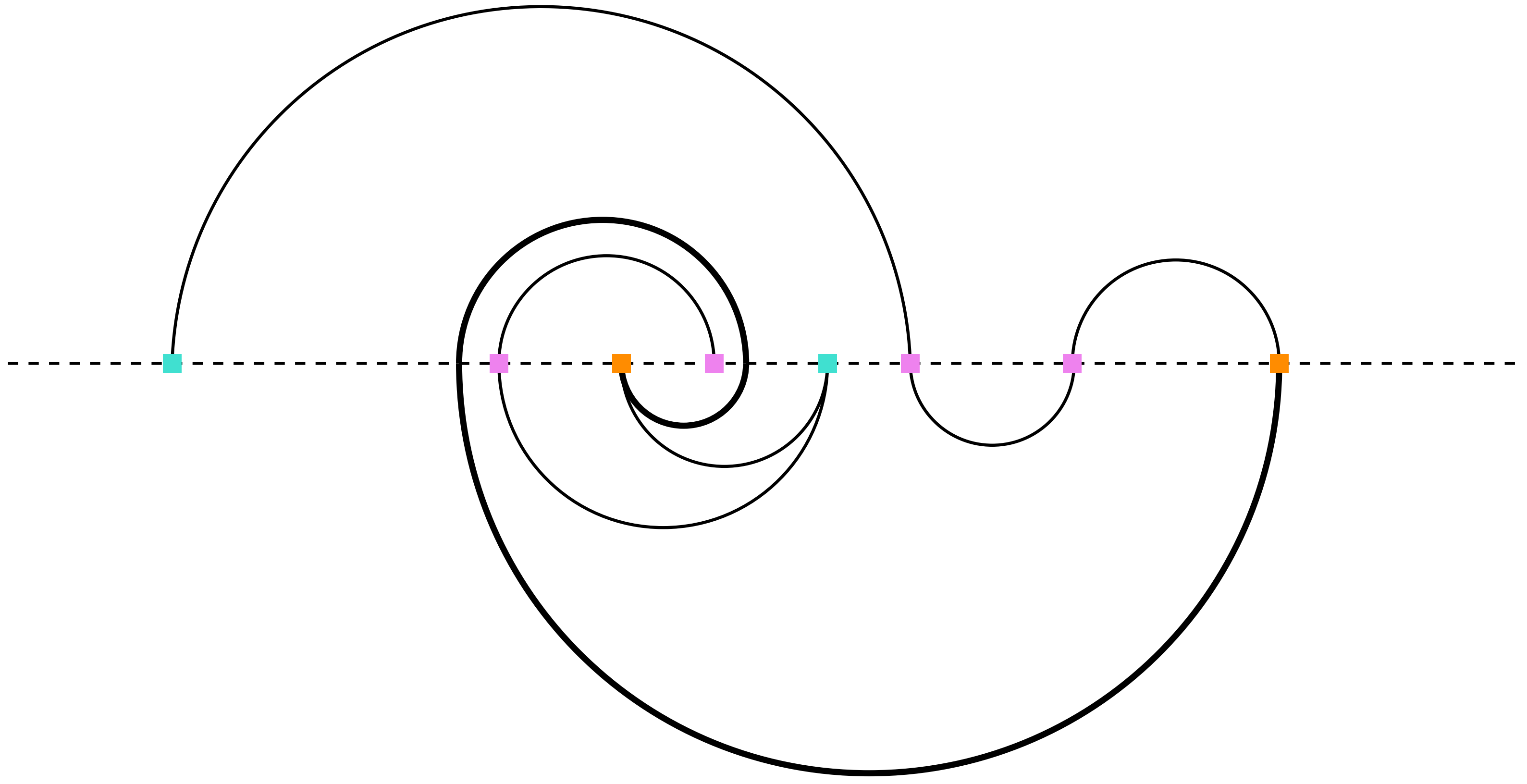


Sequence of colors

Compute a 2-page topological book embedding consistent with the sequence of colors and with at most 2 spine crossings per edges

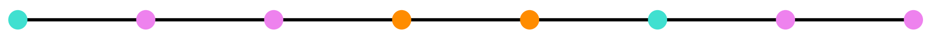
Proof approach

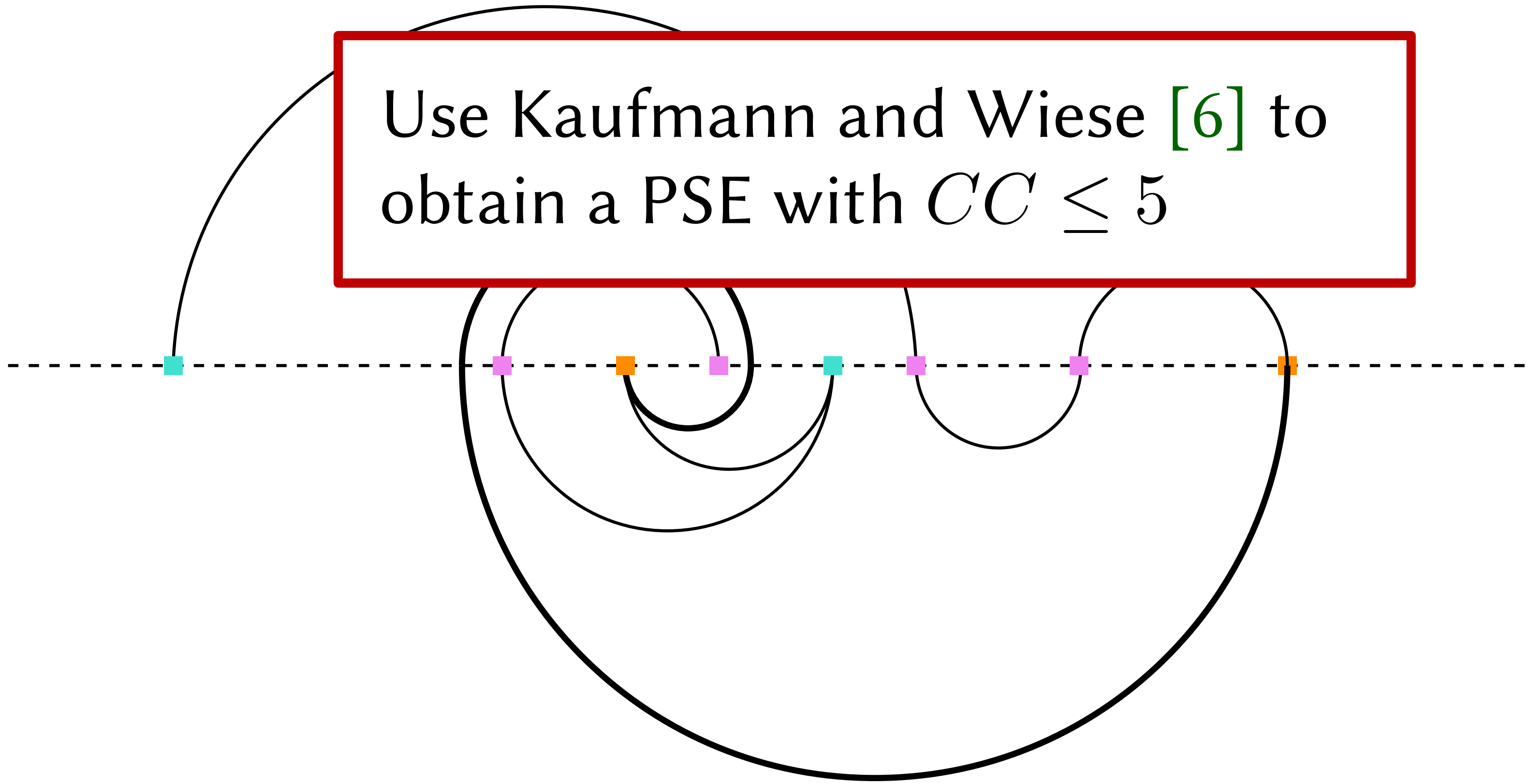
Path



Proof approach

Path



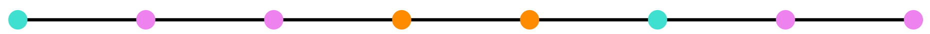


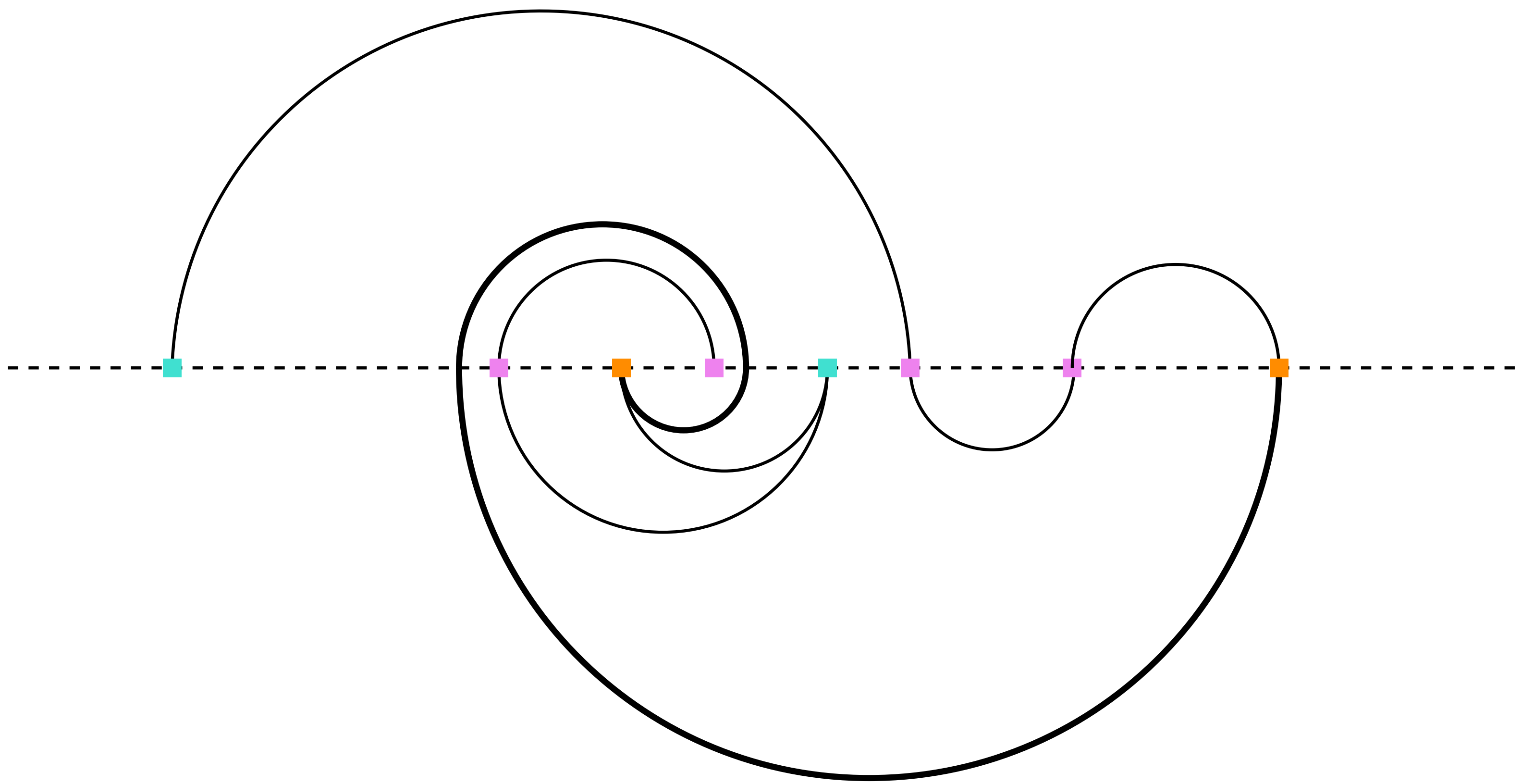
Use Kaufmann and Wiese [6] to obtain a PSE with $CC \leq 5$

[6] Kaufmann, Wiese JGAA 2002

Proof approach

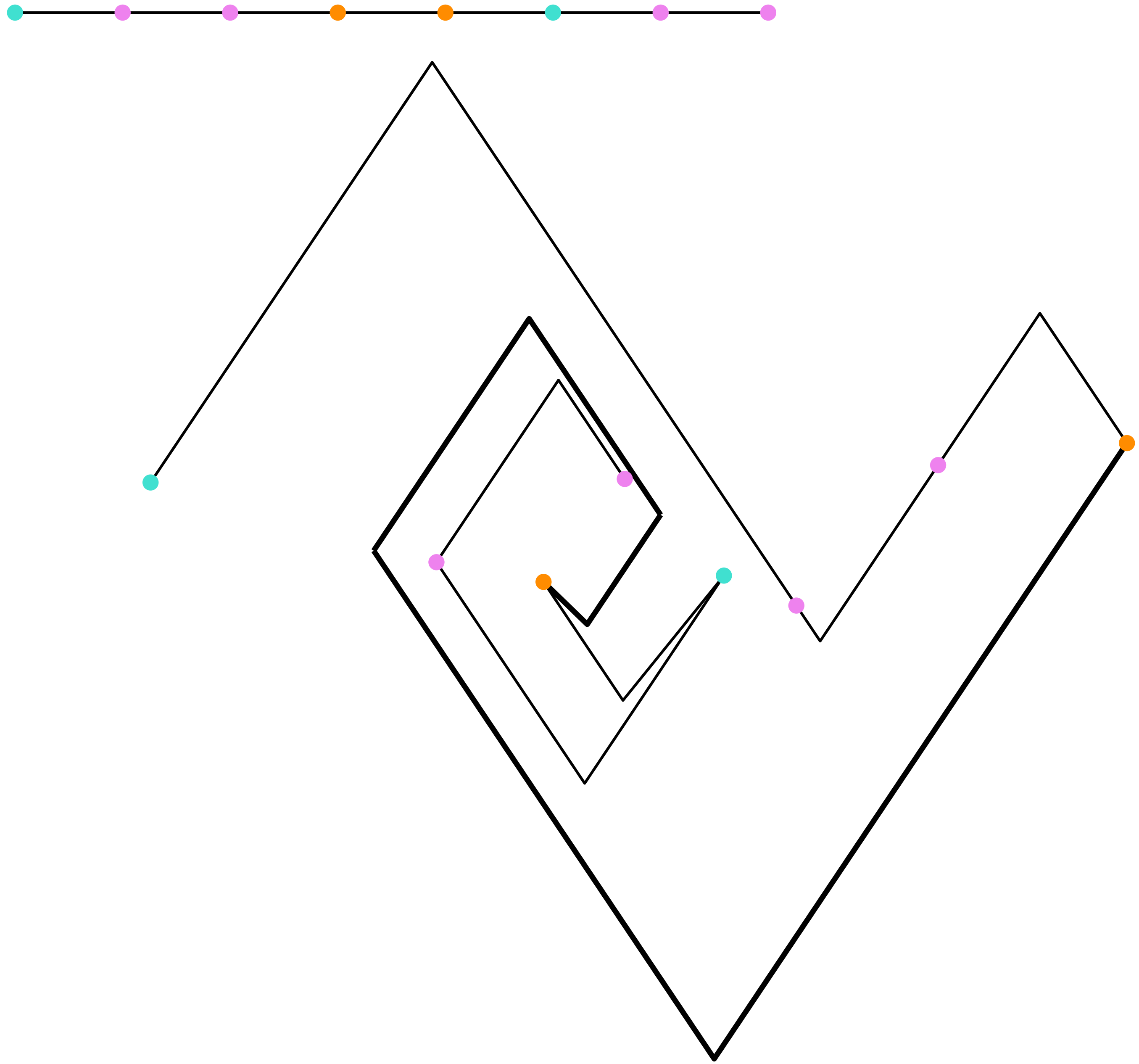
Path





Proof approach

Path



2-page topological book embedding of paths

Every 3-colored path admits a 2-page topological book embedding with at most 2 spine crossing per edge for any given sequence of colors

2-page topological book embedding of paths

Path P

Sequence of colors σ

2-page topological book embedding of paths

Path P

Sequence of colors σ

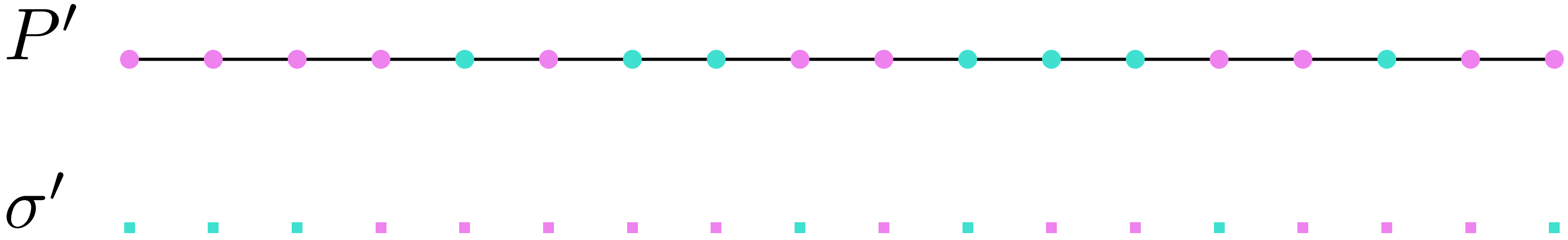
Remove the third color

2-page topological book embedding of paths

Path P'

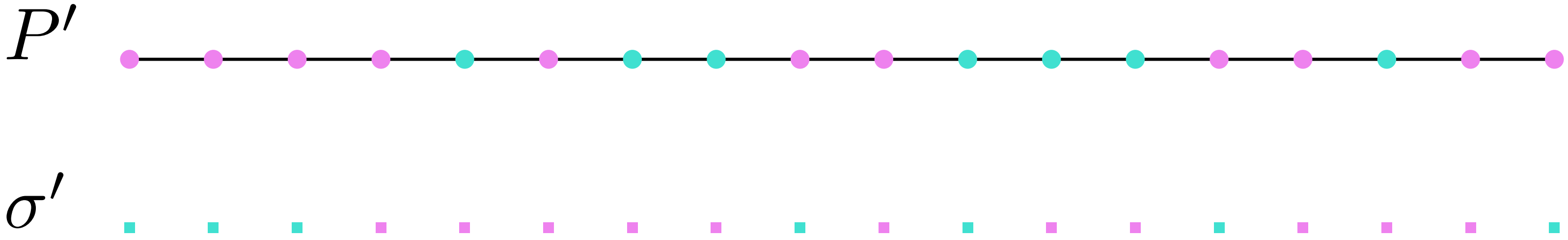
Sequence of colors σ'

2-page topological book embedding of paths



2-page topological book embedding of paths

Consider a prefix P'' of P' and the corresponding prefix σ'' of σ'



2-page topological book embedding of paths

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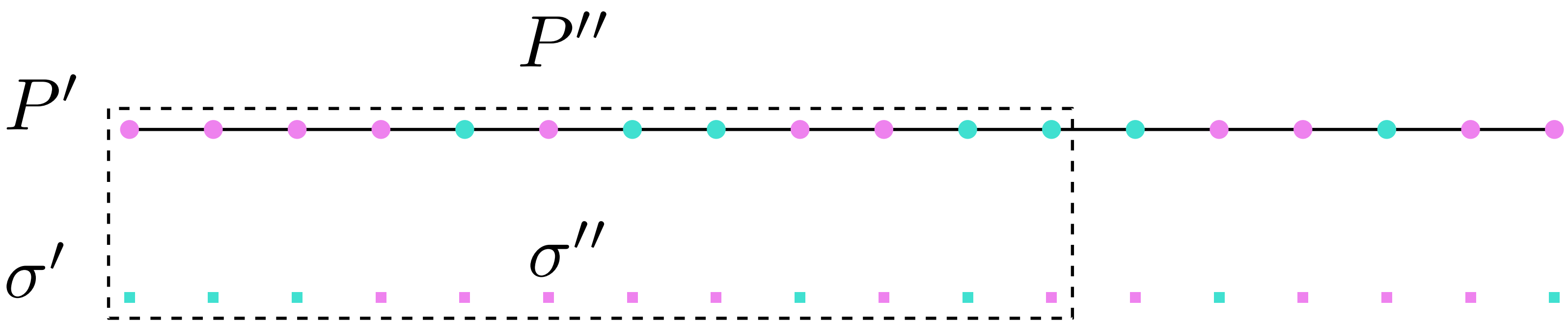
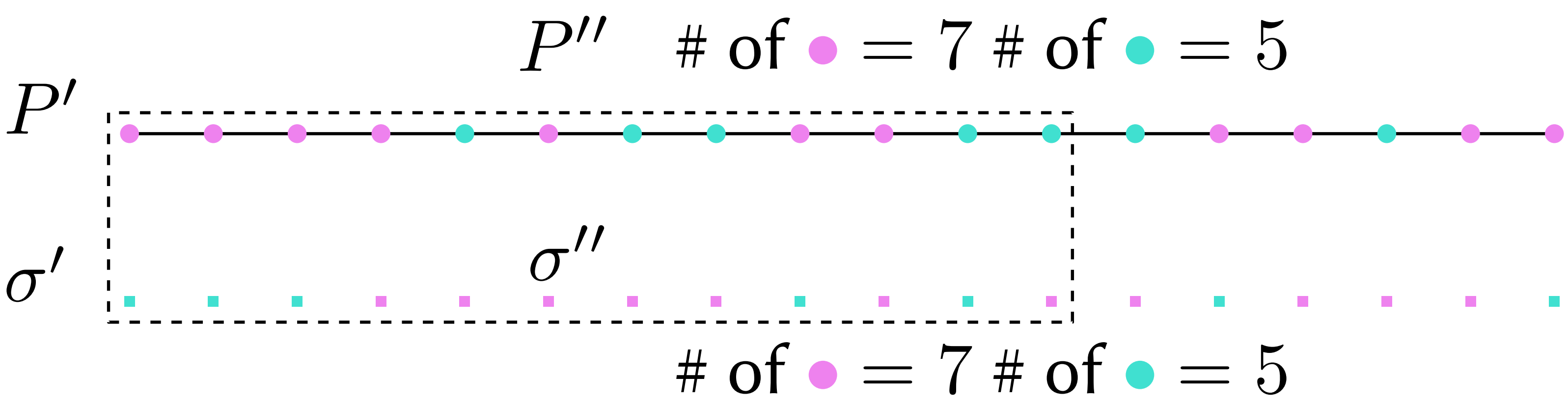


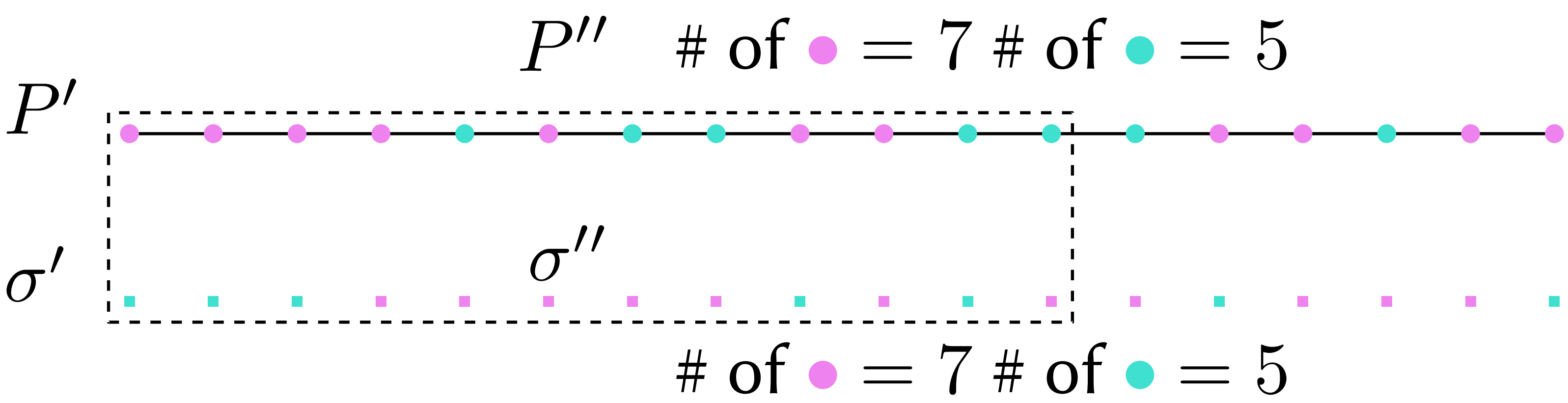
2-page topological book embedding of paths

If the # of ● in P'' = # of ● in σ''
AND the # of ● in P'' = # of ● in σ''
we say that P'' and σ'' are balanced



2-page topological book embedding of paths

If no prefix of P'' and σ'' are balanced we say that P'' and σ'' are minimally balanced



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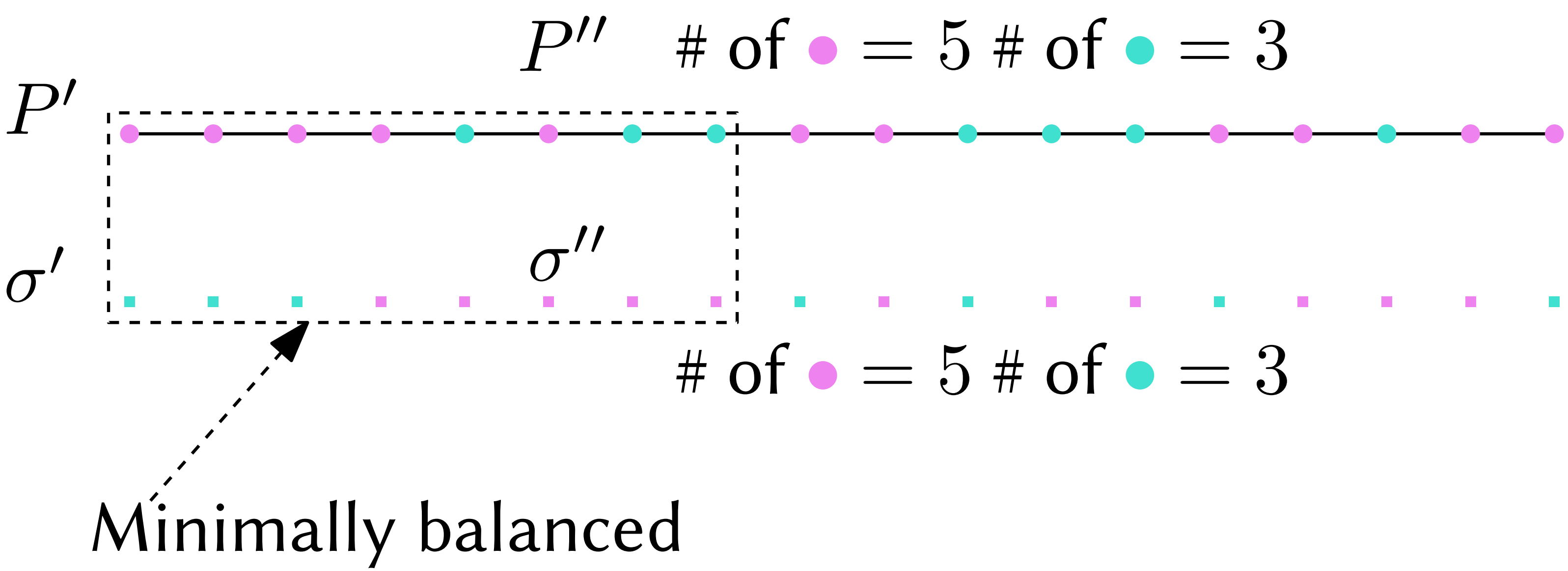
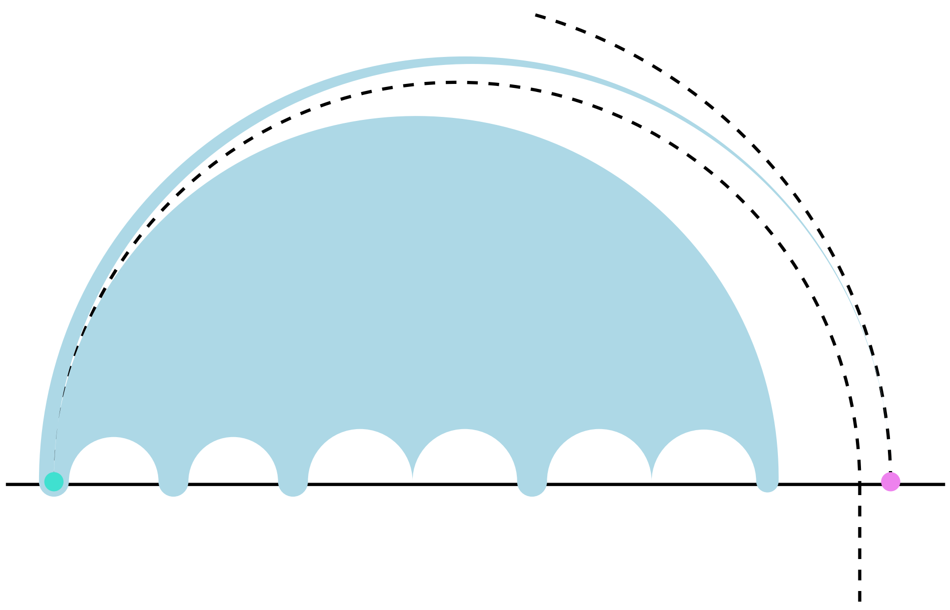


2-page topological book embedding of paths

We prove that P' admits a 2-page topological book embedding consistent with σ' s.t.

- there are at most 2 spine crossings per edge
- the first vertex is accessible from above without spine crossings
- the last vertex is accessible from below with one spine crossing



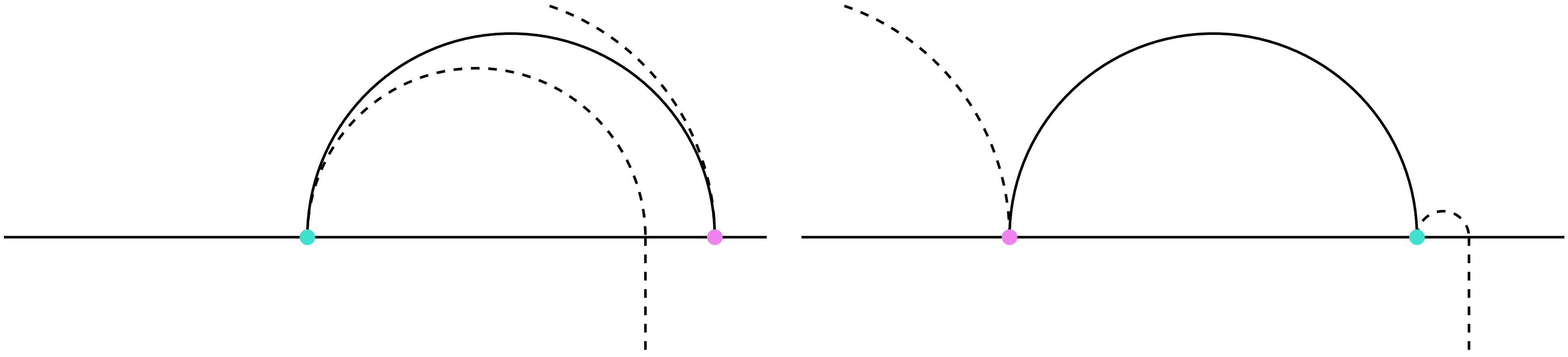
2-page topological book embedding of paths

Proof by induction on the number of vertices

2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n = 1, 2$



2-page topological book embedding of paths

Proof by induction on the number of vertices

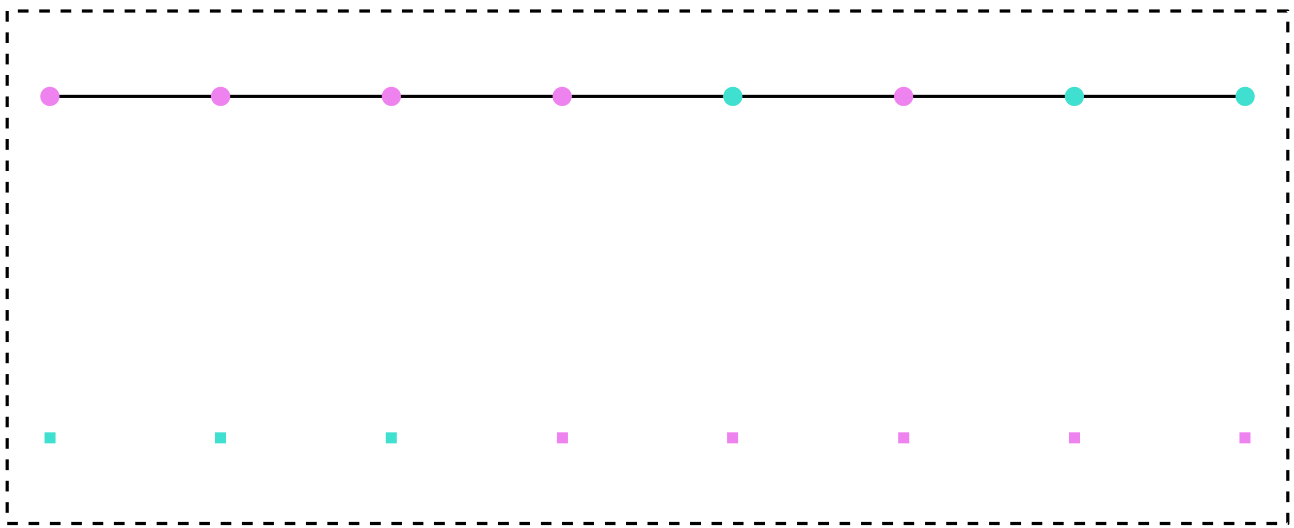
Base case $n > 2$

2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n > 2$

Case 1: P' and σ' are minimally balanced



2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n > 2$

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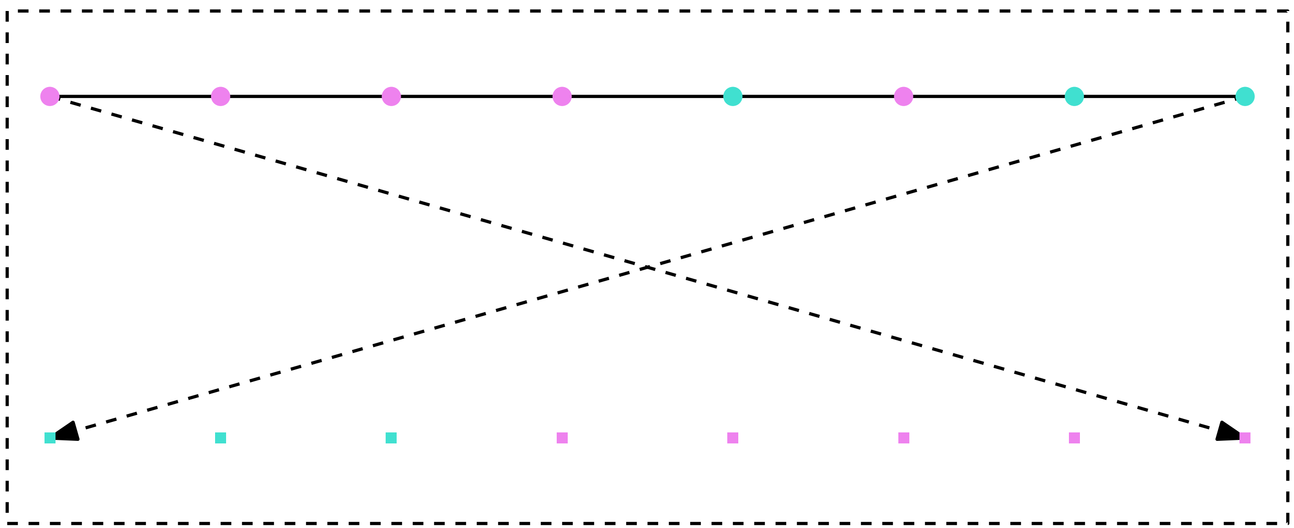
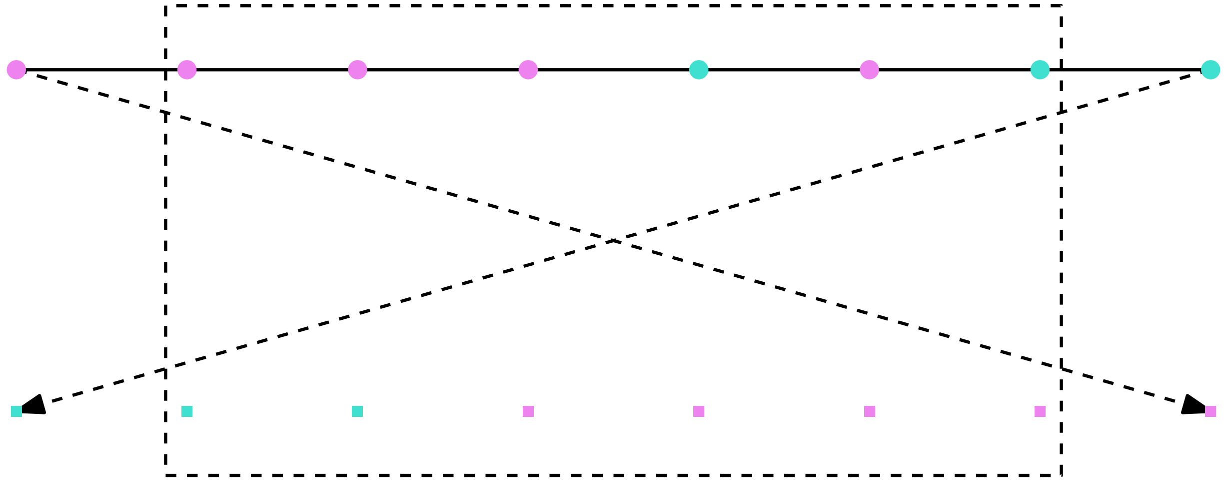


2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n > 2$

Case 1: P' and σ' are minimally balanced

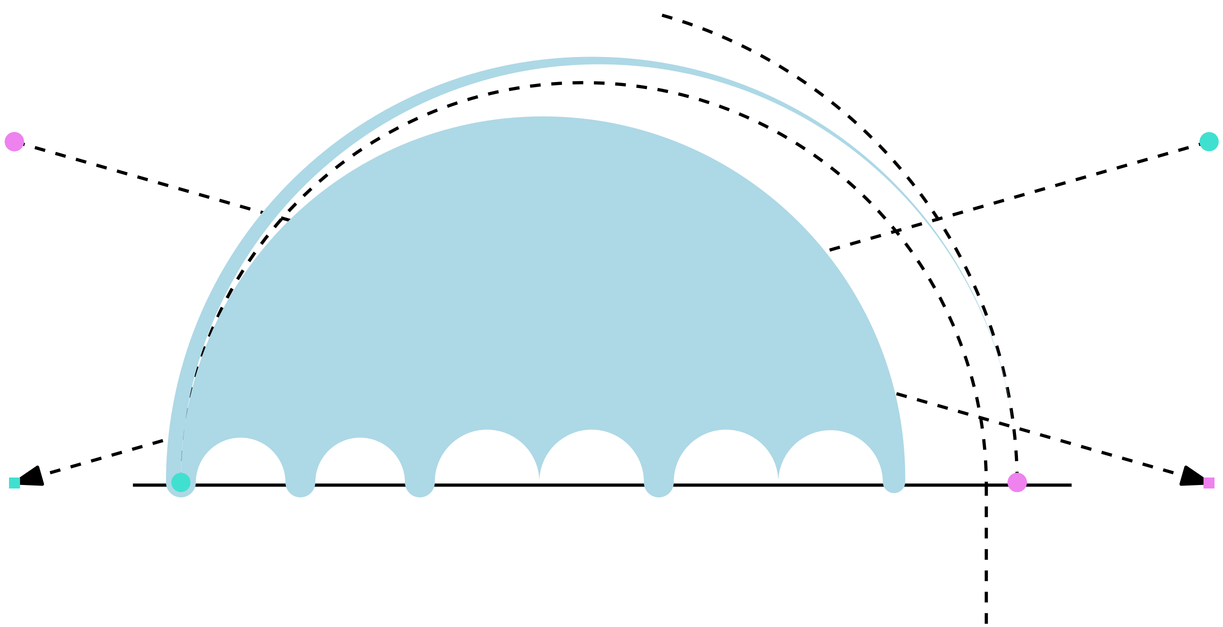


balanced

2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n > 2$
Case 1: P' and σ' are minimally balanced

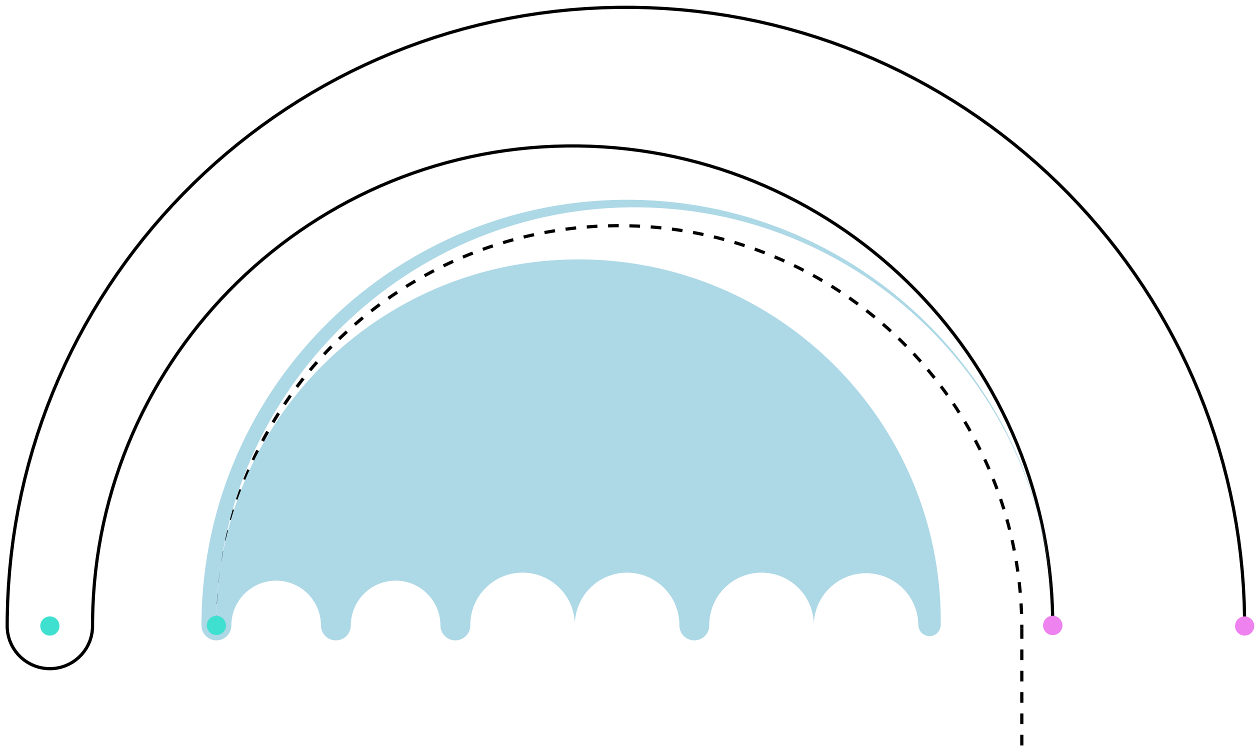


2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n > 2$

Case 1: P' and σ' are minimally balanced



2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n > 2$

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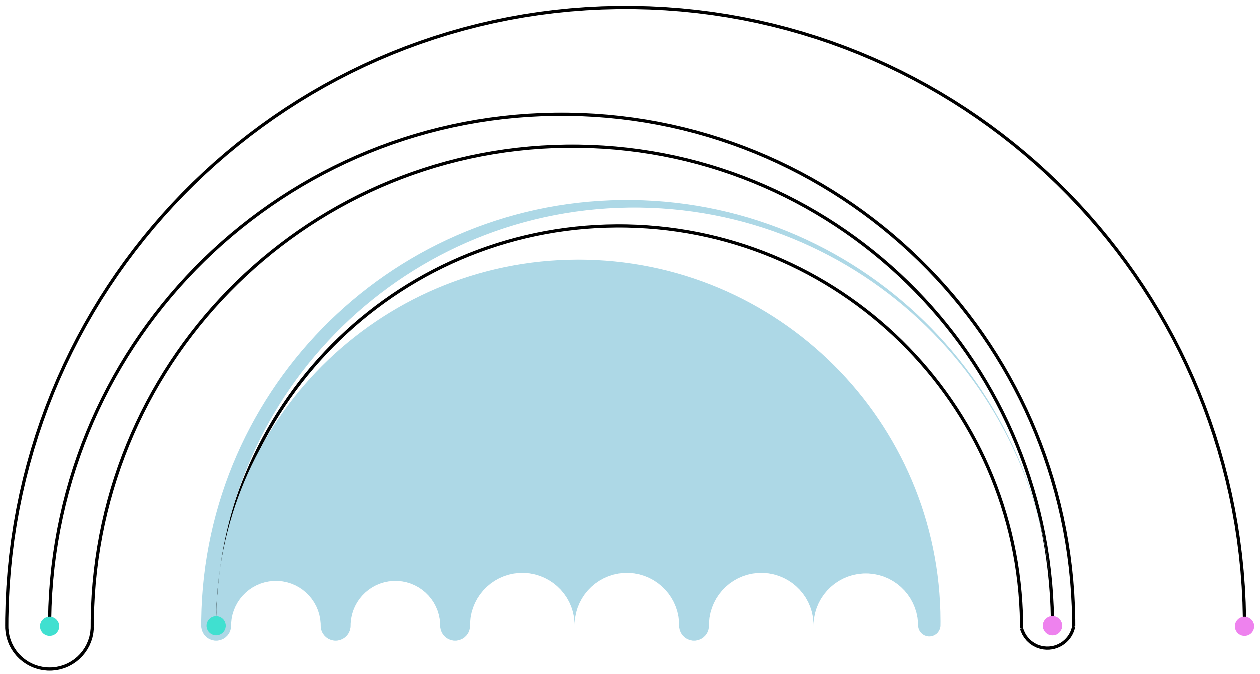

2-page topological book embedding of paths

Proof by induction on the number of vertices

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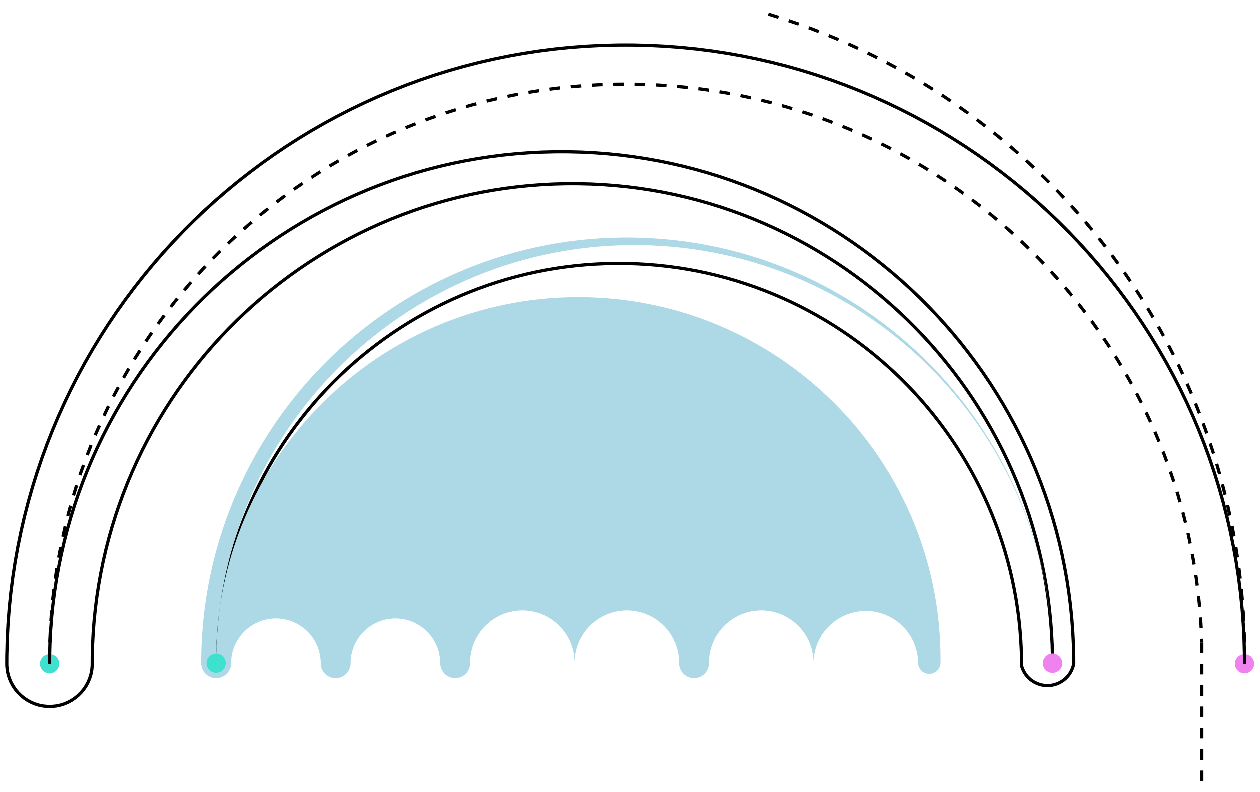
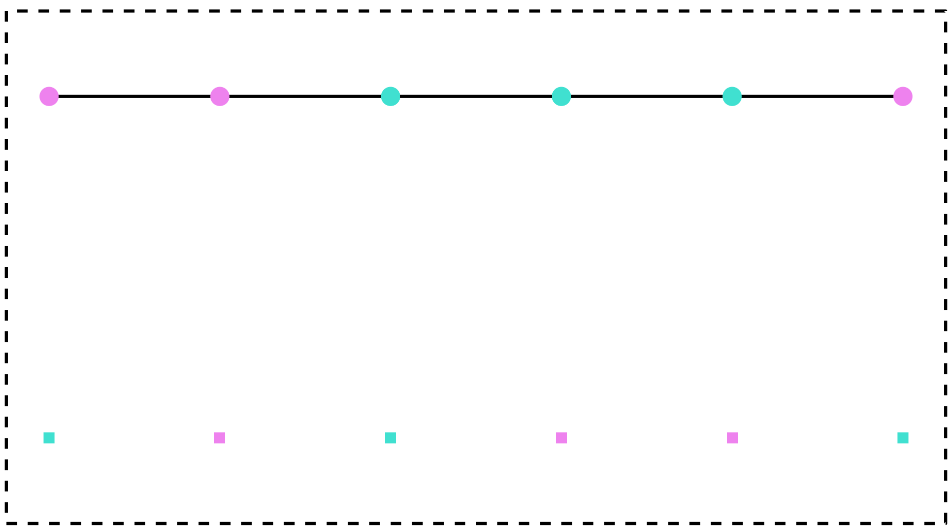


2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n > 2$

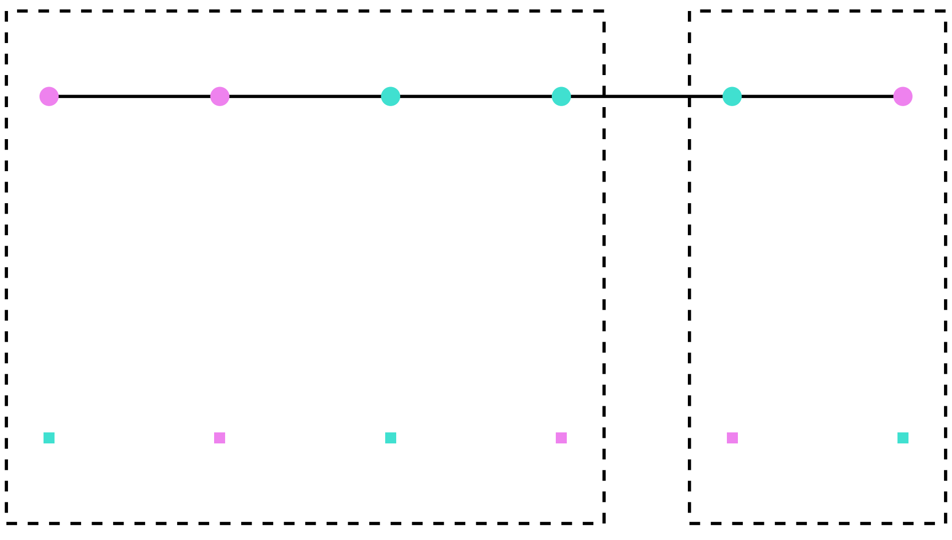
Case 2: P' and σ' are not minimally balanced



2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n > 2$
Case 2: P' and σ' are not minimally balanced

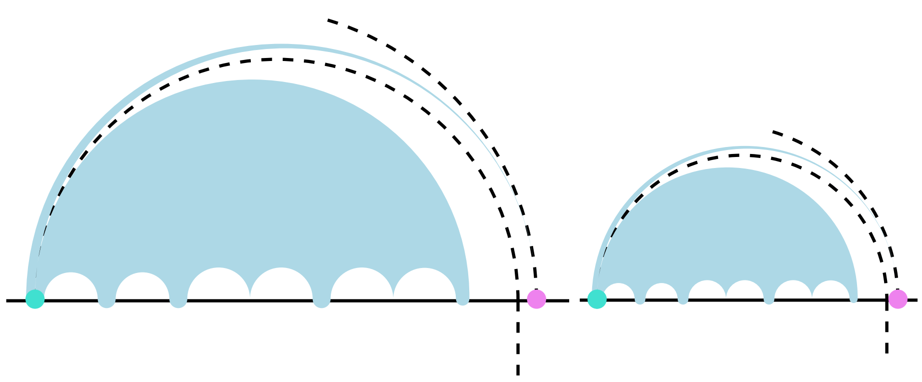


2-page topological book embedding of paths

Proof by induction on the number of vertices

Base case $n > 2$

Case 2: P' and σ' are not minimally balanced



2-page topological book embedding of paths

Proof by induction on the number of vertices

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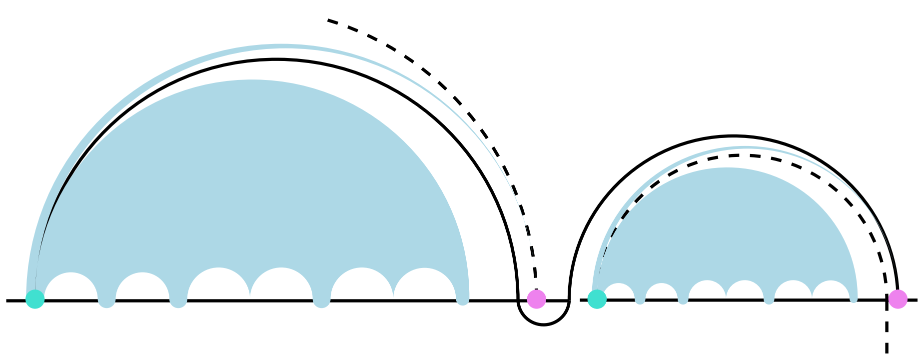
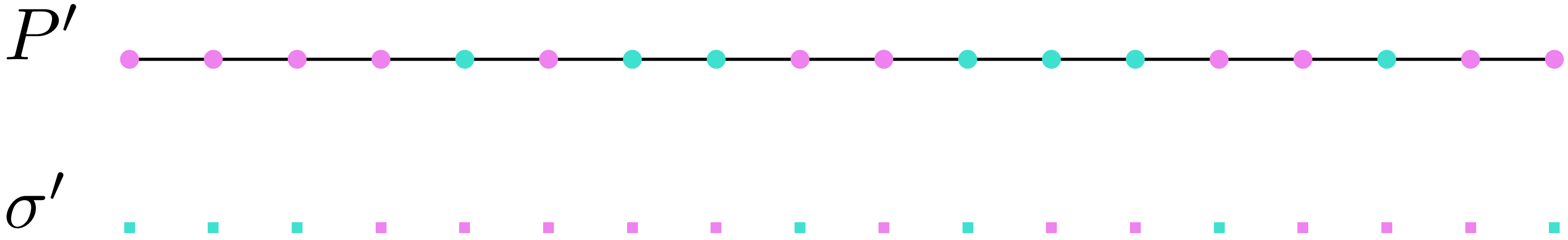
Case 2: P' and σ' are not minimally balanced



2-page topological book embedding of paths



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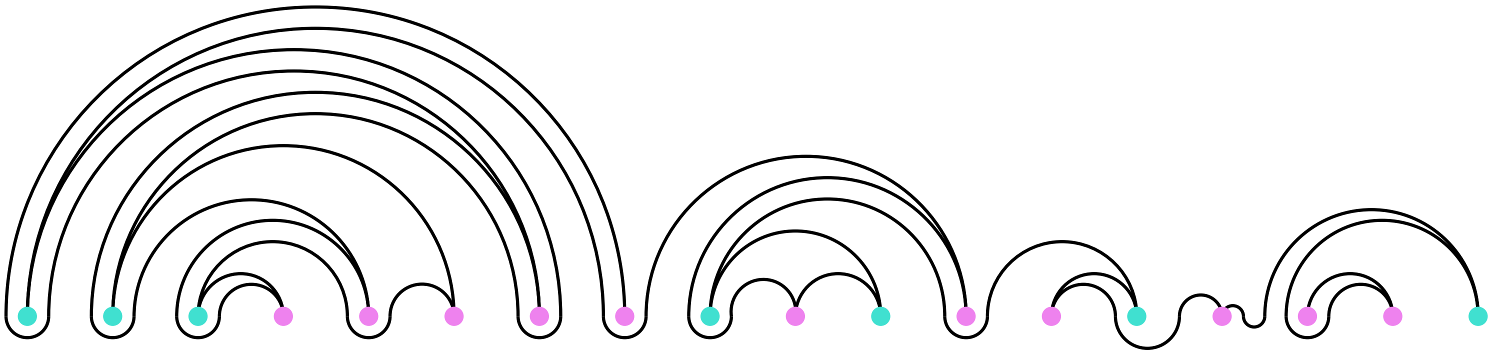
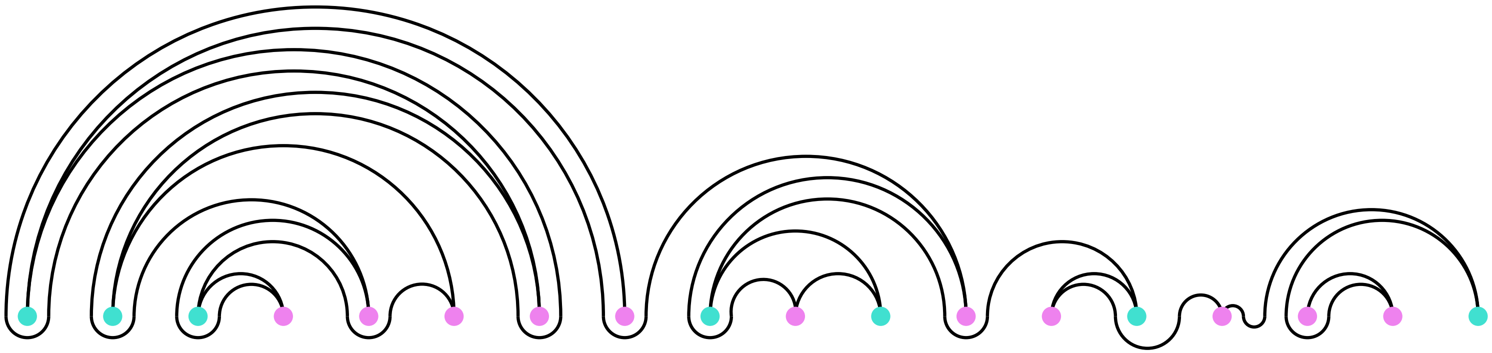


2-page topological book embedding of paths

Let's add back the third color

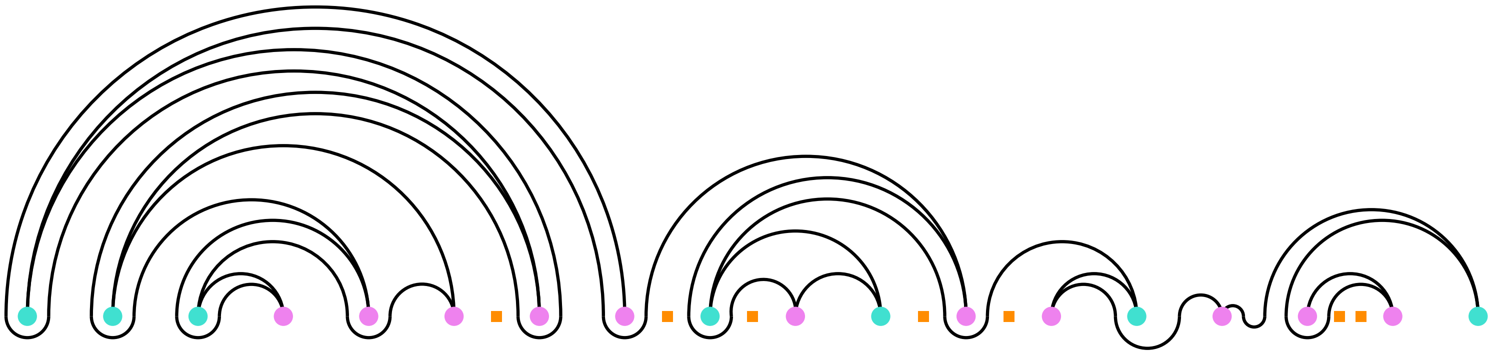
2-page topological book embedding of paths

Path P



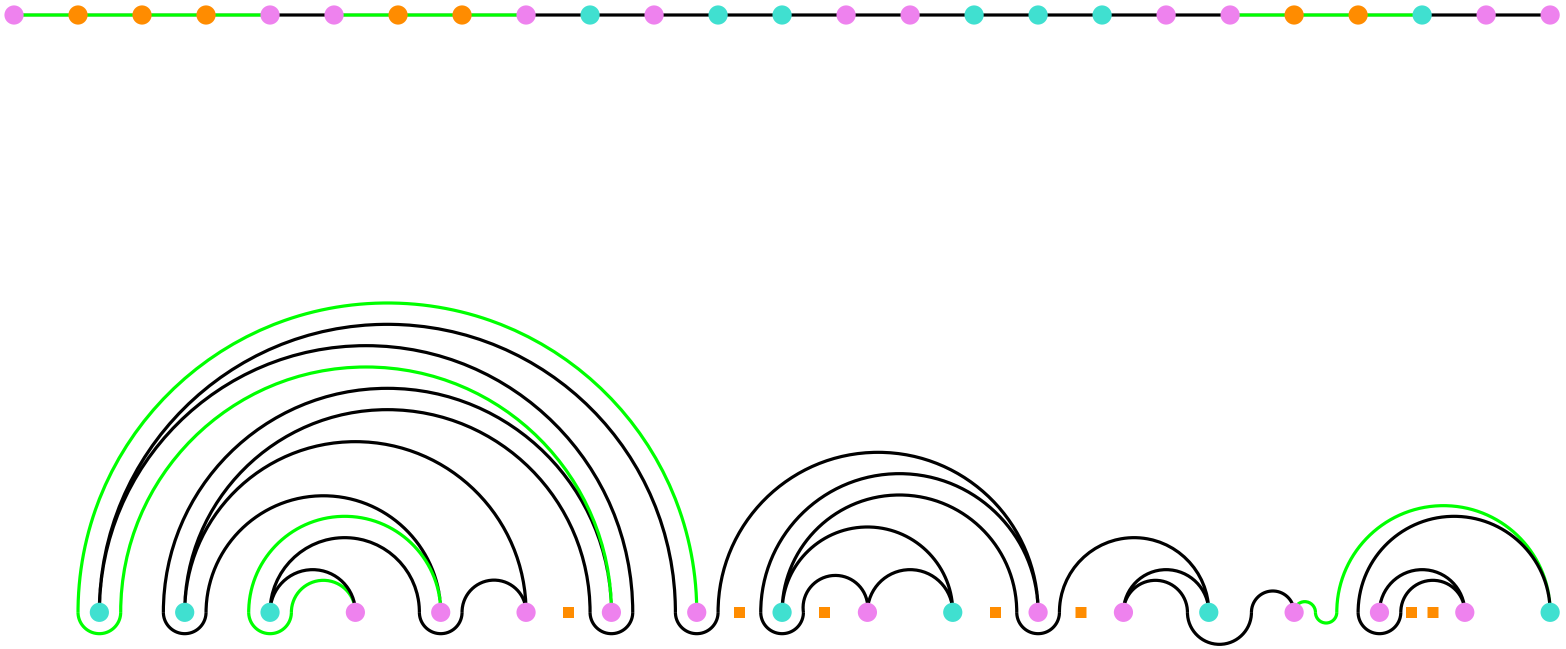
2-page topological book embedding of paths

Path P



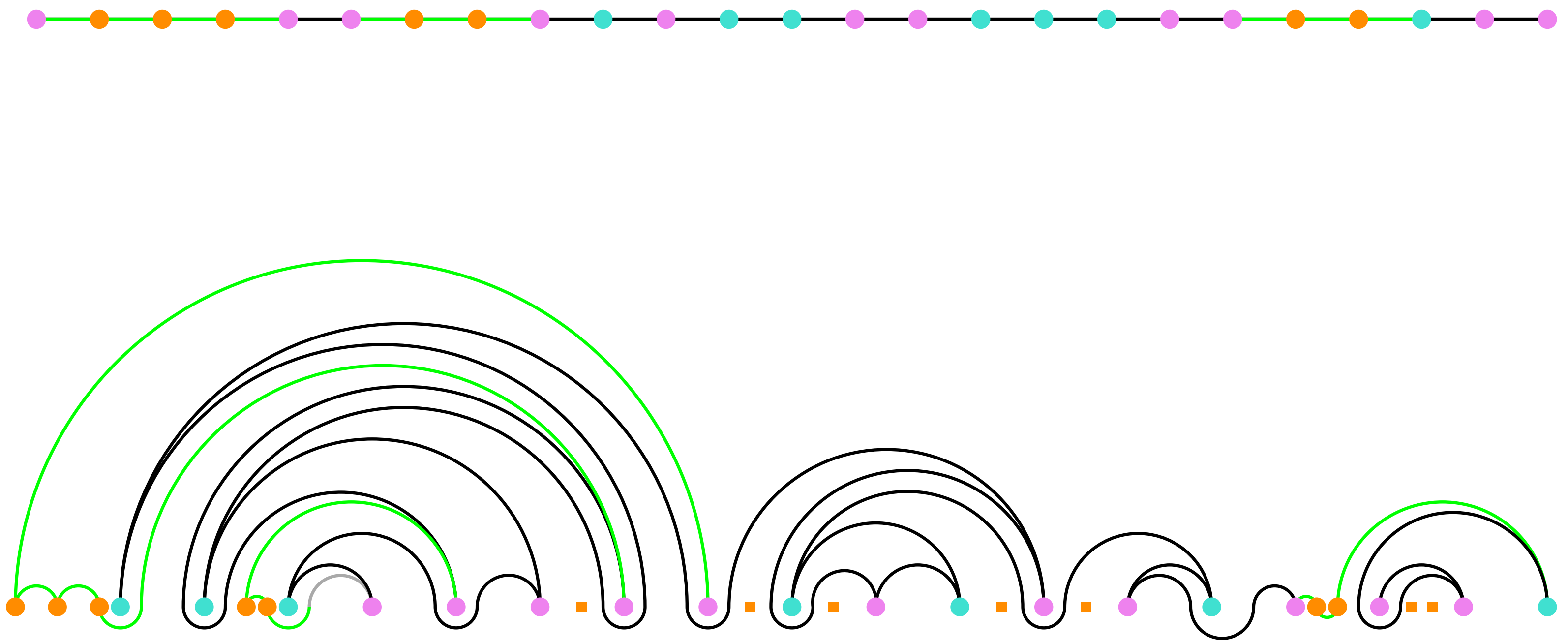
2-page topological book embedding of paths

Path P



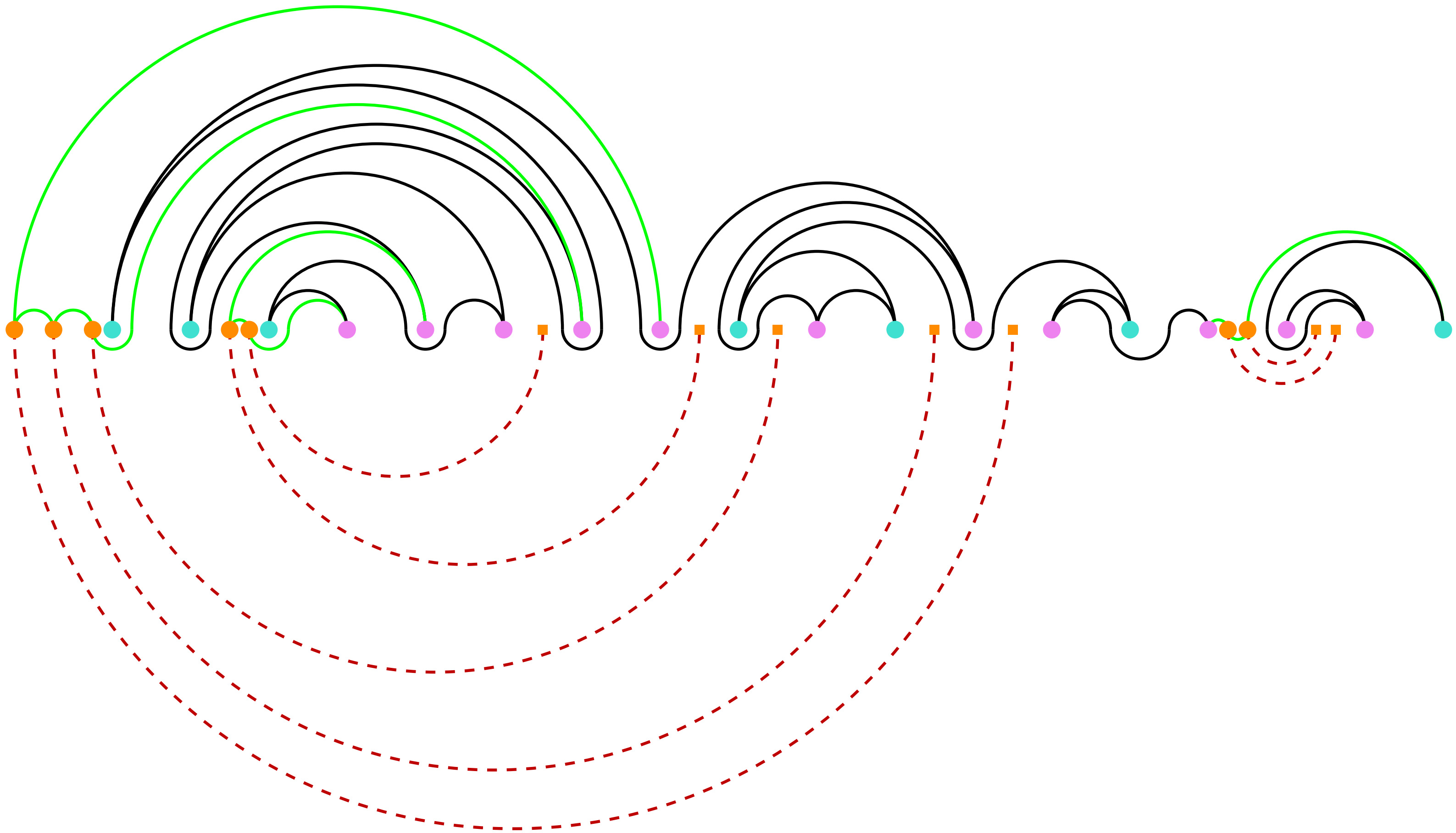
2-page topological book embedding of paths

Path P



2-page topological book embedding of paths

Path P



2-page topological book embedding of paths

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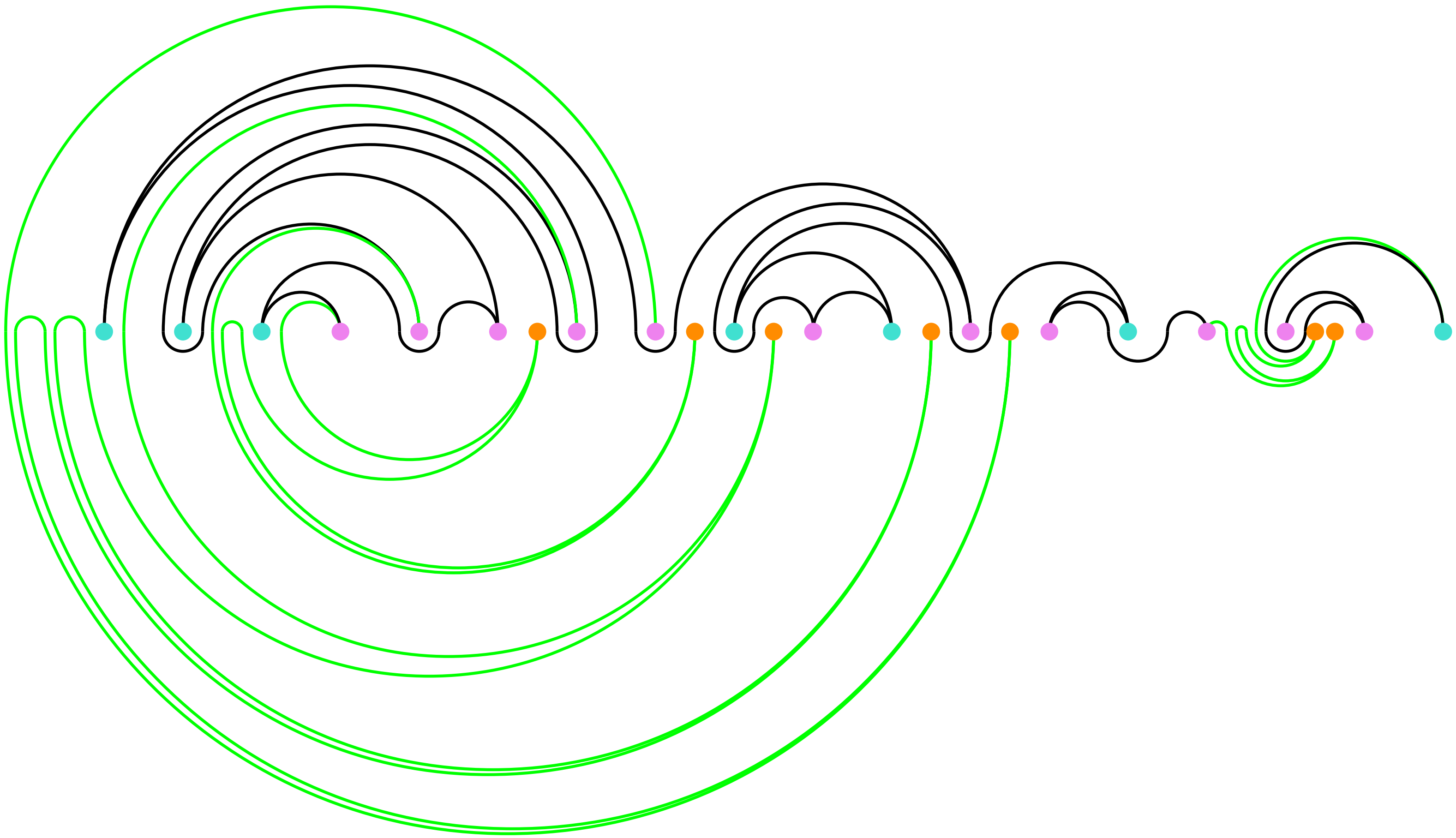

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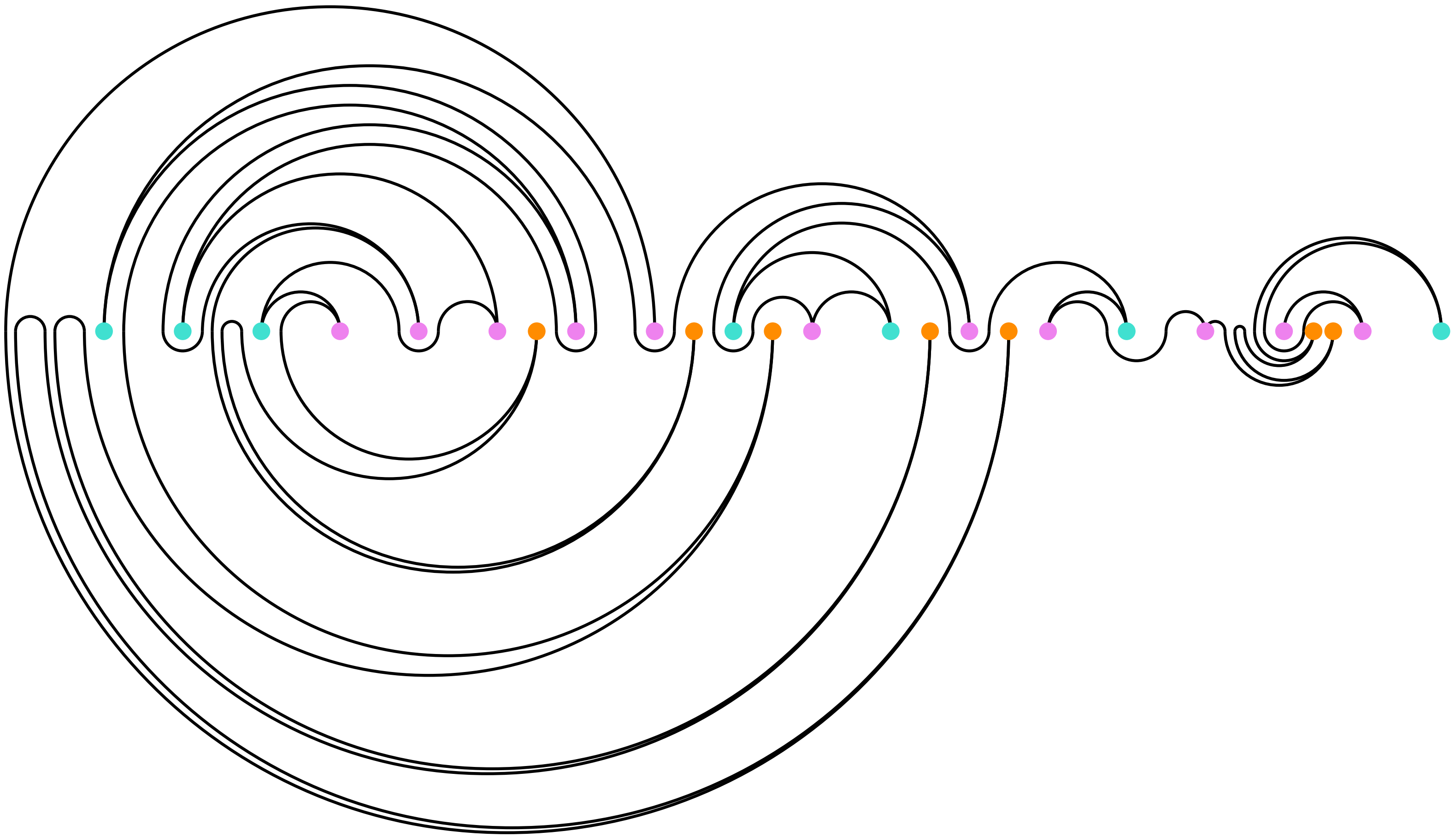

Open problems

Investigate whether the lower bound for the 3-colored forest of stars is tight.

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Characterize the 3-colored caterpillars that admit a 3-colored point-set embedding with constant curve complexity on any given set of points.

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Characterize the 3-colored caterpillars that admit a 3-colored point-set embedding with constant curve complexity on any given set of points.

Study whether constant curve complexity can always by guaranteed for 4-colored paths.

THANK YOU