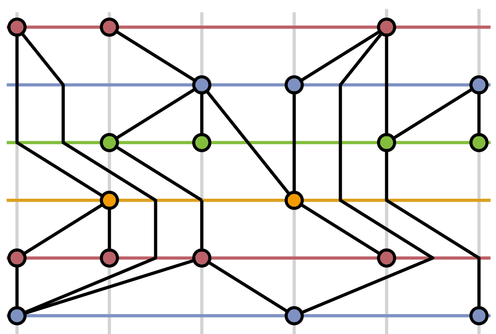
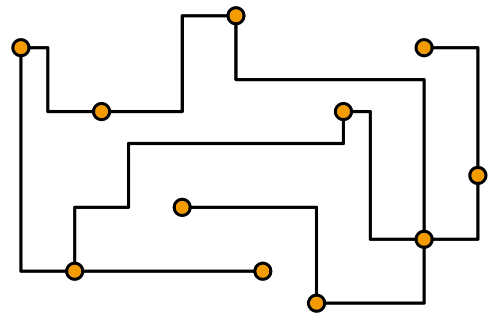
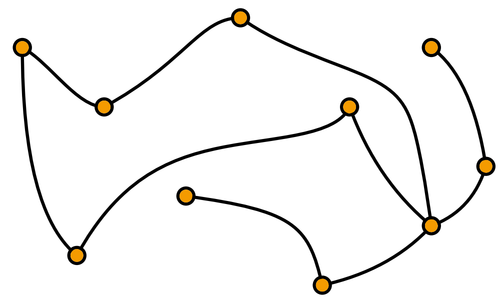
Ordered Level Planarity, Geodesic Planarity and Bi-Monotonicity

Boris Klemz
Günter Rote

Freie Universität Berlin

GD 2017, Boston

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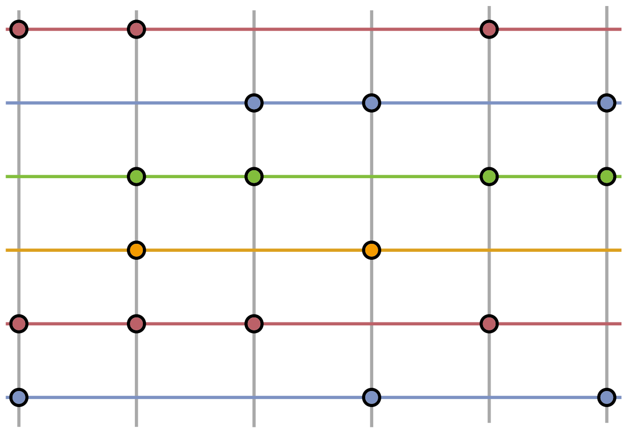

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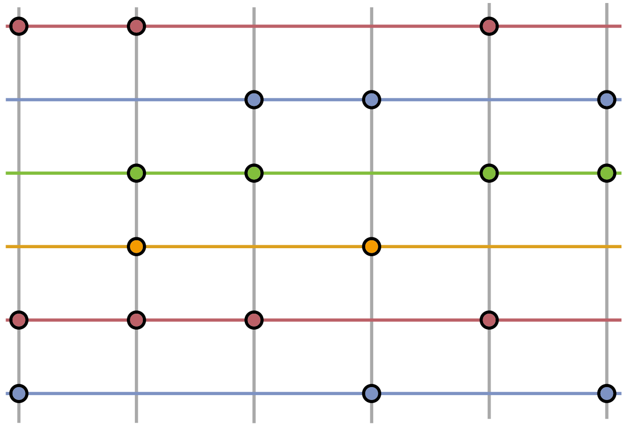

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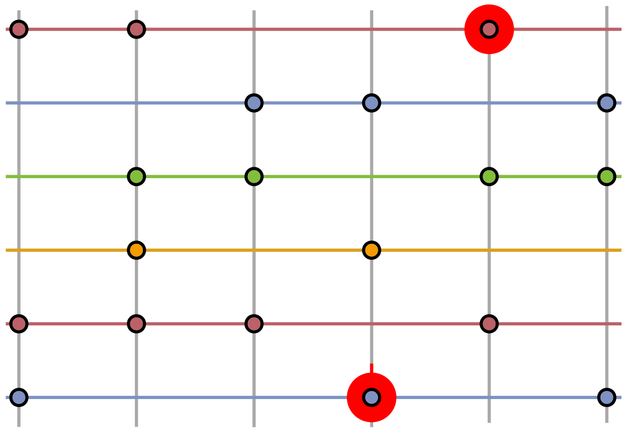

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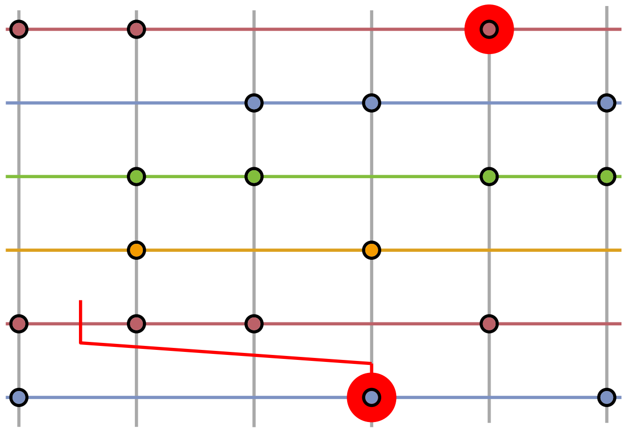

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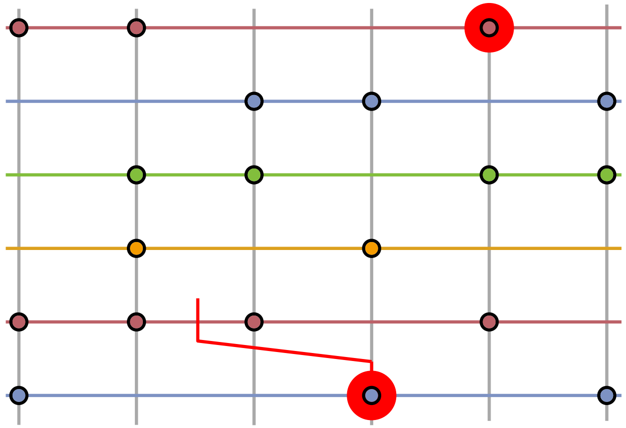

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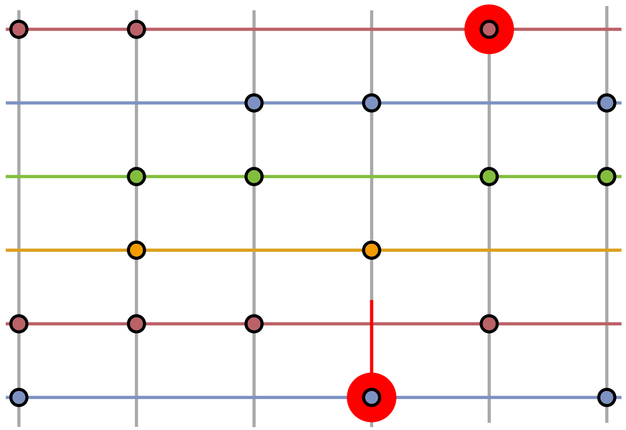

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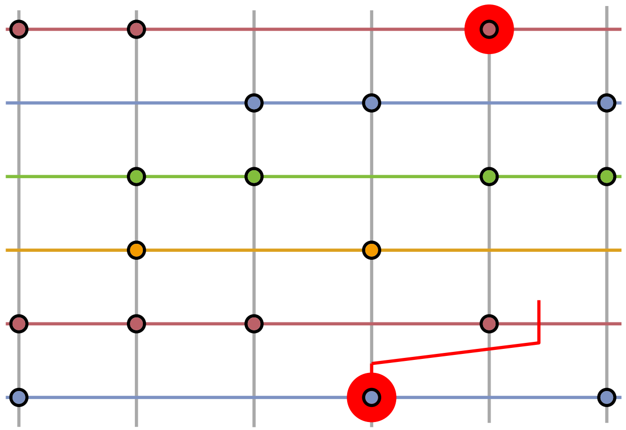

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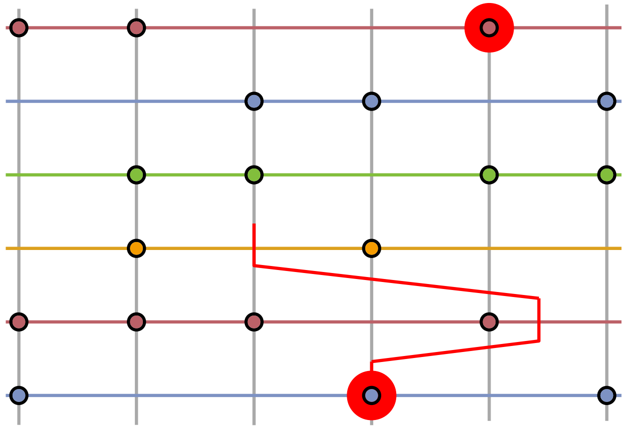

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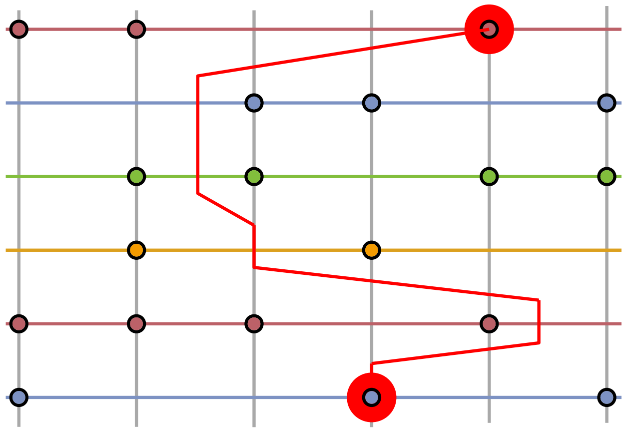

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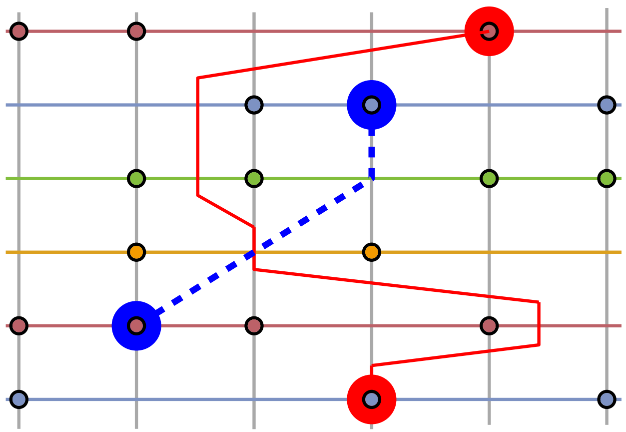

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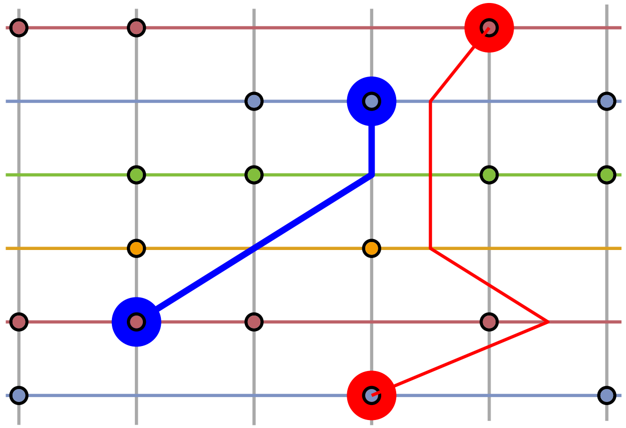

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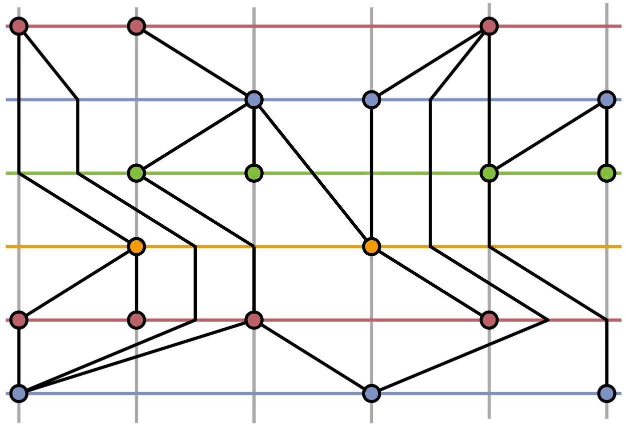

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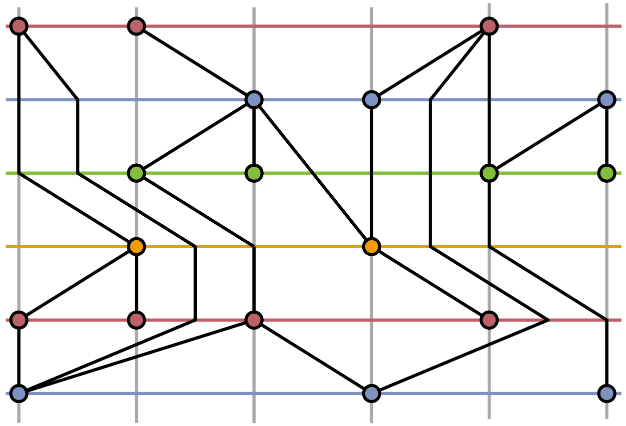

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Realizability and Orderings

The y-coordinates encode a partial order.

On each level, the x-coordinates encode a total order.

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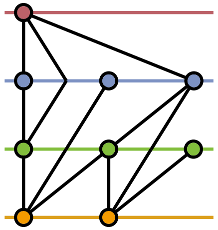

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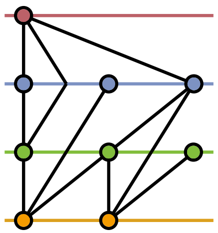

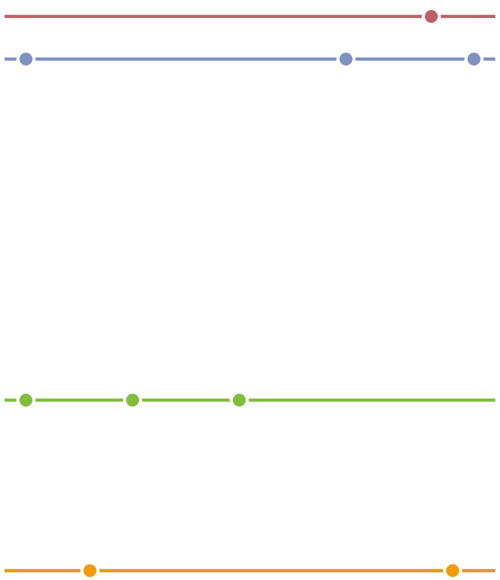

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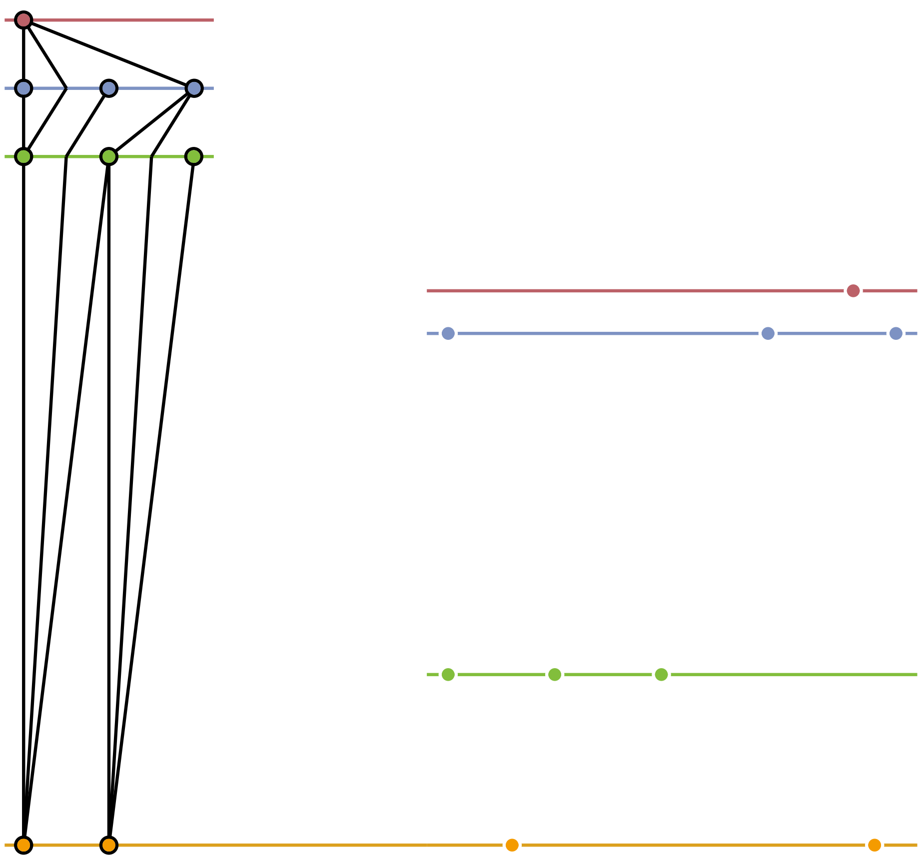

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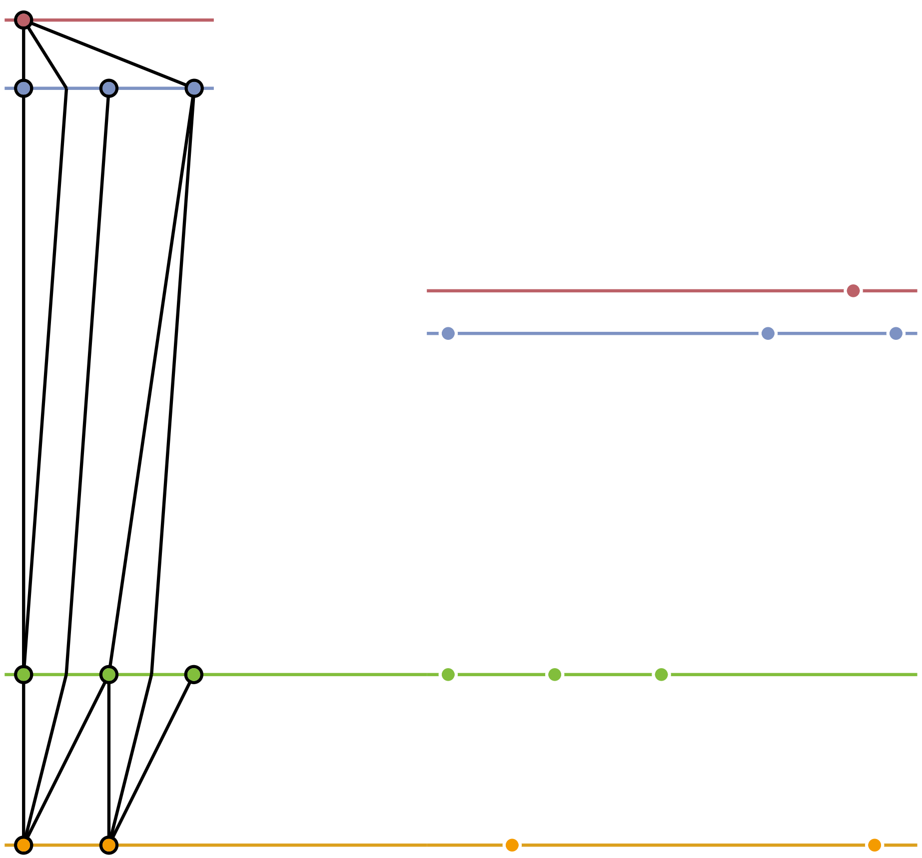

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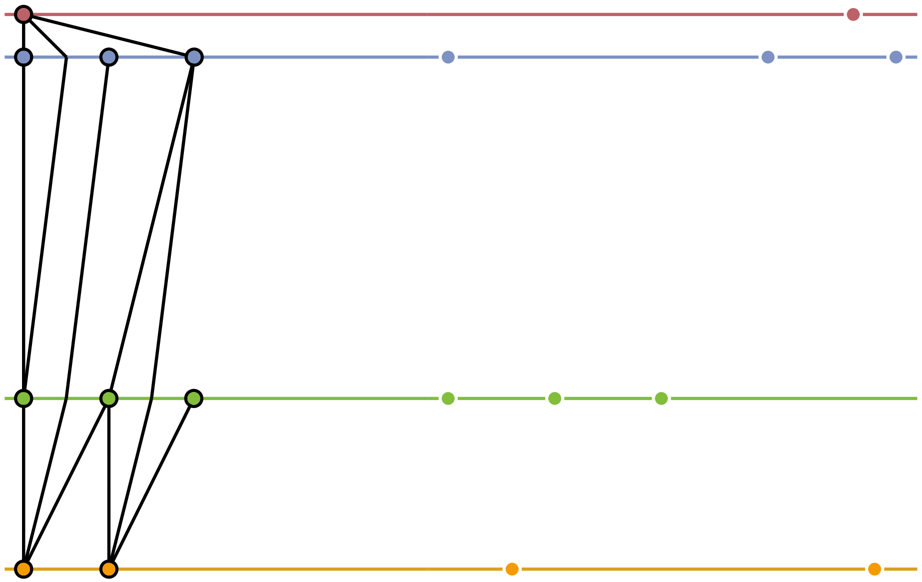

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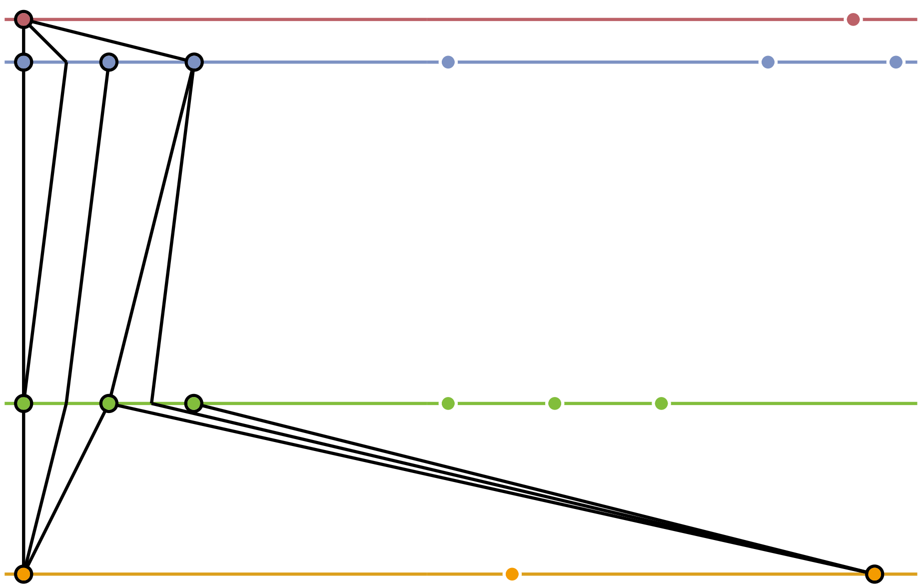

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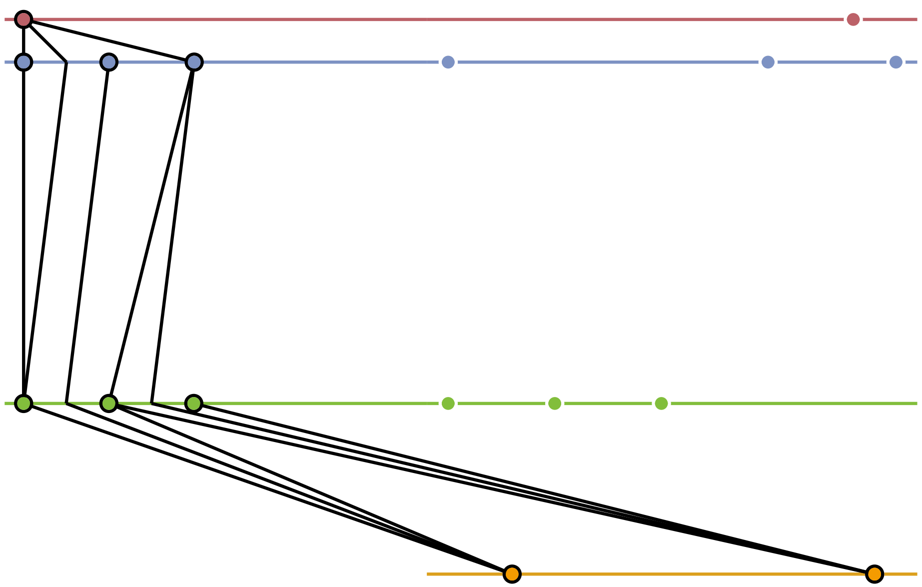

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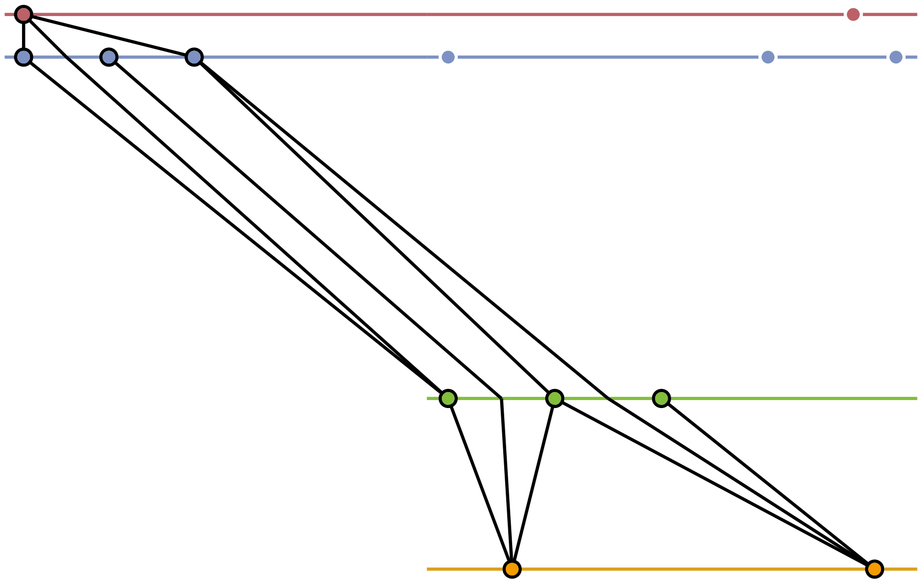

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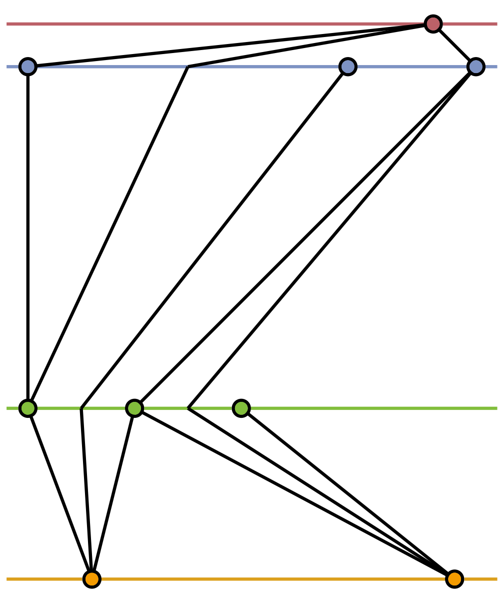

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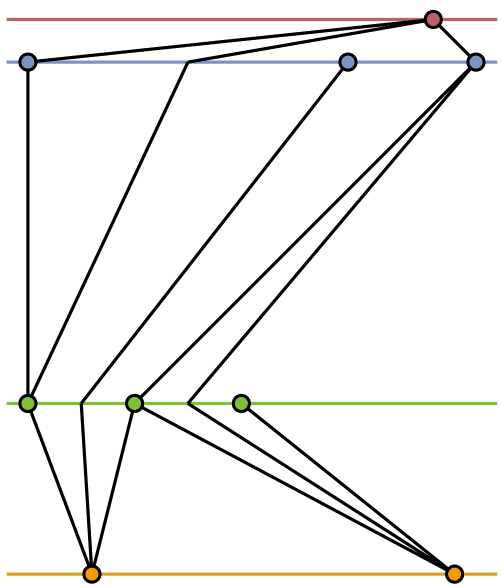
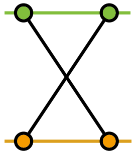


Not realizable:



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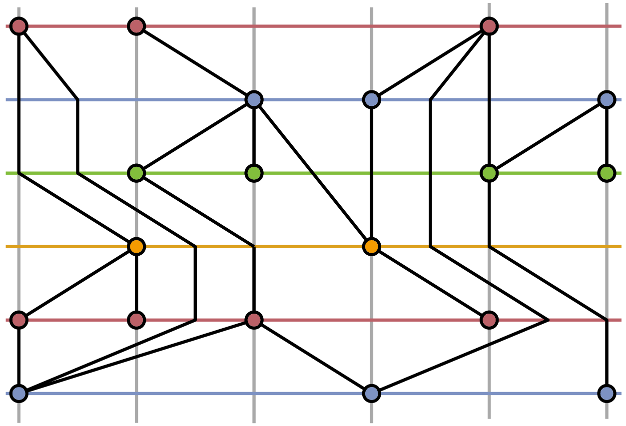

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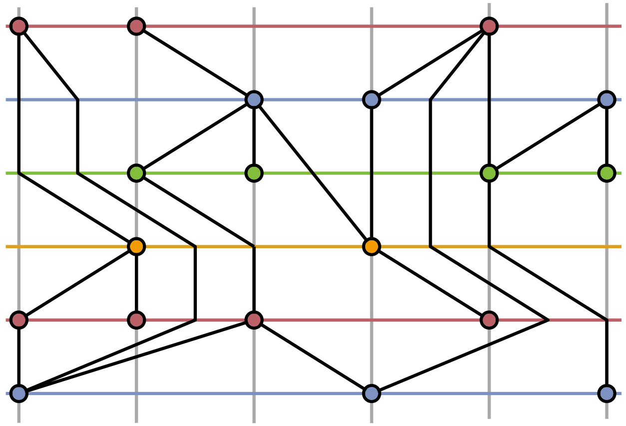

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Linear time [Jünger, Leipert, Mutzel'98]

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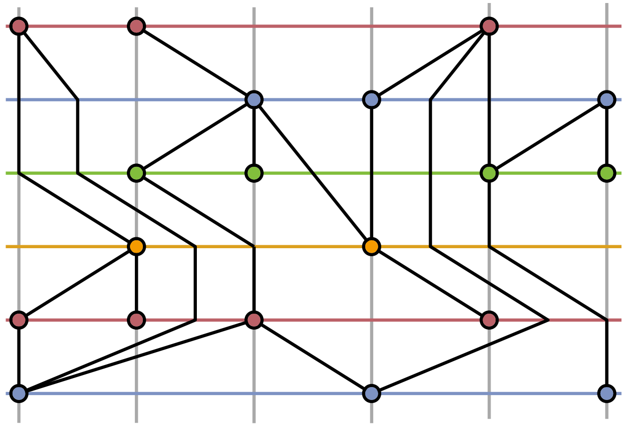

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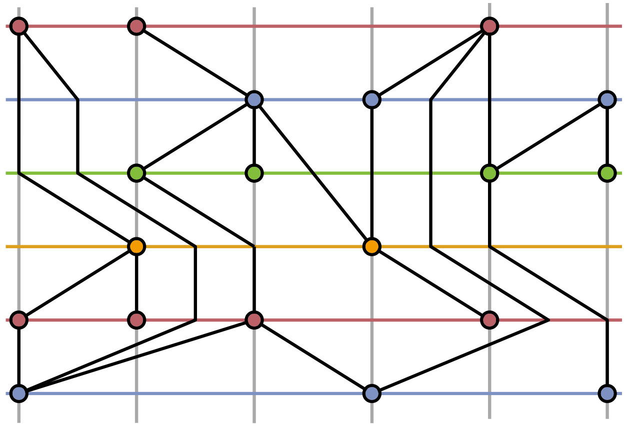

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NP-complete even in constrained cases

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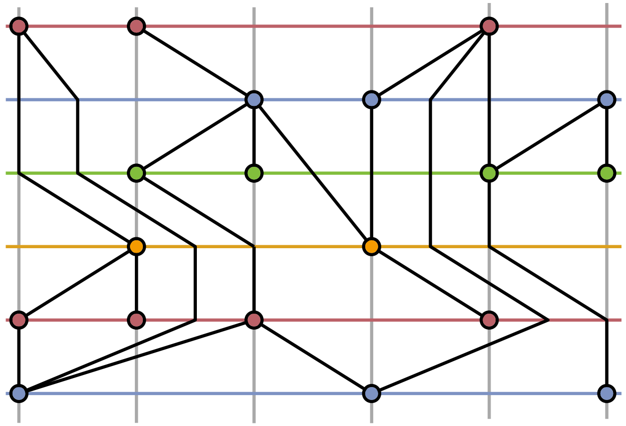

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Motivation: special case of many point-set embedding problems

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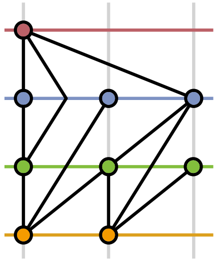
Result Overview

level-width λ = max. #vertices per level

Ordered Level Planarity



NP-complete even for $\Delta = \lambda = 2$

Polytime for $\Delta_{in} = \Delta_{out} = 1$ or $\lambda = 1$

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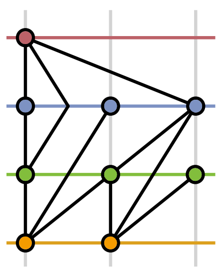

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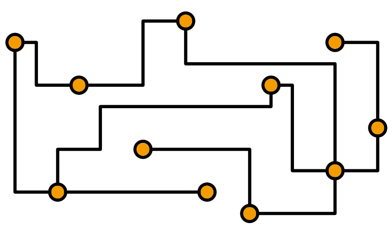
Geodesic Planarity



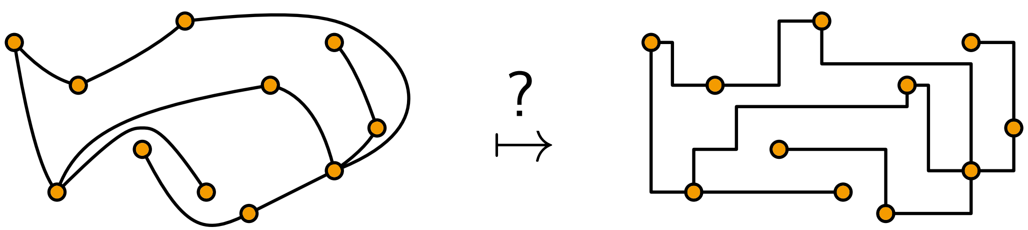
NP-hard even for matchings in general position

Problem Definition

Manhattan Geodesic Planarity

Given: planar graph, vertex coordinates, $\Delta \leq 4$

Want: plane drawing with rectilinear L_1-geodesic edges
and vertices at prescribed positions



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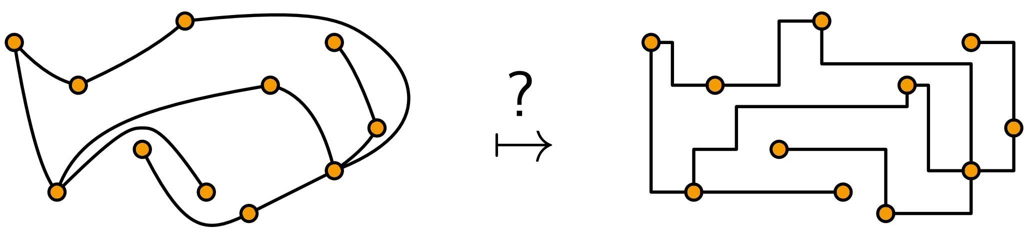
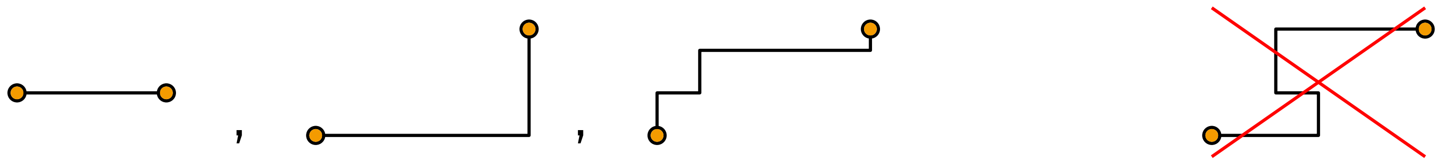


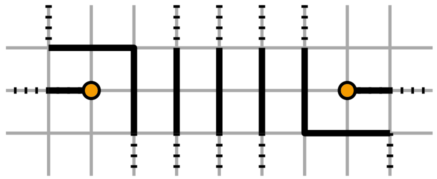
Edge representations:



Previous Work

MANHATTAN GEODESIC PLANARITY is NP-hard even for matchings if drawings are restricted to a grid. [Katz, Krug, Rutter, Wolff GD'09]

Idea: space between vertices is bounded



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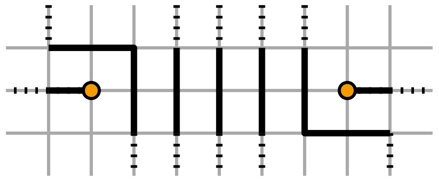
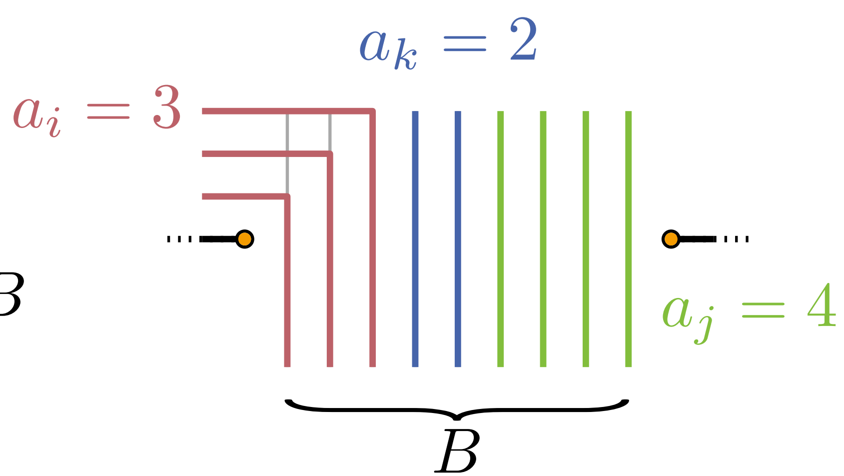


Reduction from 3-PARTITION:

Given: $a_1, \ldots, a_{3n} \in \mathbb{N}$, $\sum a_i = 3B$
Want: partition into n triples of sum B



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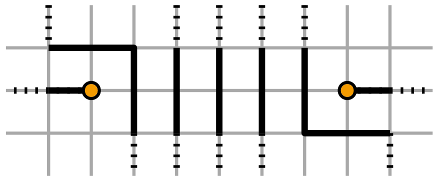

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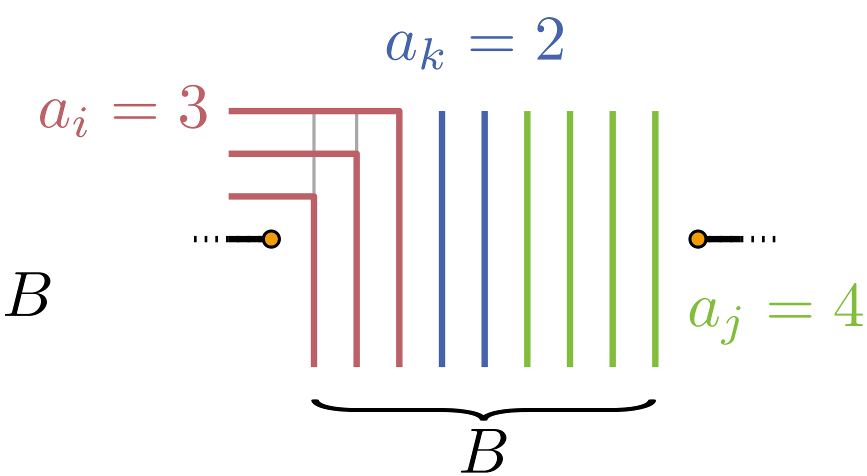


Non-grid version:

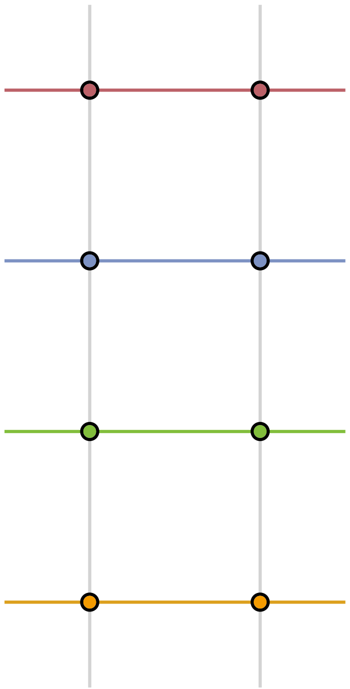
Claim: polytime for matchings [Katz, Krug, Rutter, Wolff GD'09]

Our result: NP-hardness for matchings
via reduction from ORDERED LEVEL PLANARITY

The Reduction

$$\textsc{Ordered Level Planarity} \leq_p \textsc{Manhattan Geodesic Planarity}$$

$\Delta = \lambda = 2$ matching, general position



$\mapsto$

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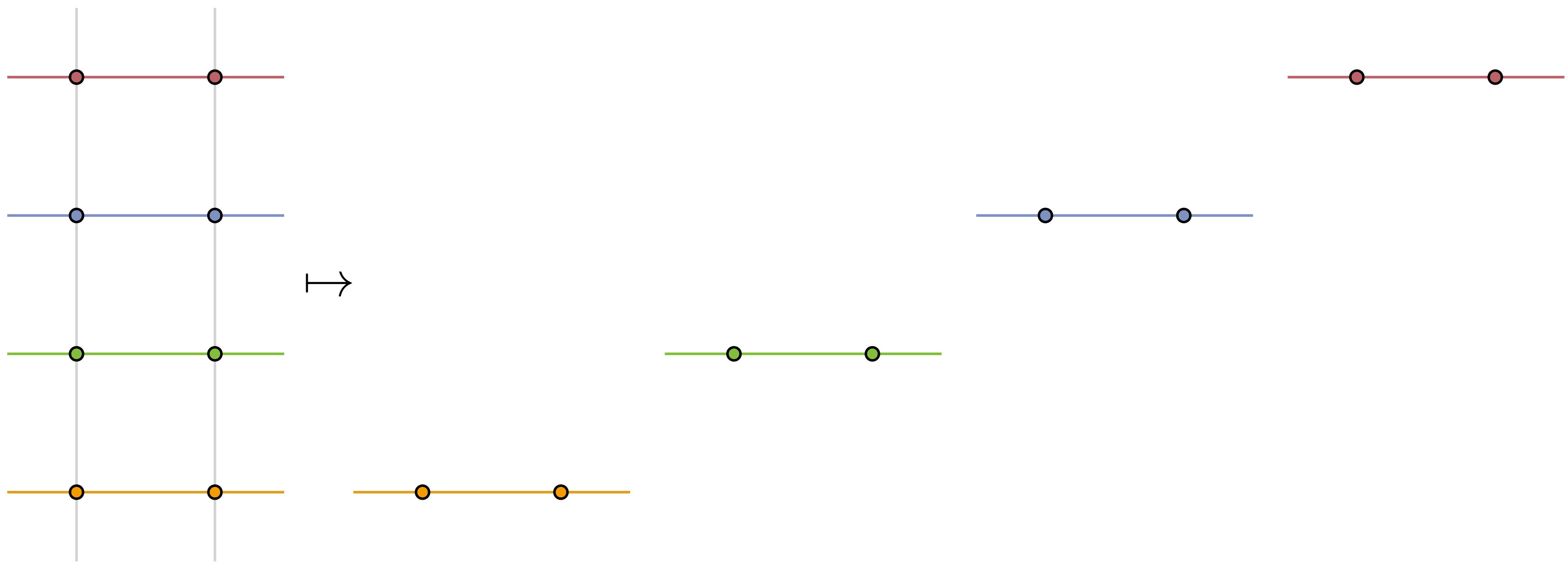

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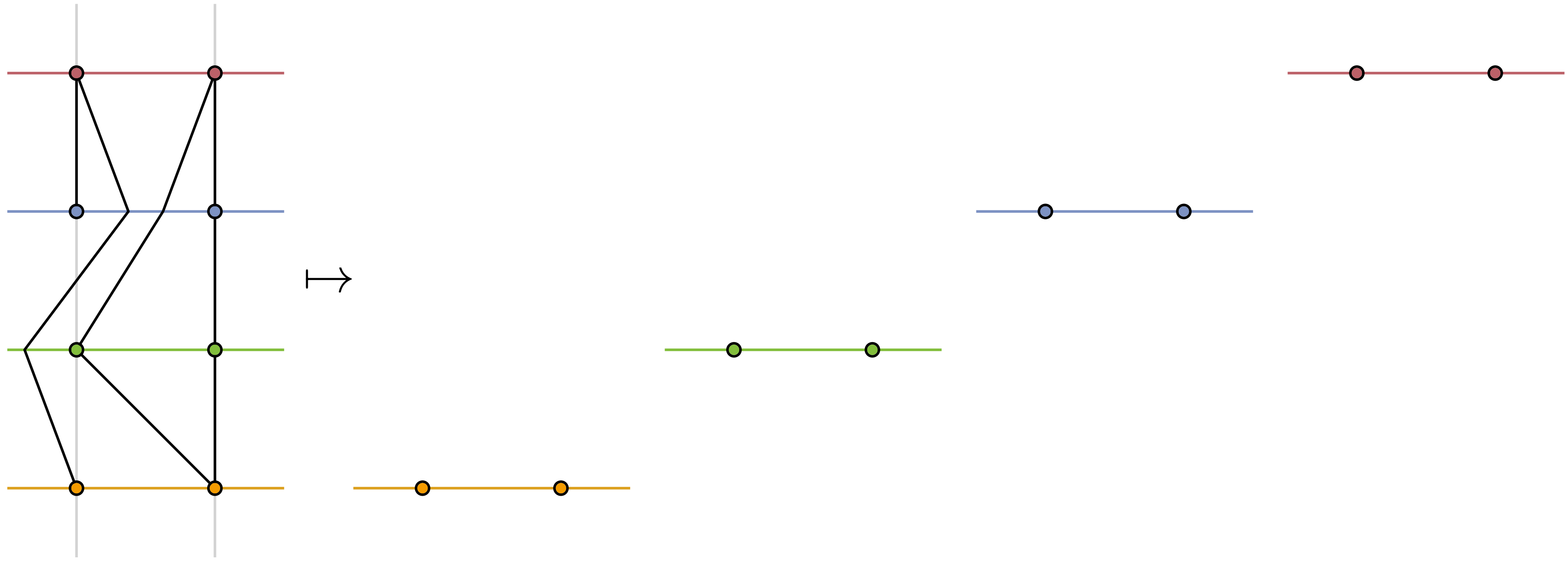

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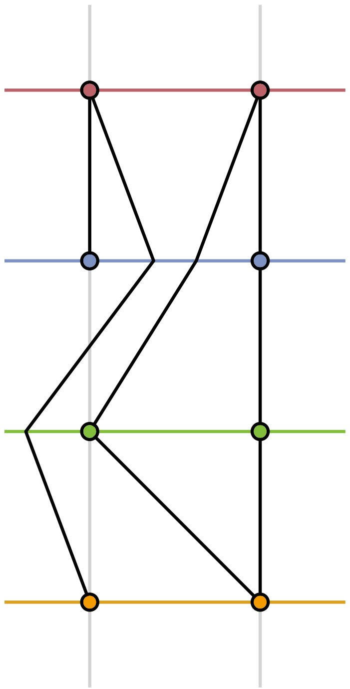

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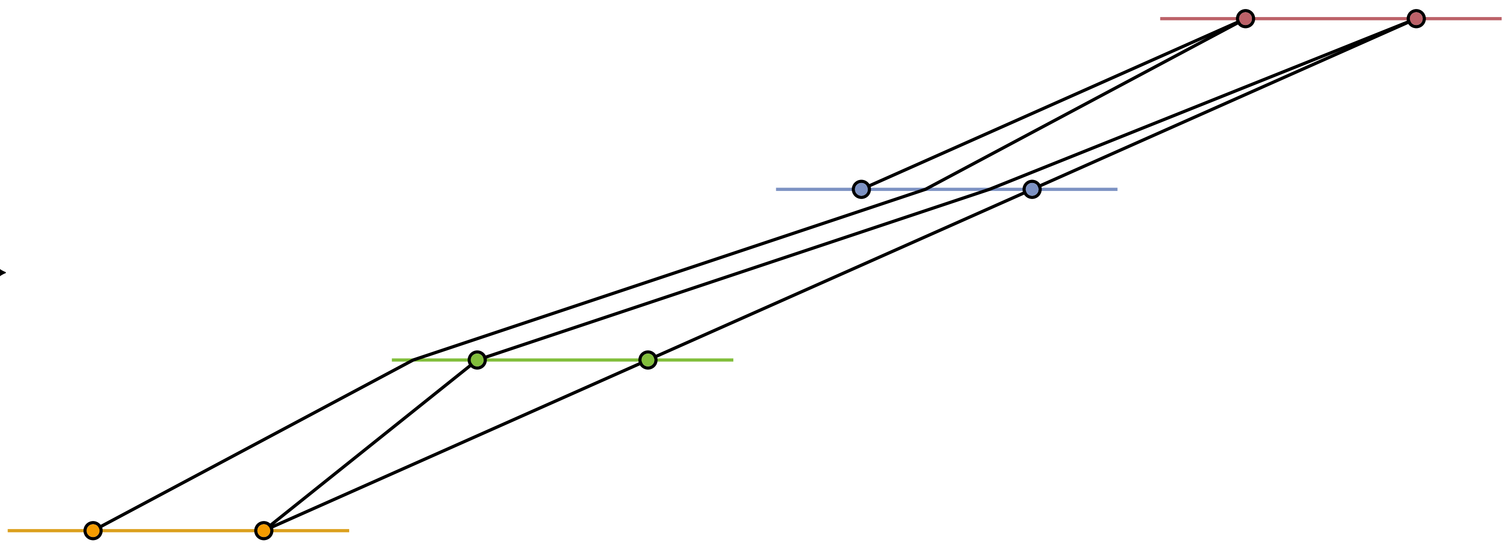

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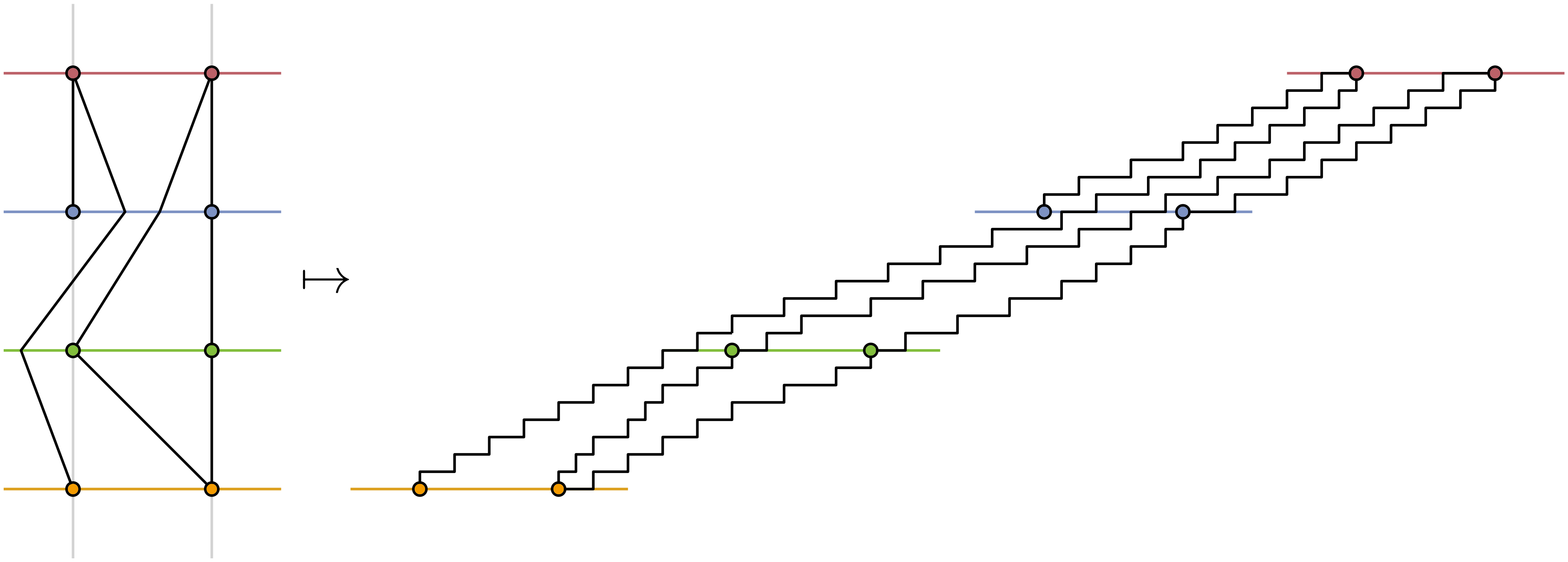

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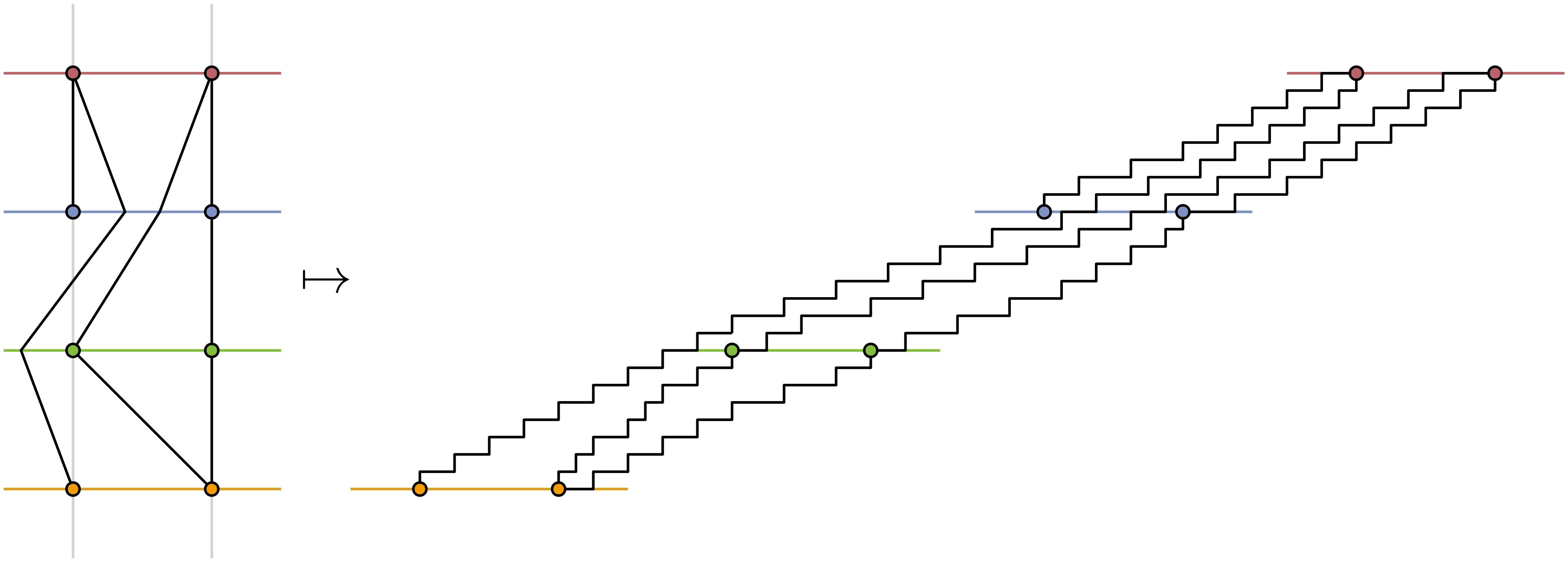
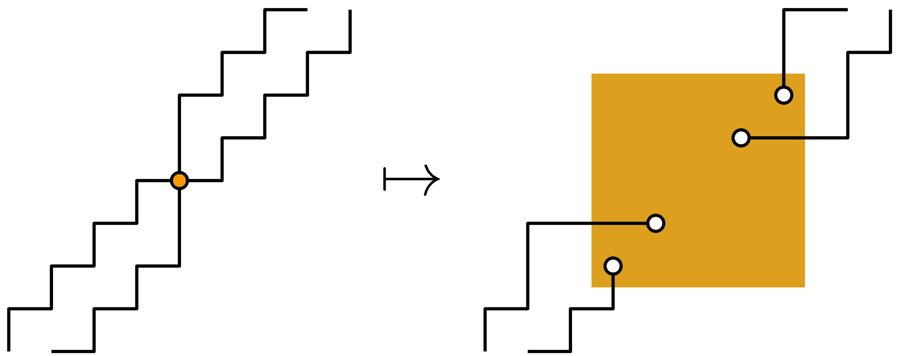


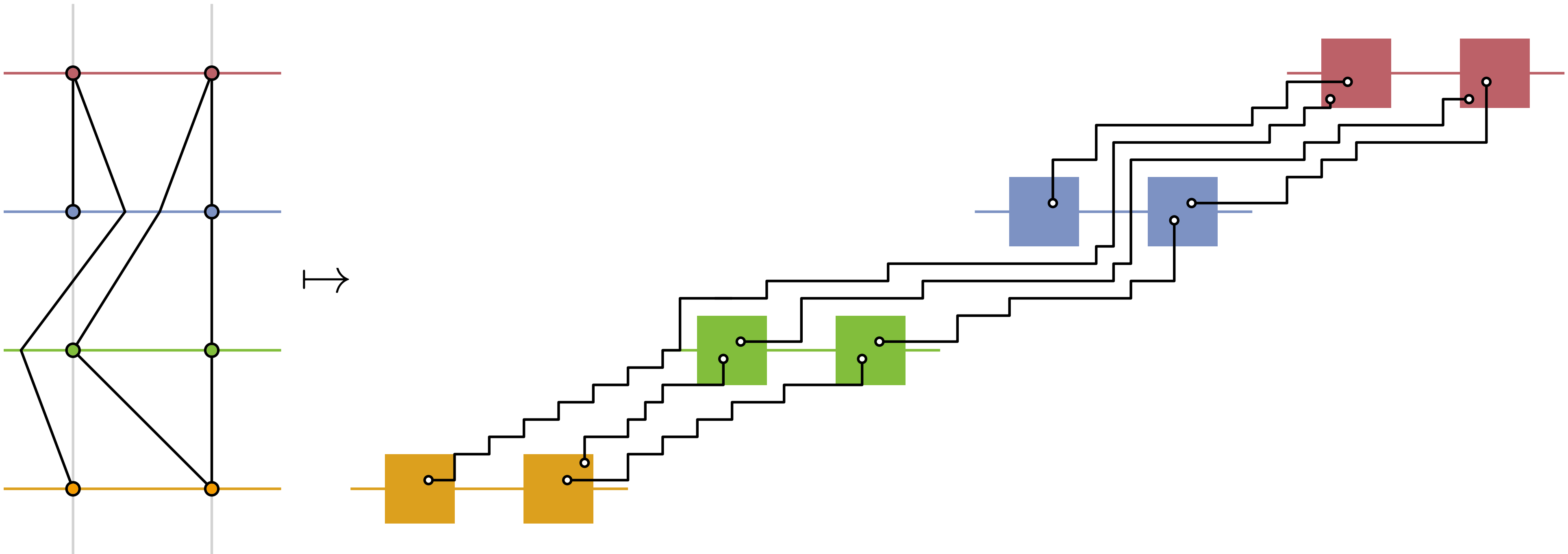
splitting gadget:



The Reduction

ORDERED LEVEL PLANARITY $\leq_p$ MANHATTAN GEODESIC PLANARITY

$\Delta = \lambda = 2$ matching, general position



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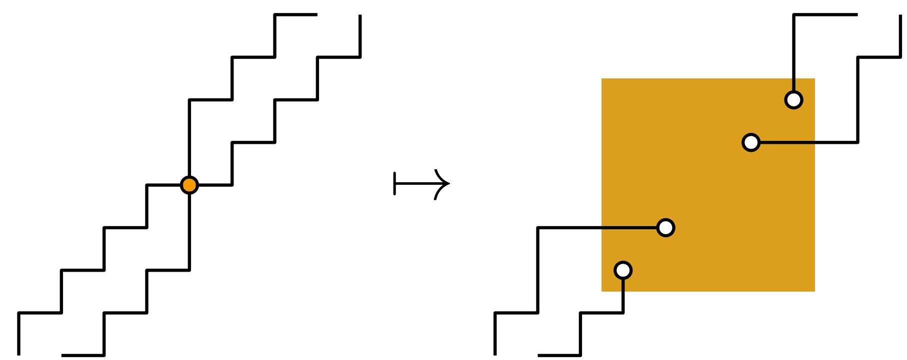

The Reduction

Ordered Level Planarity $\leq_p$ Manhattan Geodesic Planarity

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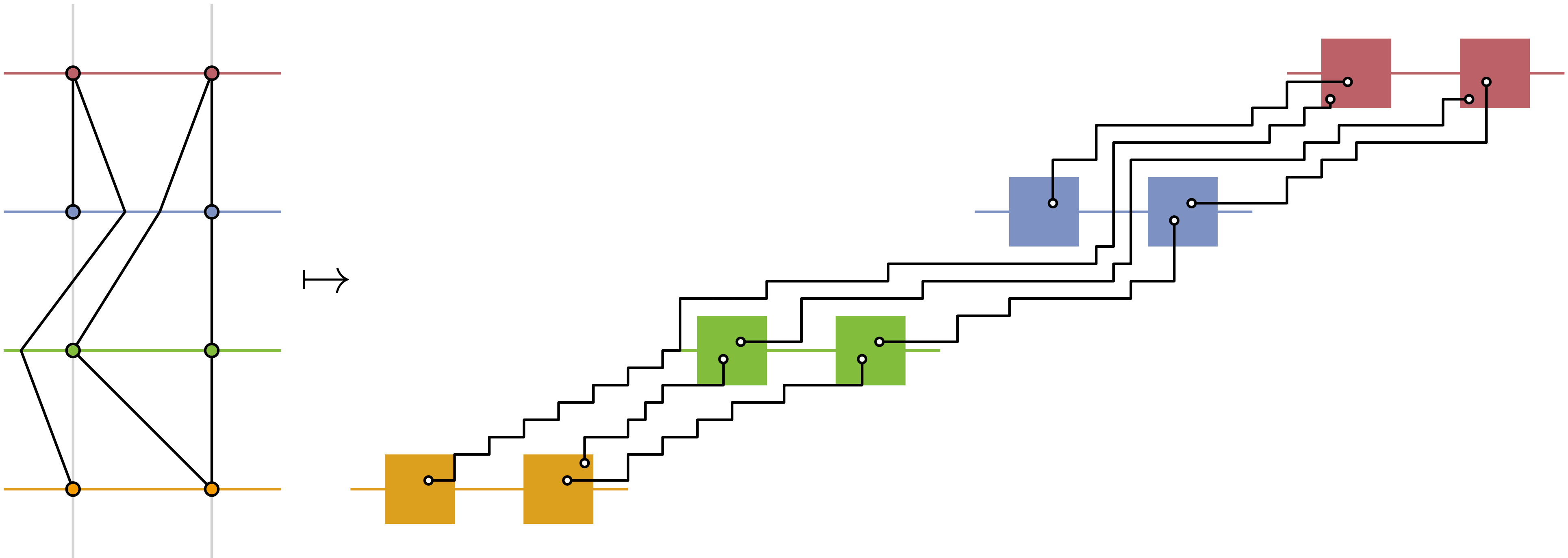

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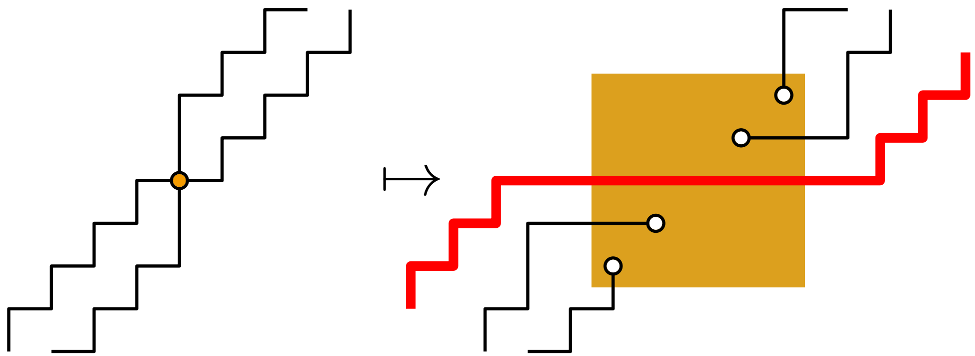

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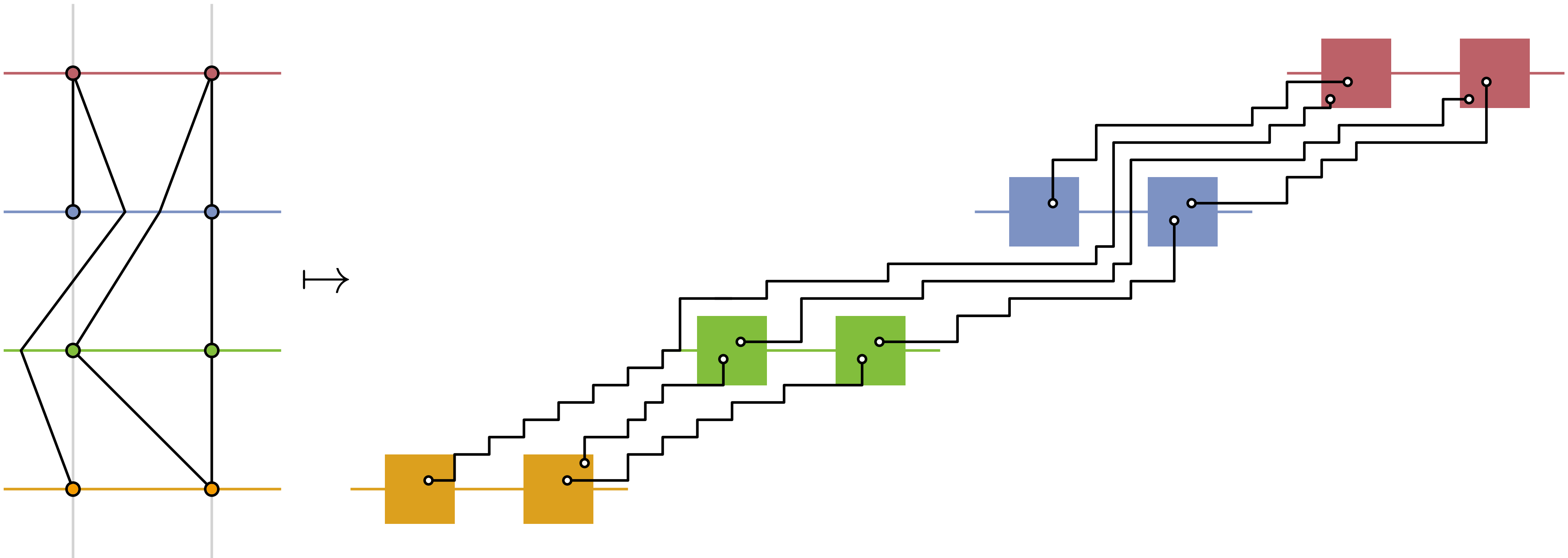

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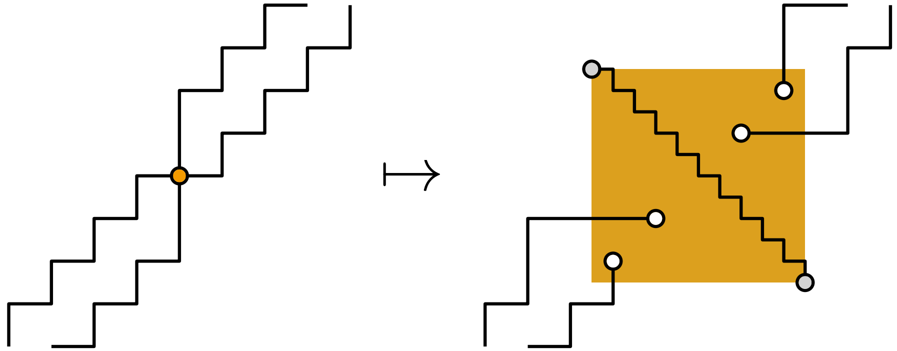

The Reduction

ORDERED LEVEL PLANARITY $\leq_p$ MANHATTAN GEODESIC PLANARITY

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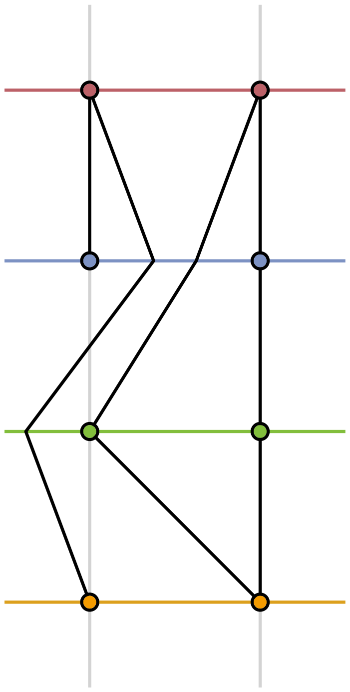
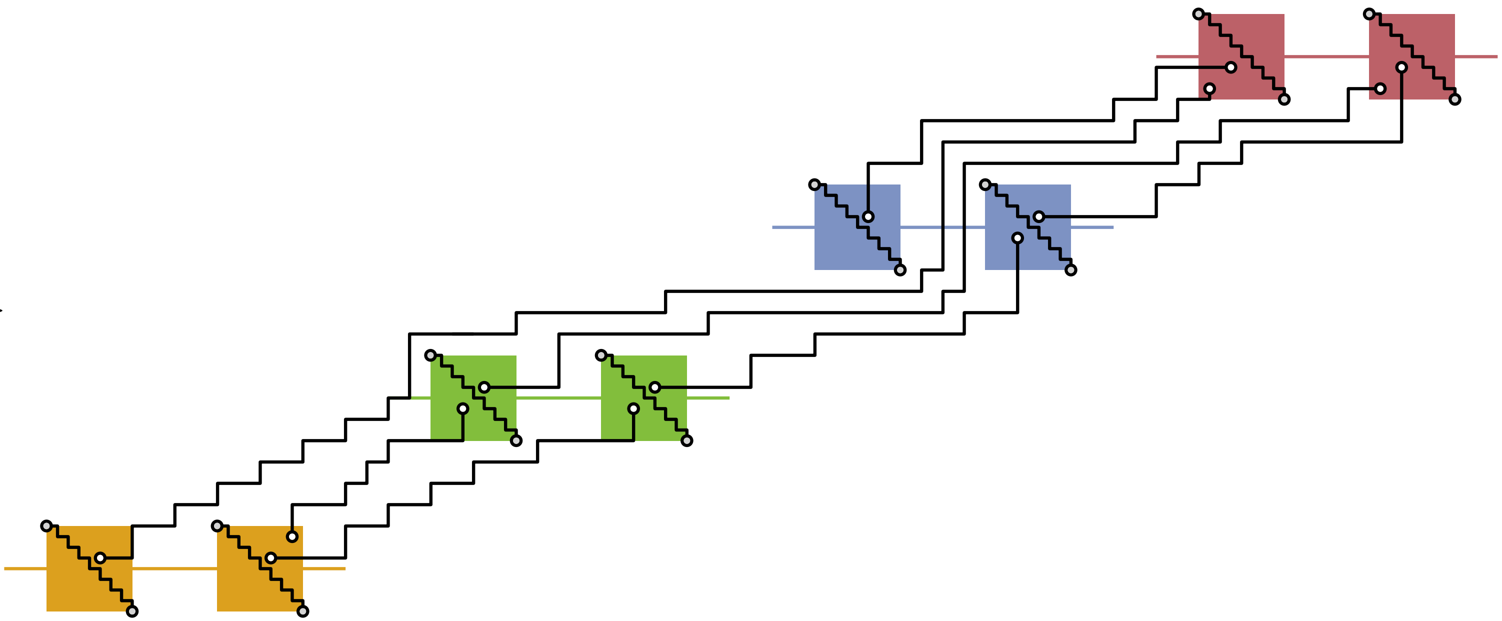


$\mapsto$



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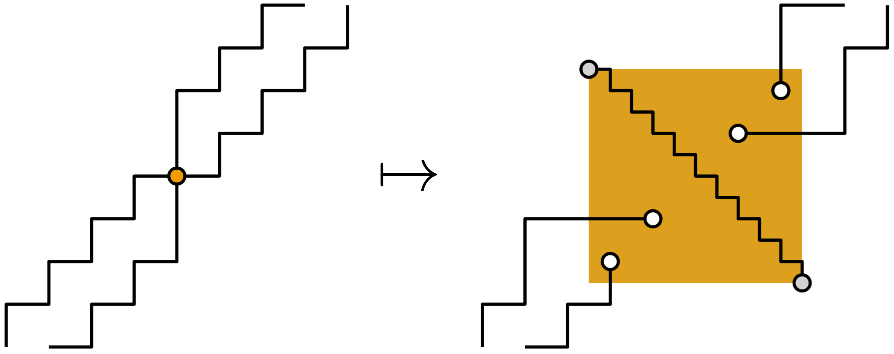
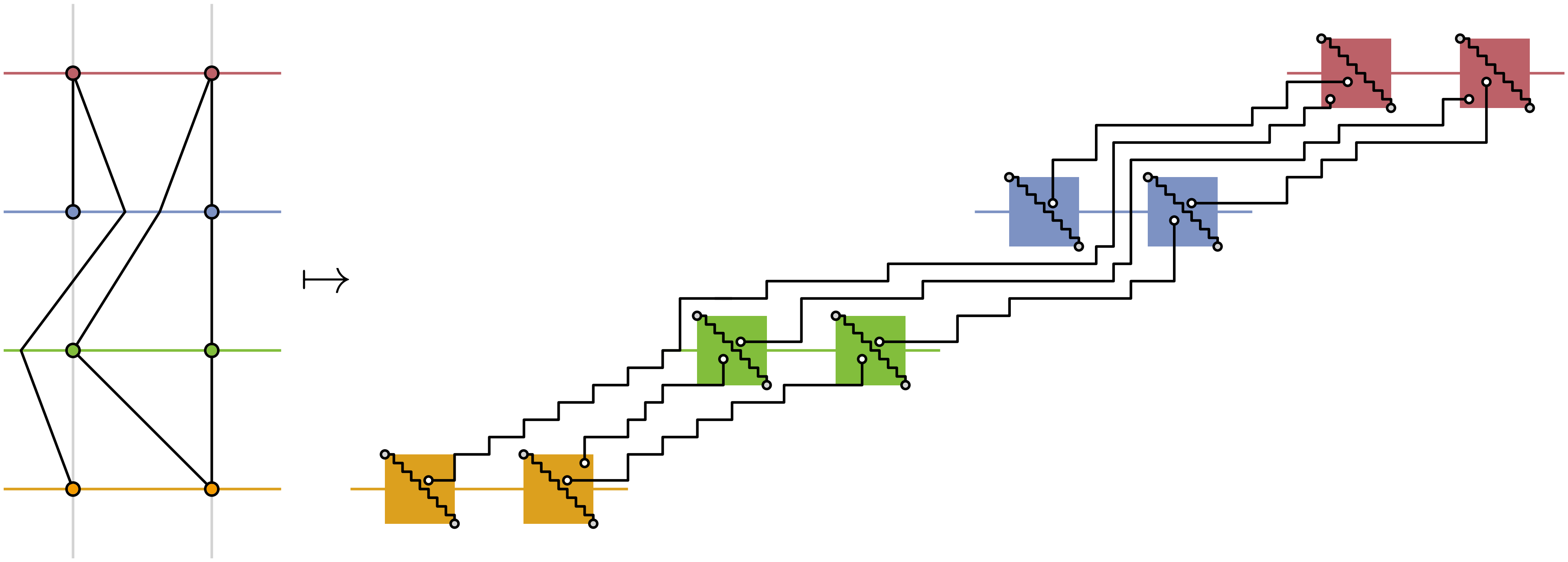


The Reduction

Ordered Level Planarity $\leq_p$ Manhattan Geodesic Planarity

$\Delta = \lambda = 2$ matching, general position ?



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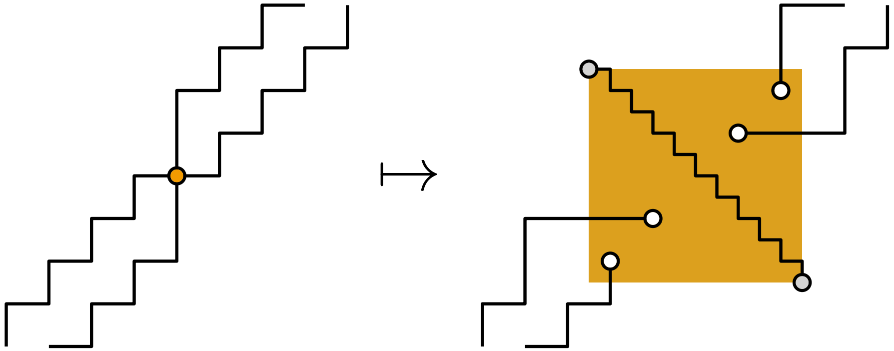

The Reduction

$\textsc{Ordered Level Planarity} \leq_p \textsc{Manhattan Geodesic Planarity}$

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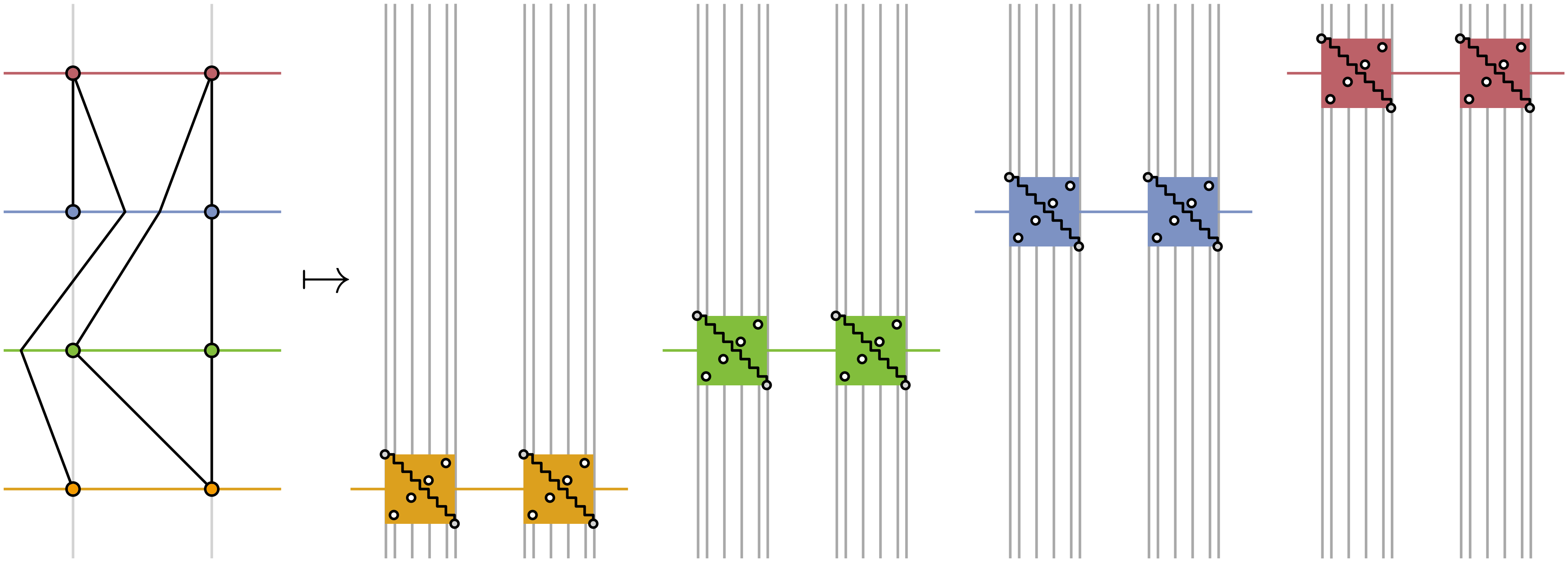

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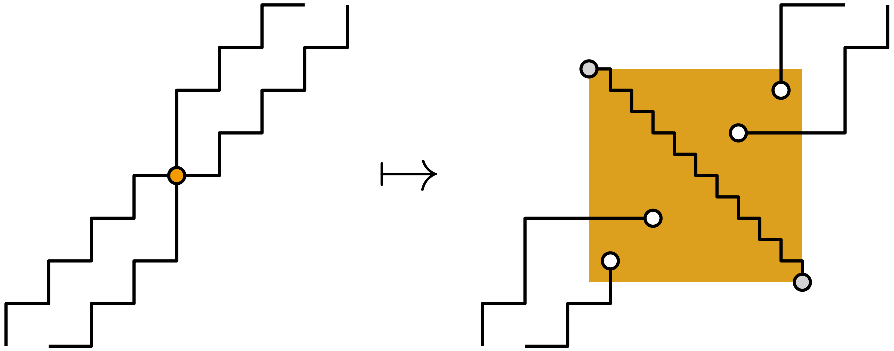

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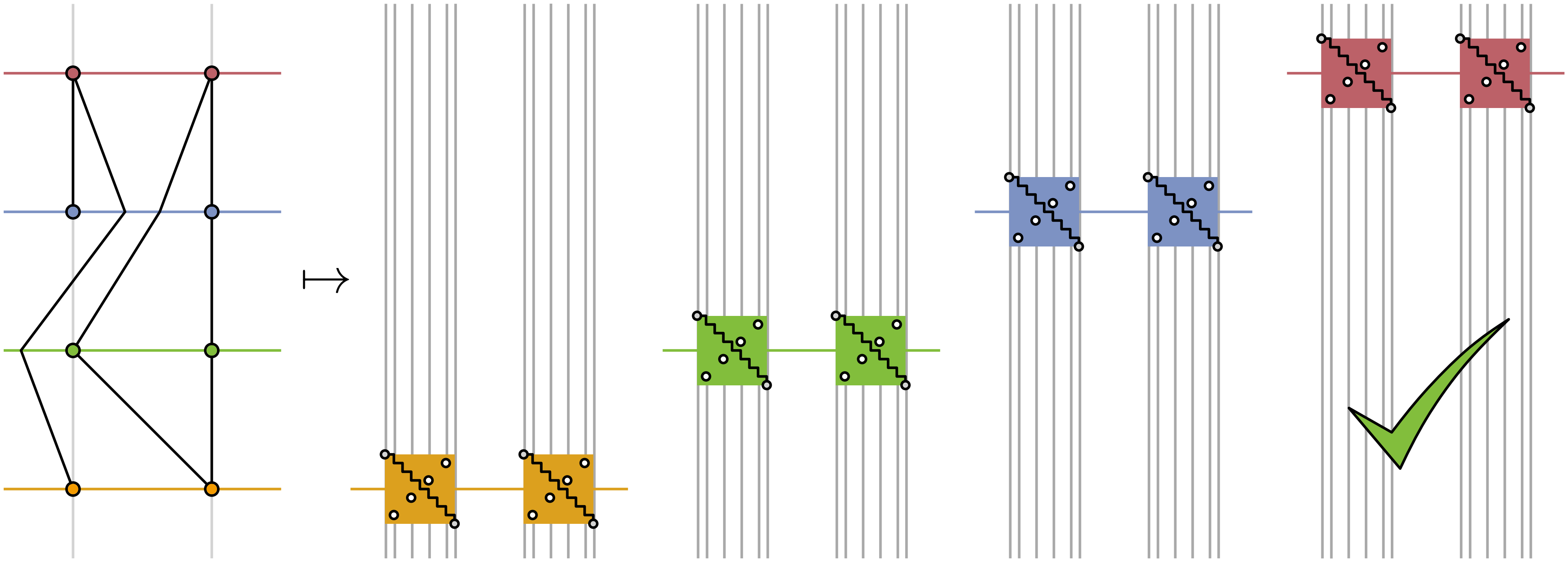

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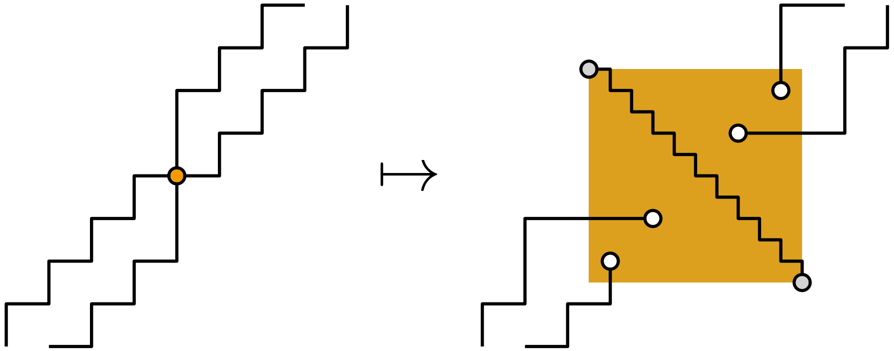

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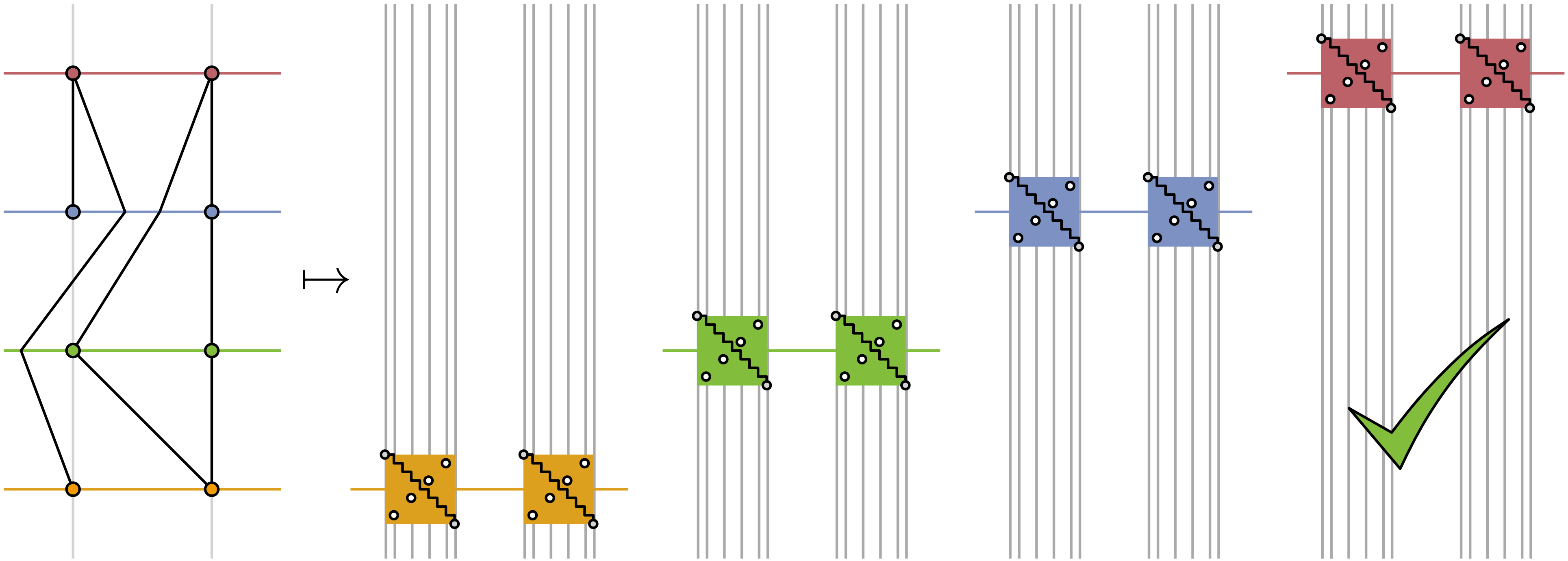

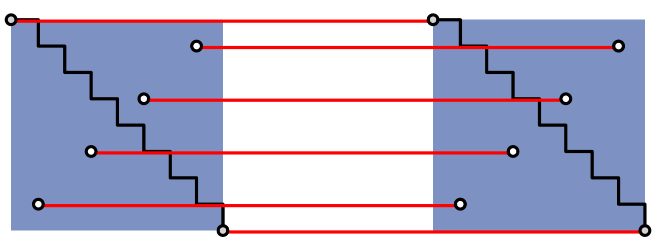

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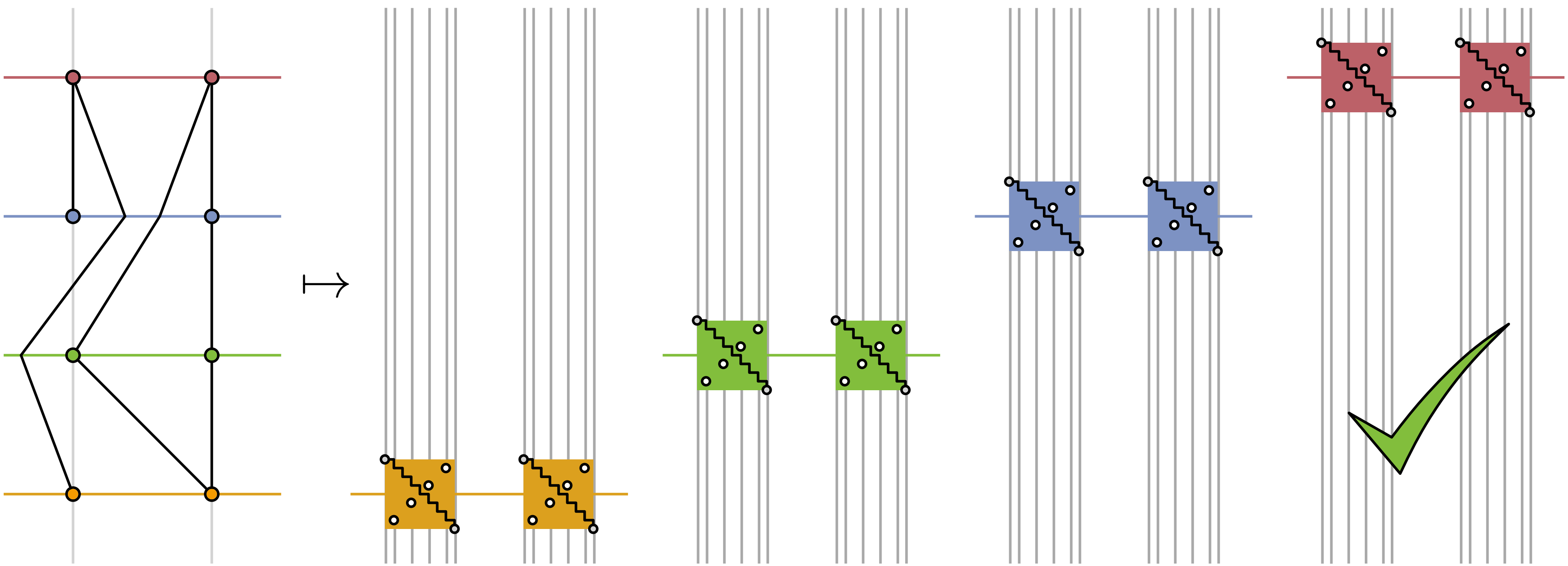

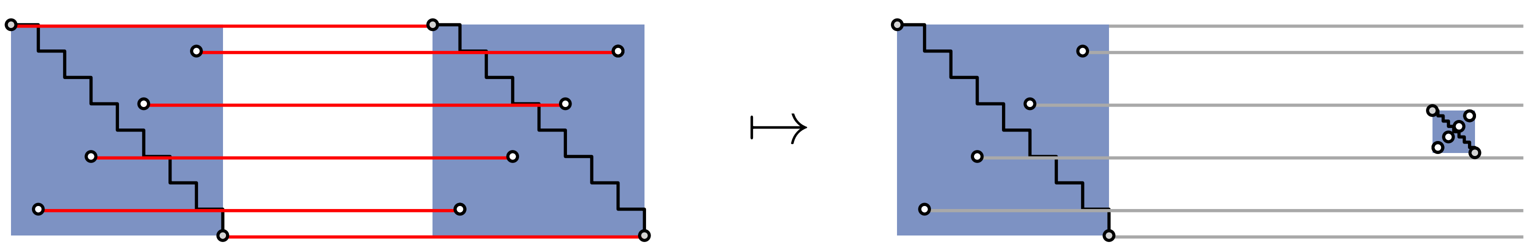

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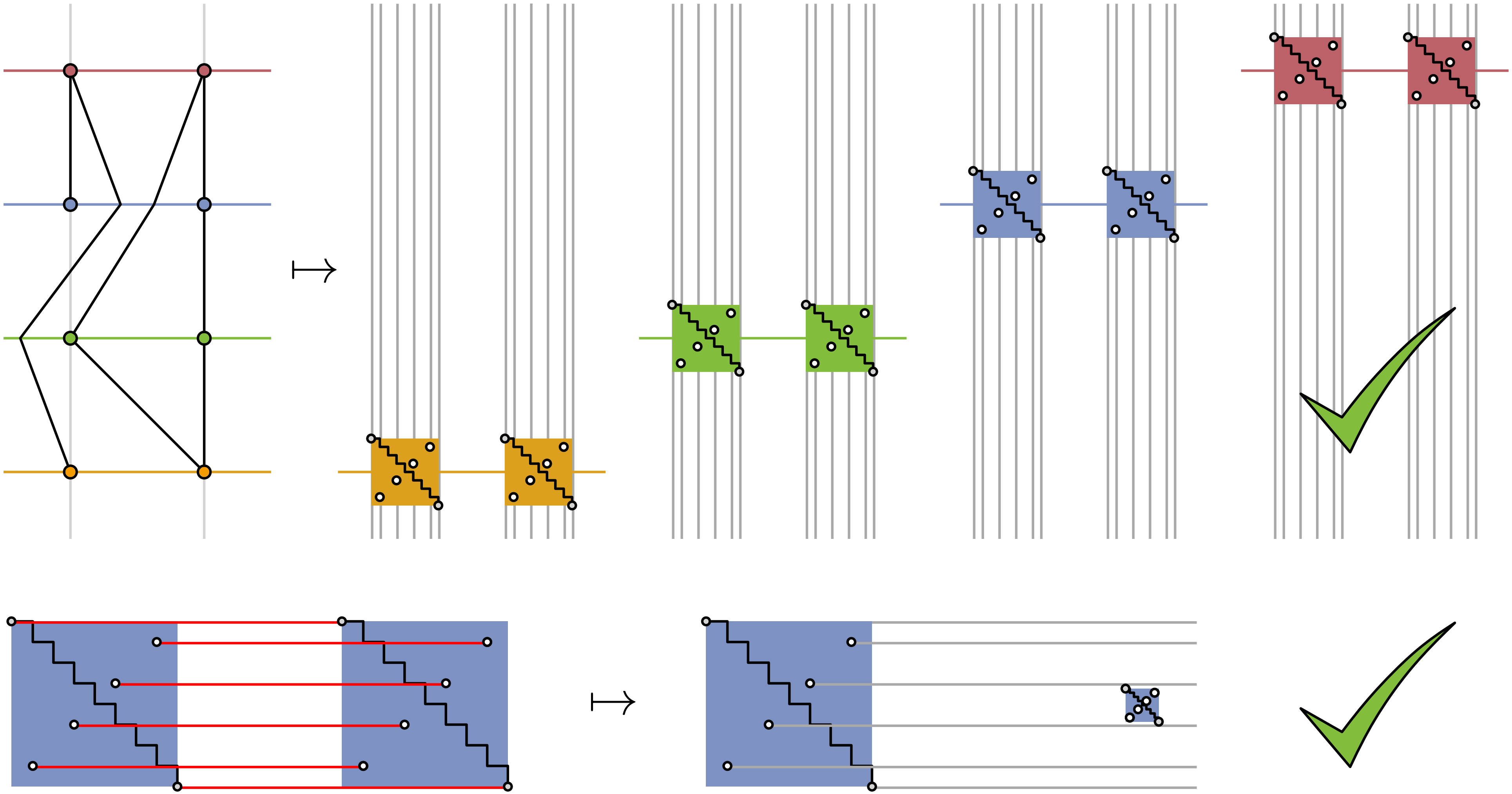
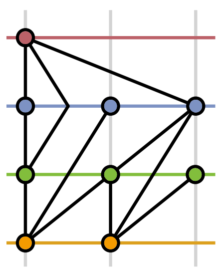


Result Overview

level-width λ = max. #vertices per level

Ordered Level Planarity

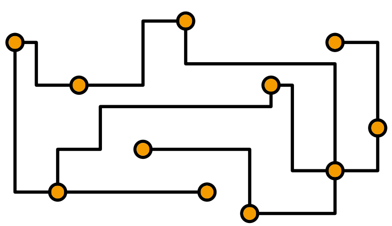


NP-complete even for $\Delta = \lambda = 2$

Polytime for $\Delta_{in} = \Delta_{out} = 1$ or $\lambda = 1$

$\leq$

Geodesic Planarity



NP-hard even
for matchings
in general position

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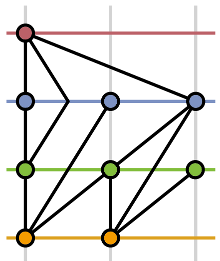

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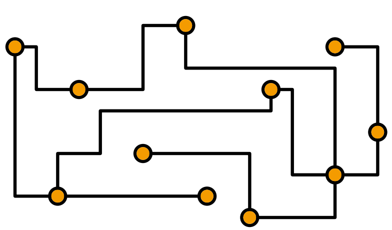
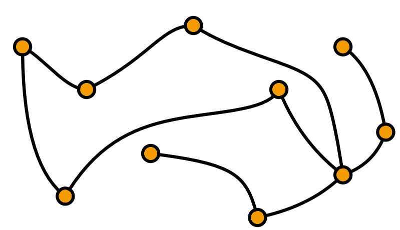


NP-hard even for matchings in general position

$\leq$

Bi-monotonicity



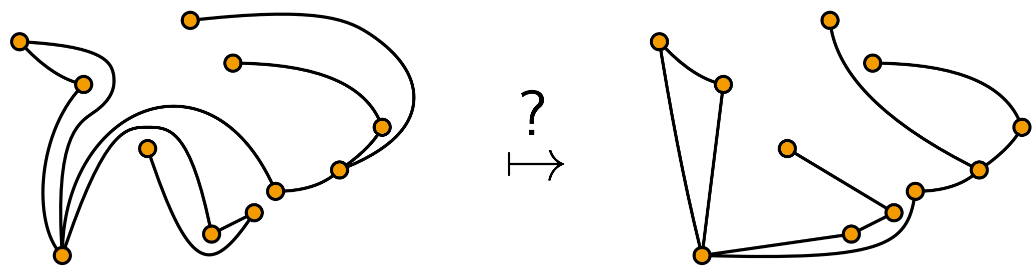
NP-hard even for matchings

Problem Definition

Bi-Monotonicity

Given: planar graph, vertex coordinates in general position

Want: plane drawing, xy-monotone edges,
and vertices at prescribed positions



Proposed by [Fulek, Pelsmajer, Schaefer, Štefankovič'11]

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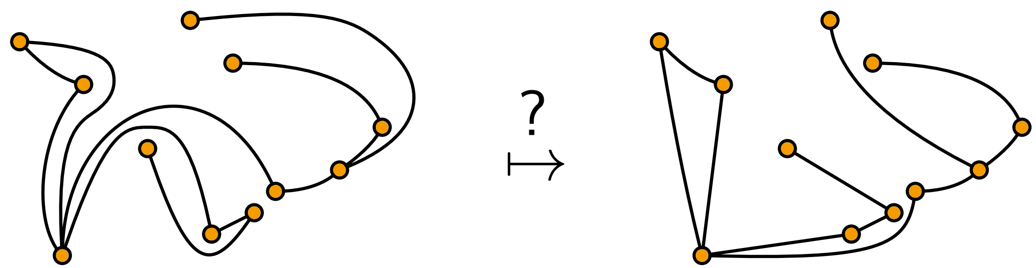

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matching, general position — matching

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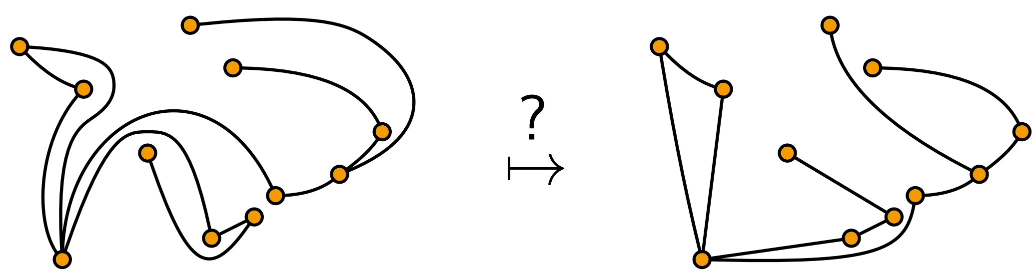

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The reduction: do nothing

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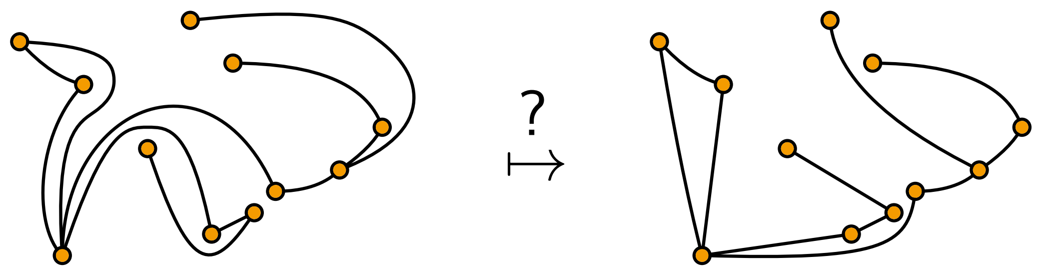

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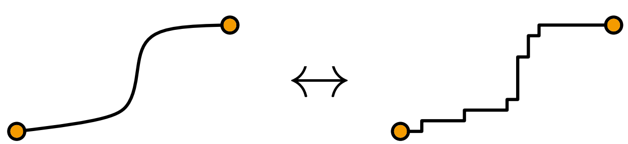

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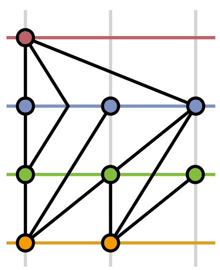
level-width λ = max. #vertices per level

Ordered Level Planarity



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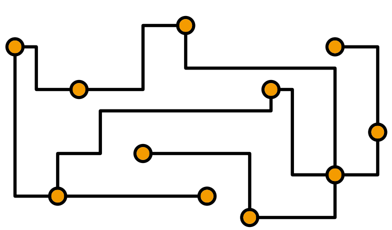

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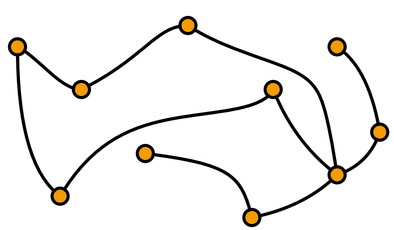

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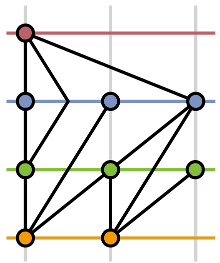

ORDERED LEVEL PLANARITY

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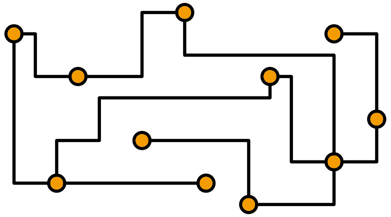
GEODESIC PLANARITY



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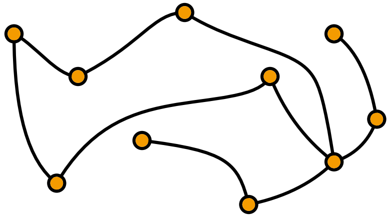
BI-MONOTONICITY



NP-hard even for matchings

NP-hardness of Ordered Level Planarity

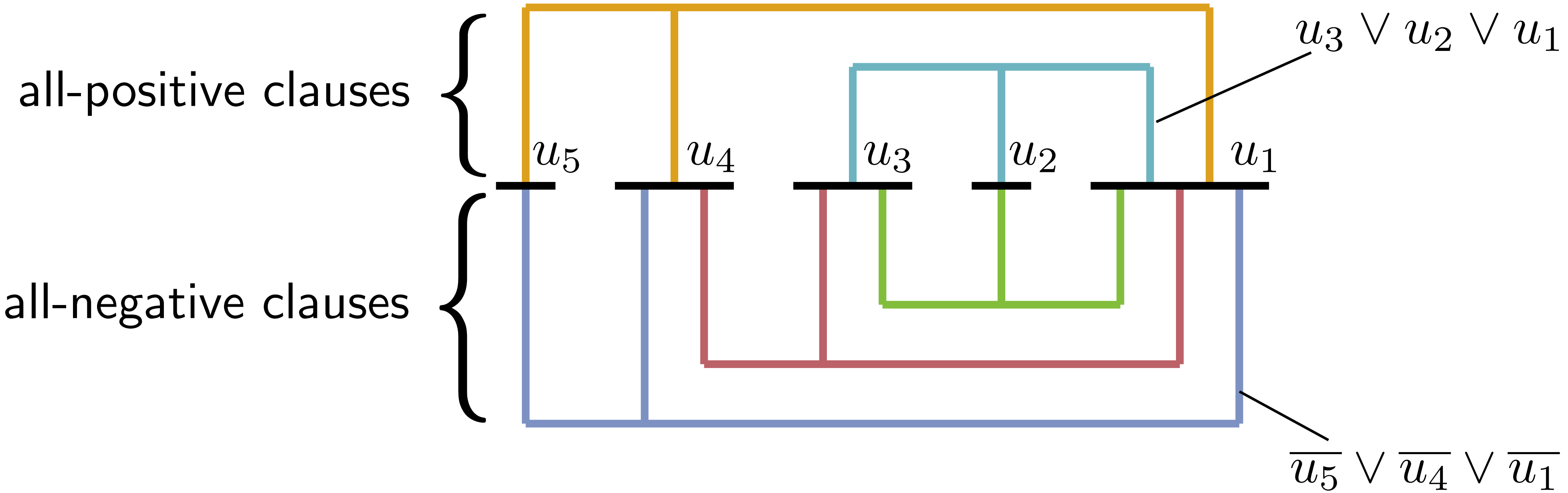
Proof via reduction from

Planar Monotone 3-SAT

3-Satisfiability restricted to instances that ...

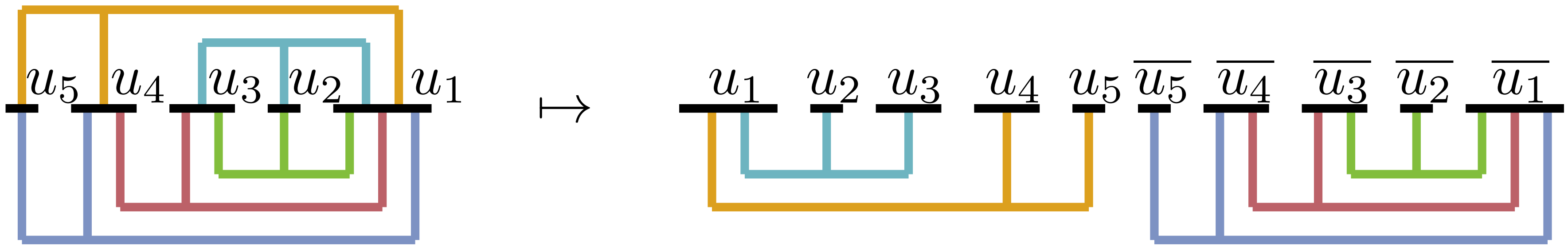
... have only all-positive and all-negative clauses

... admit a contact representation with line segments and E-shapes



NP-complete [de Berg, Khosravi'12]

The Reduction



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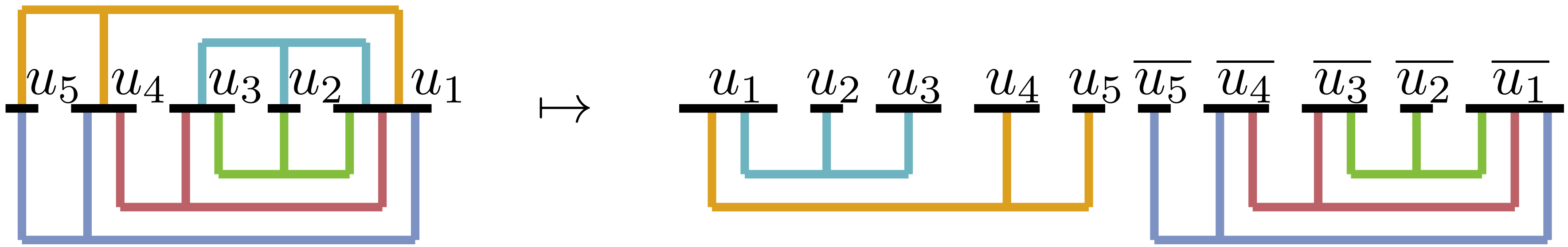

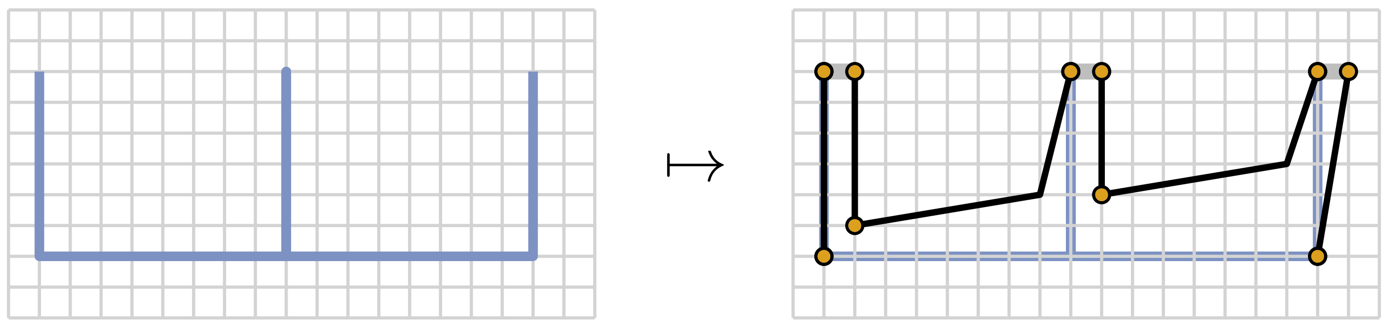

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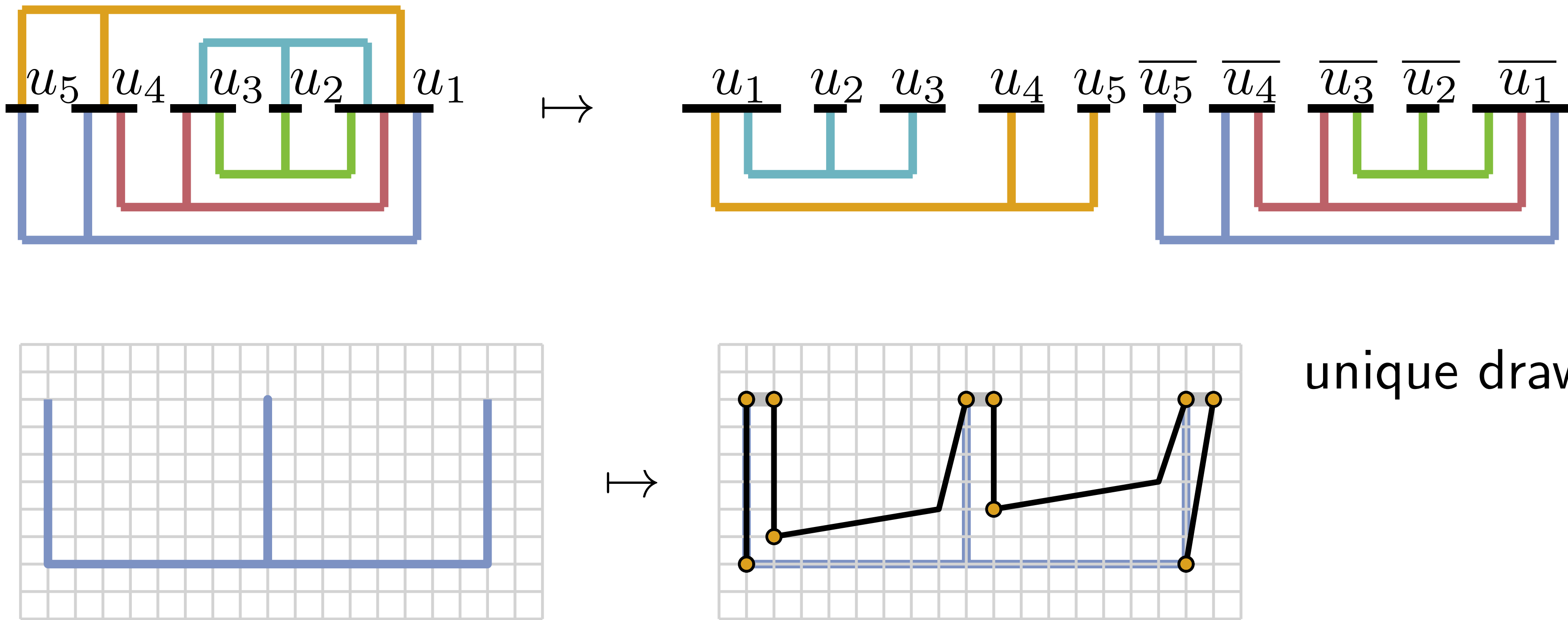
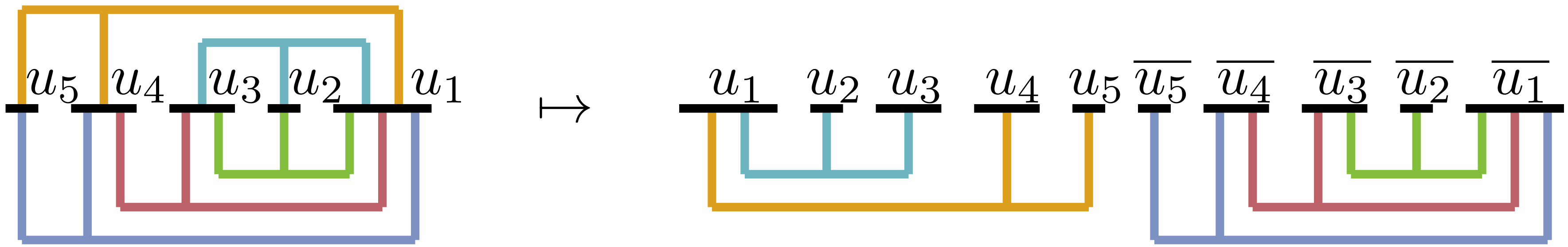
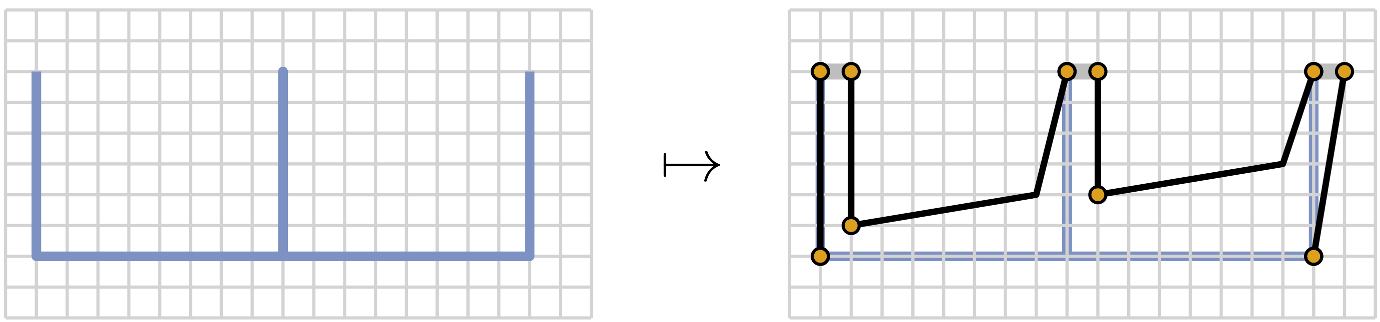


unique drawing

The Reduction





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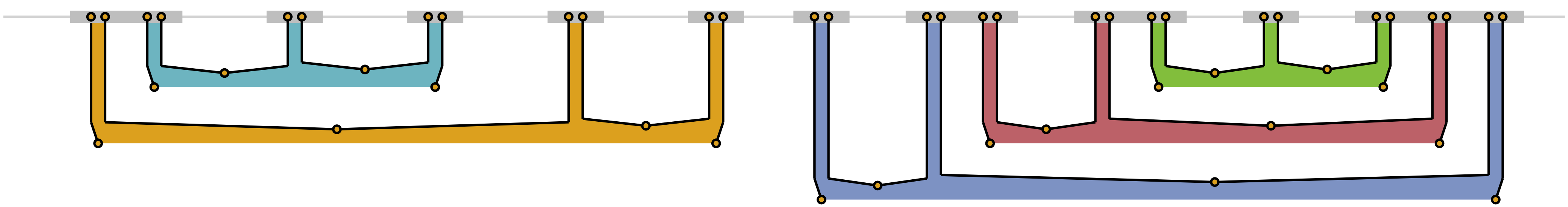

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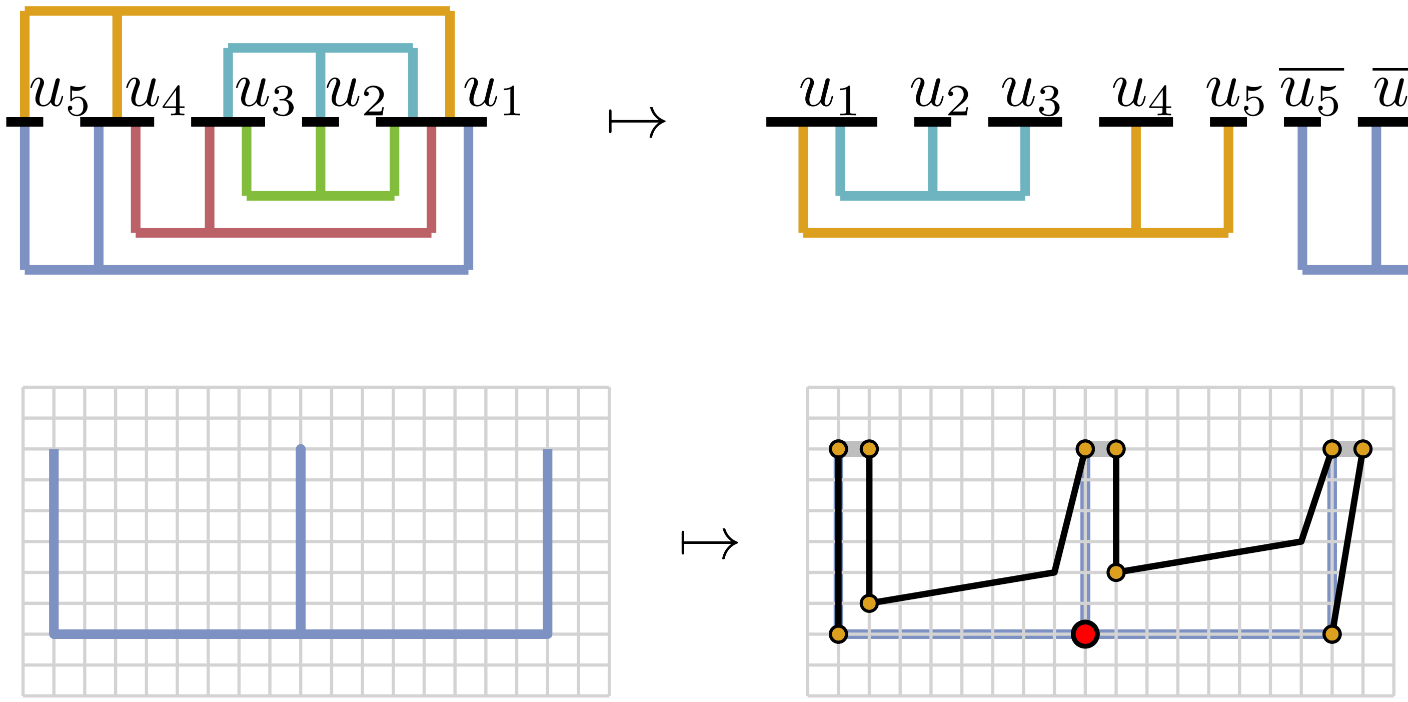
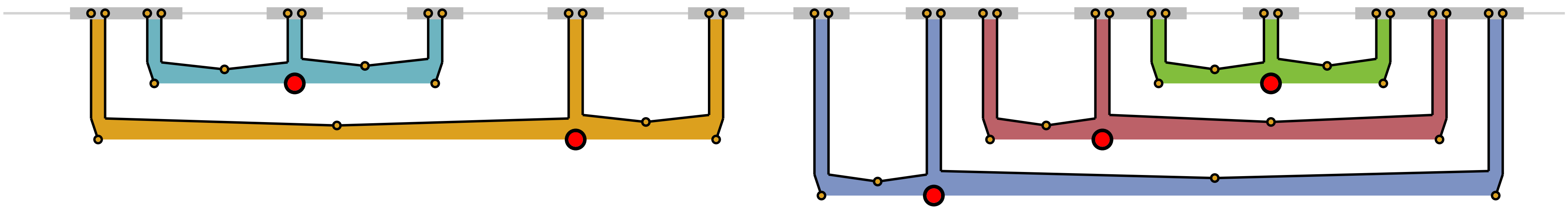


unique drawing

clause edge



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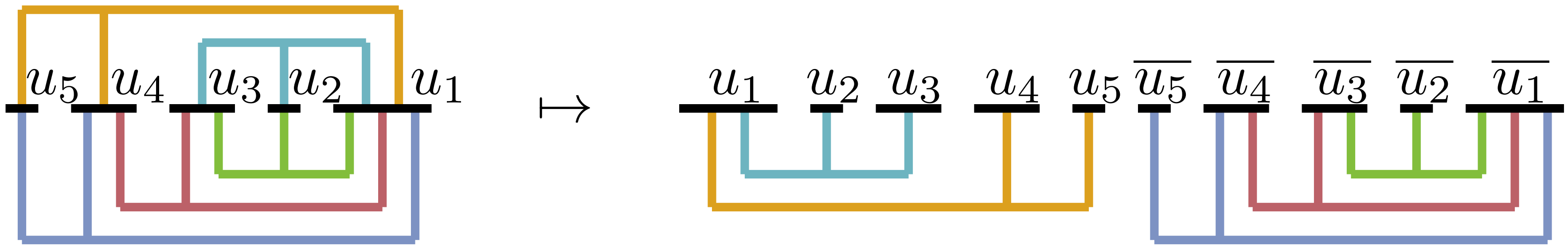

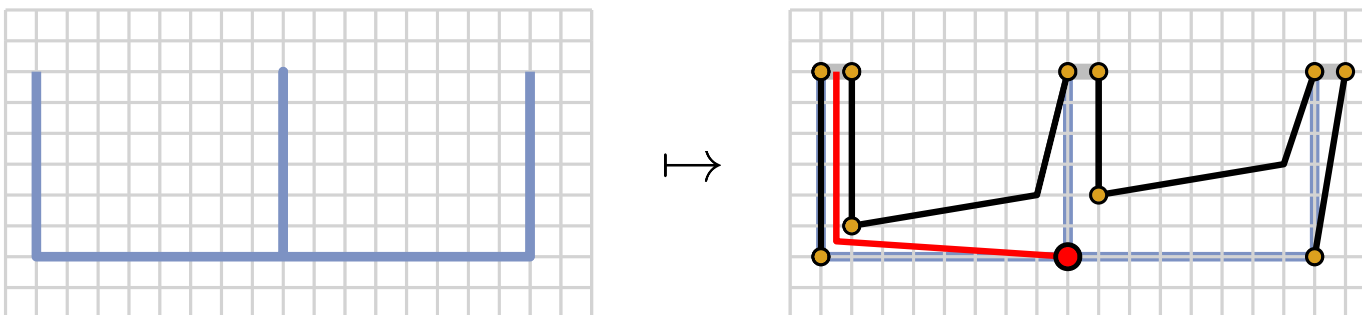

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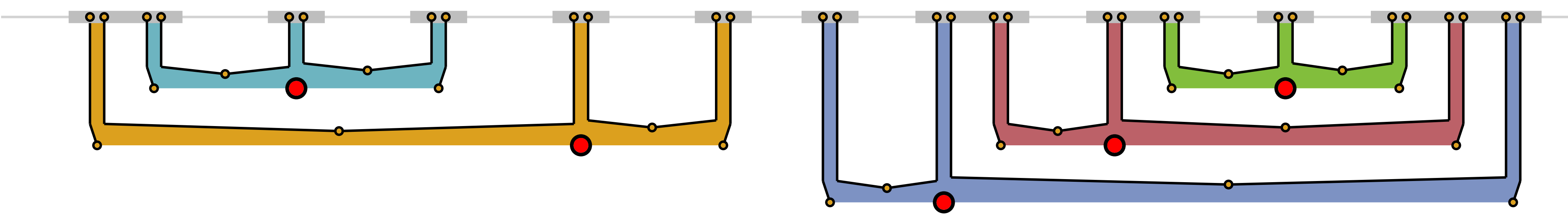

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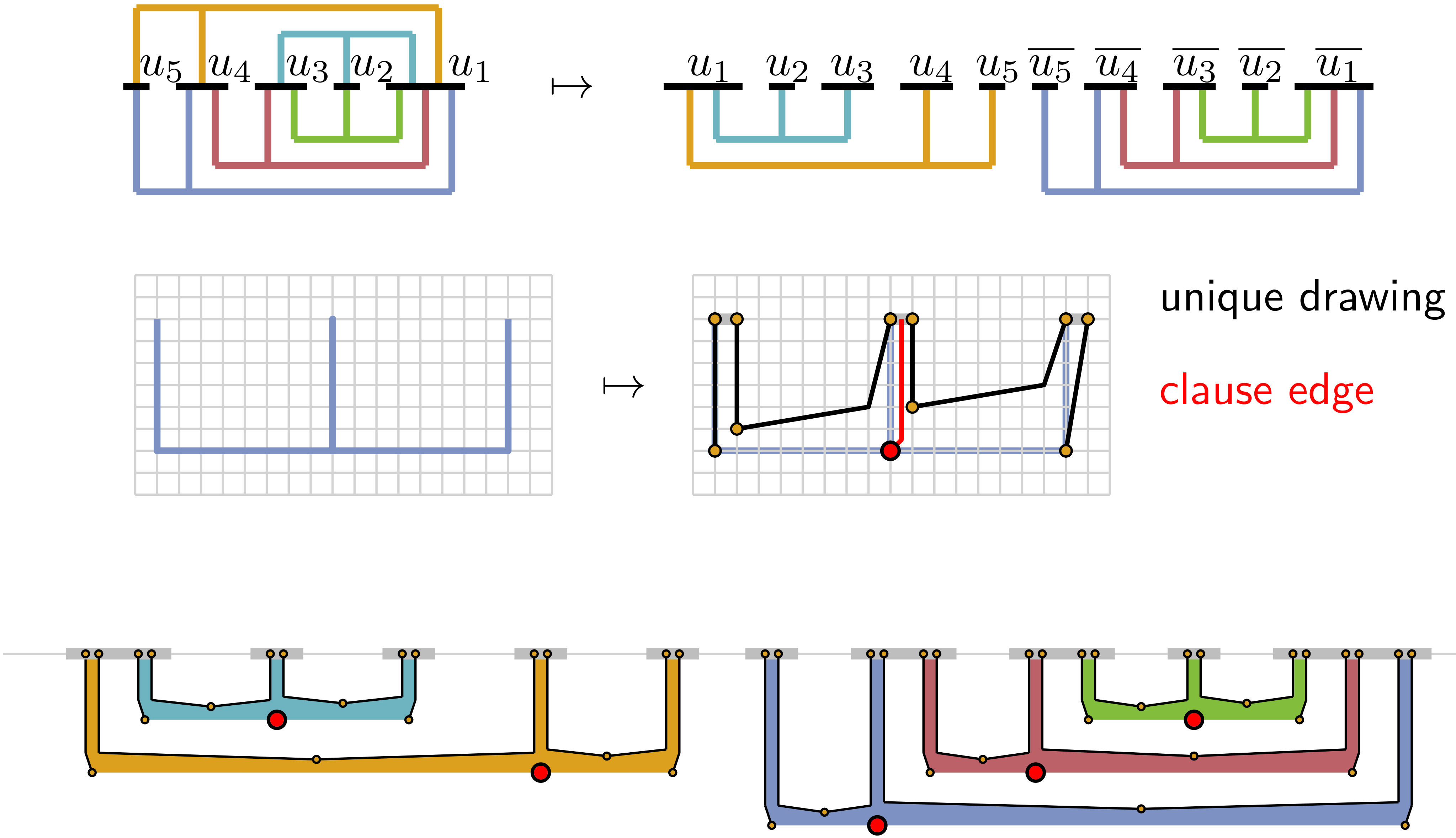

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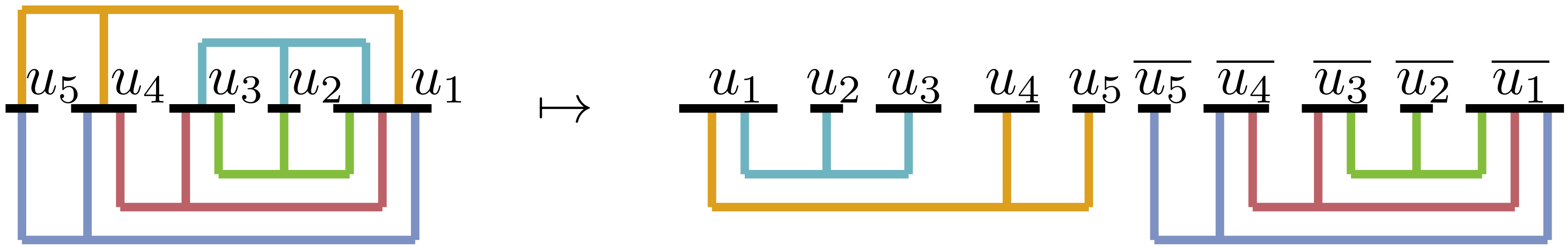

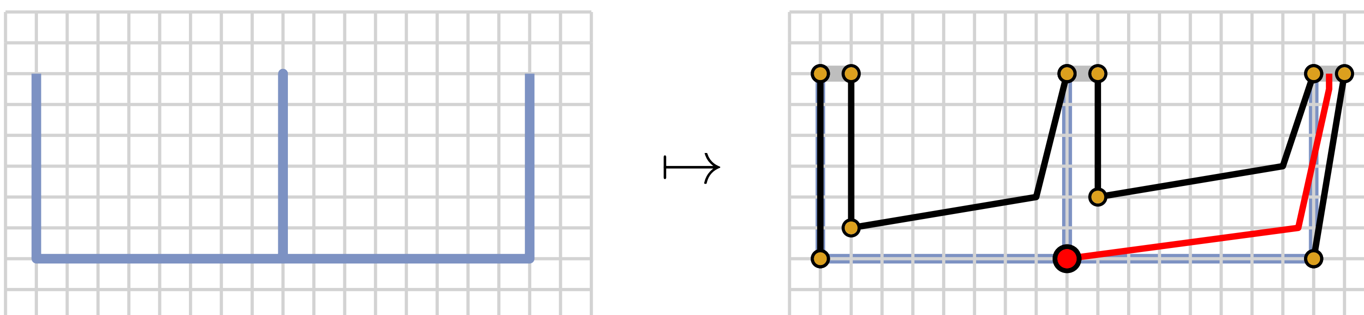

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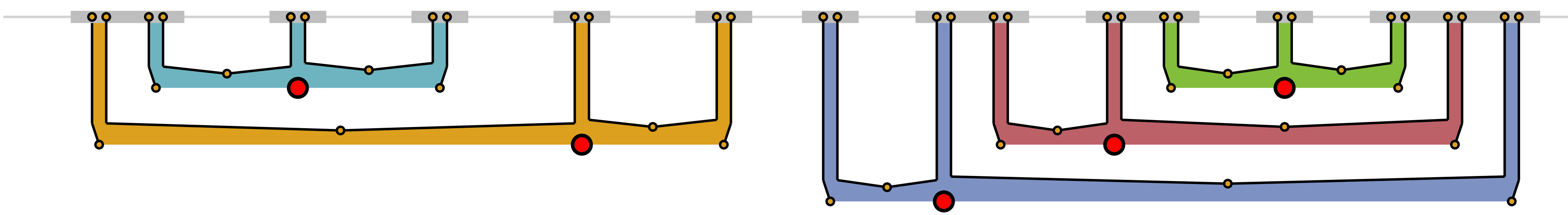

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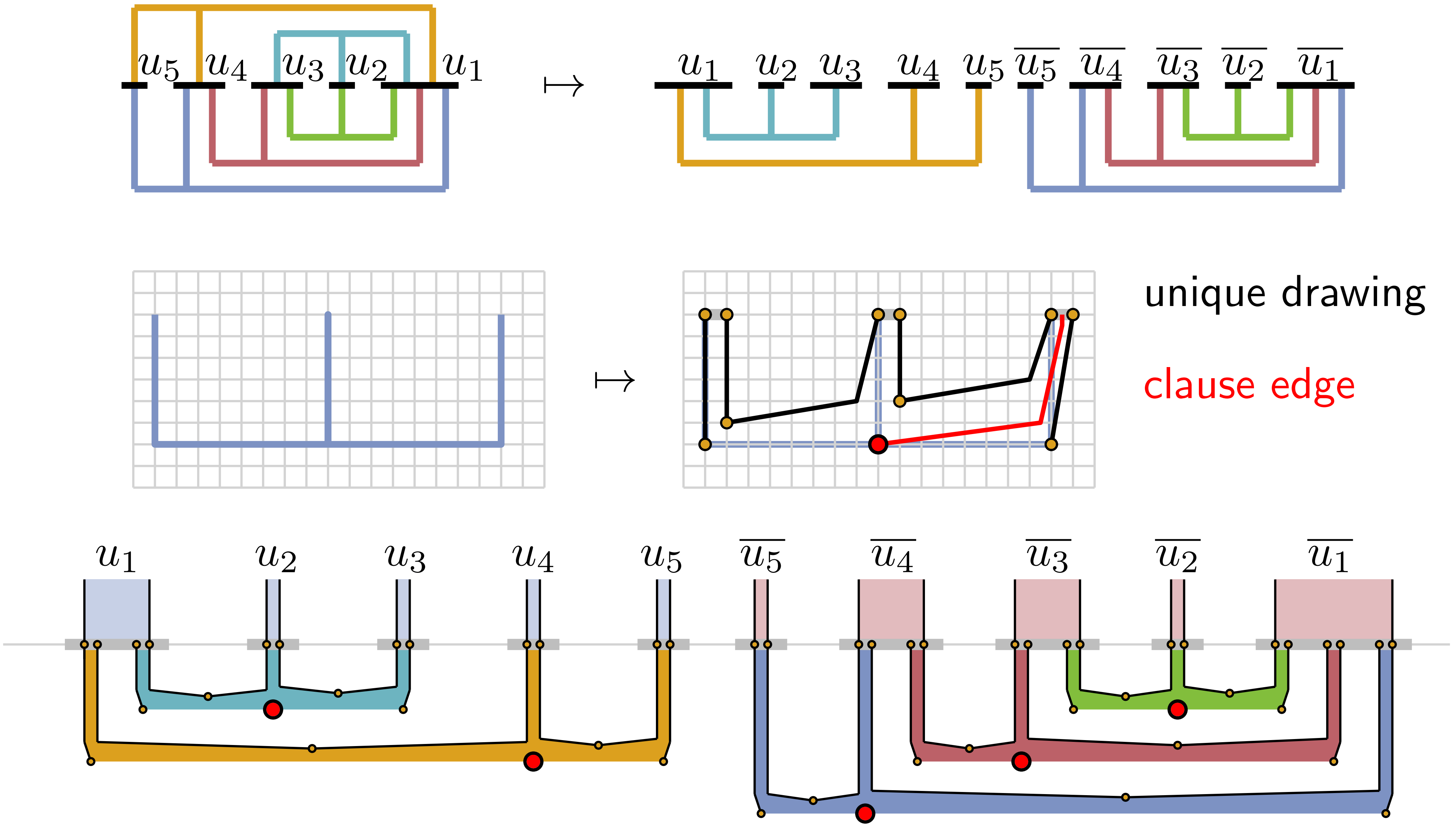

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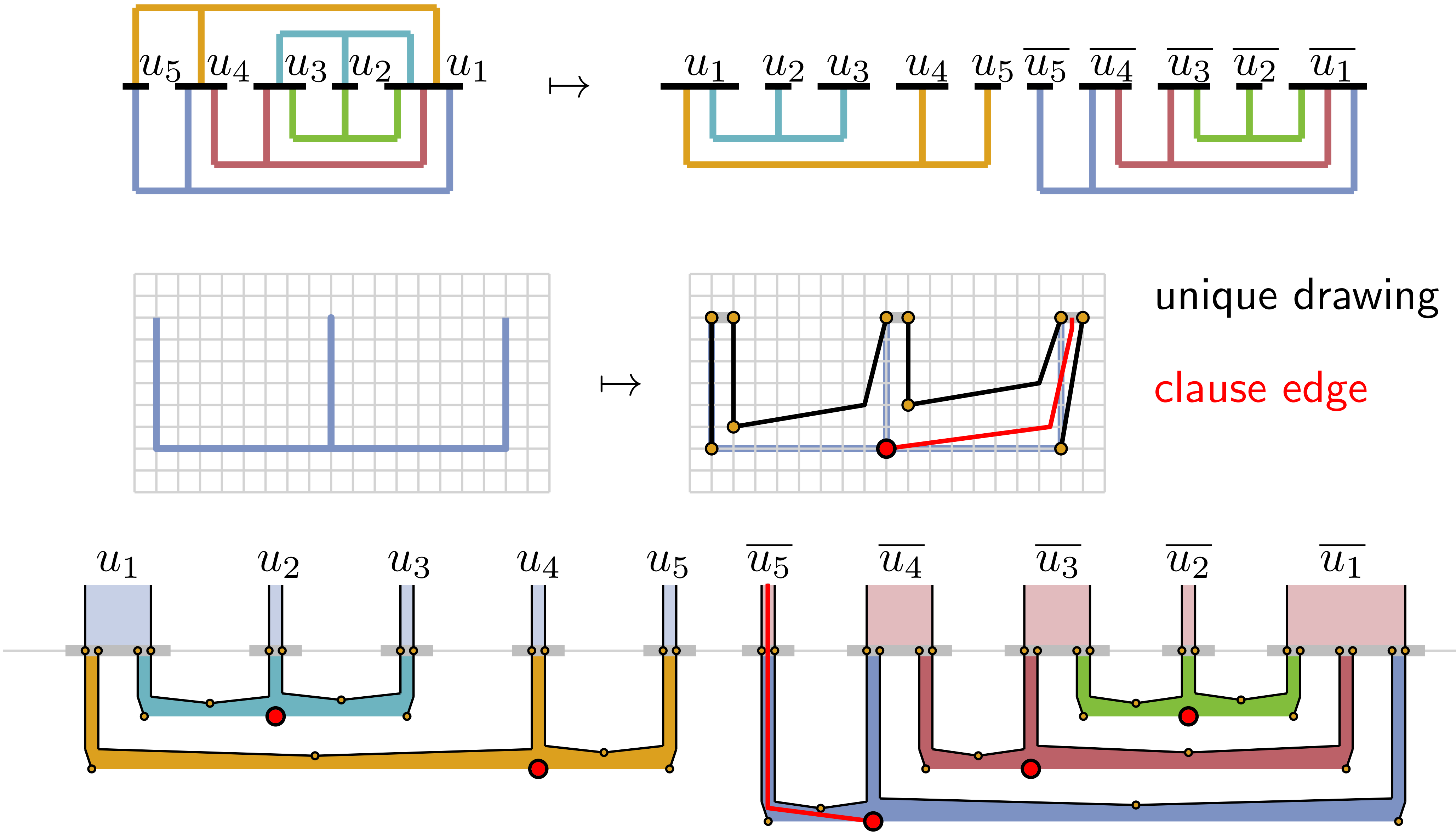

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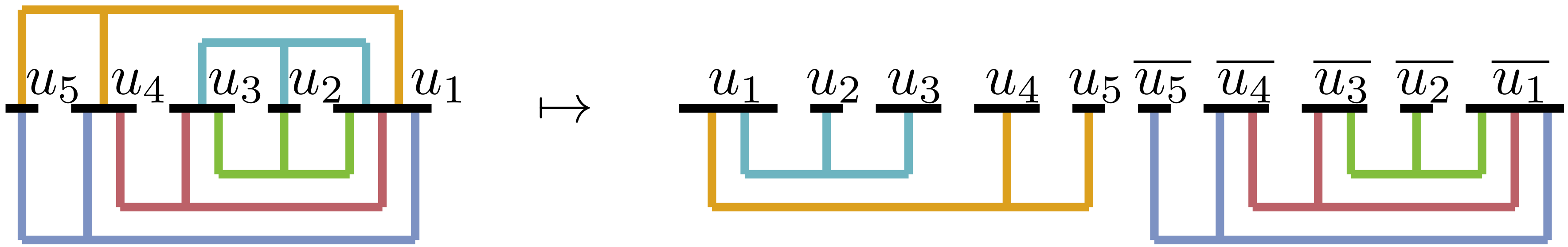

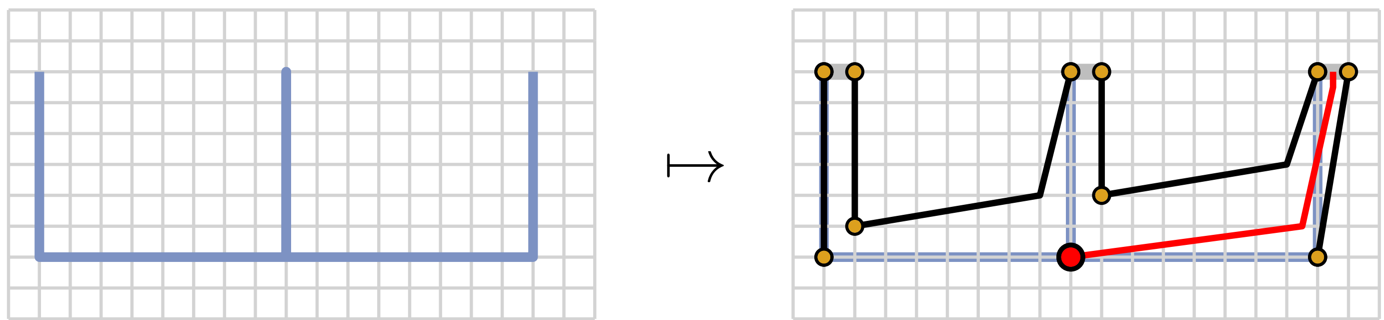

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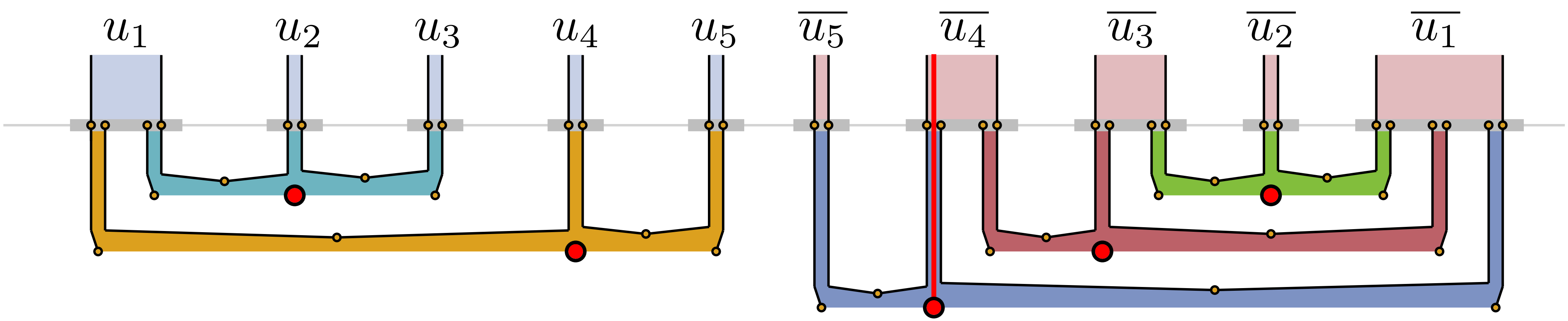

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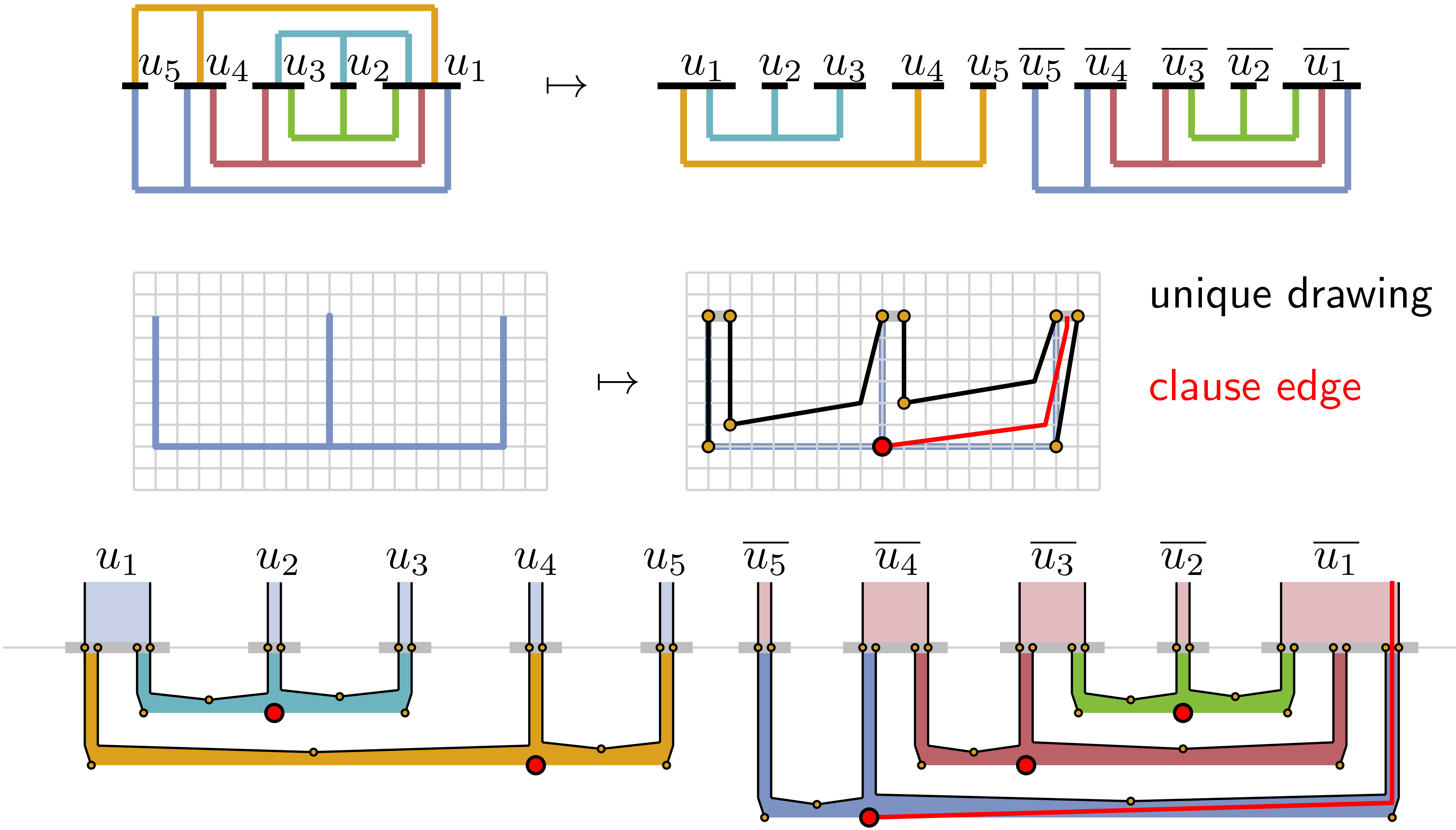

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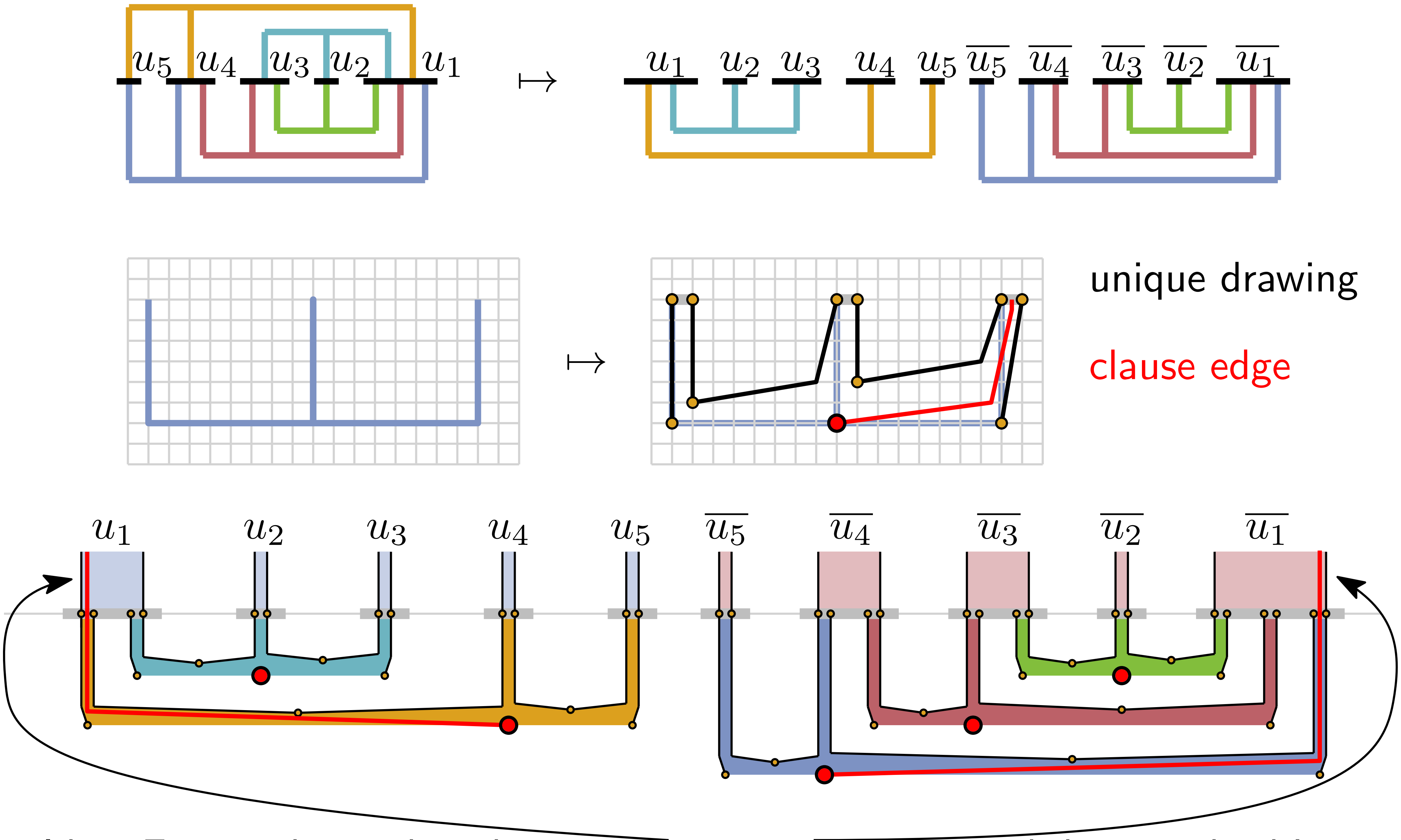

Idea: Ensure that either the tunnel u_i or $\overline{u_i}$ can be used, but not both!

Variable Gadgets

Ensure that for every i either the tunnel u_i or $\overline{u_i}$ can be used, but not both!

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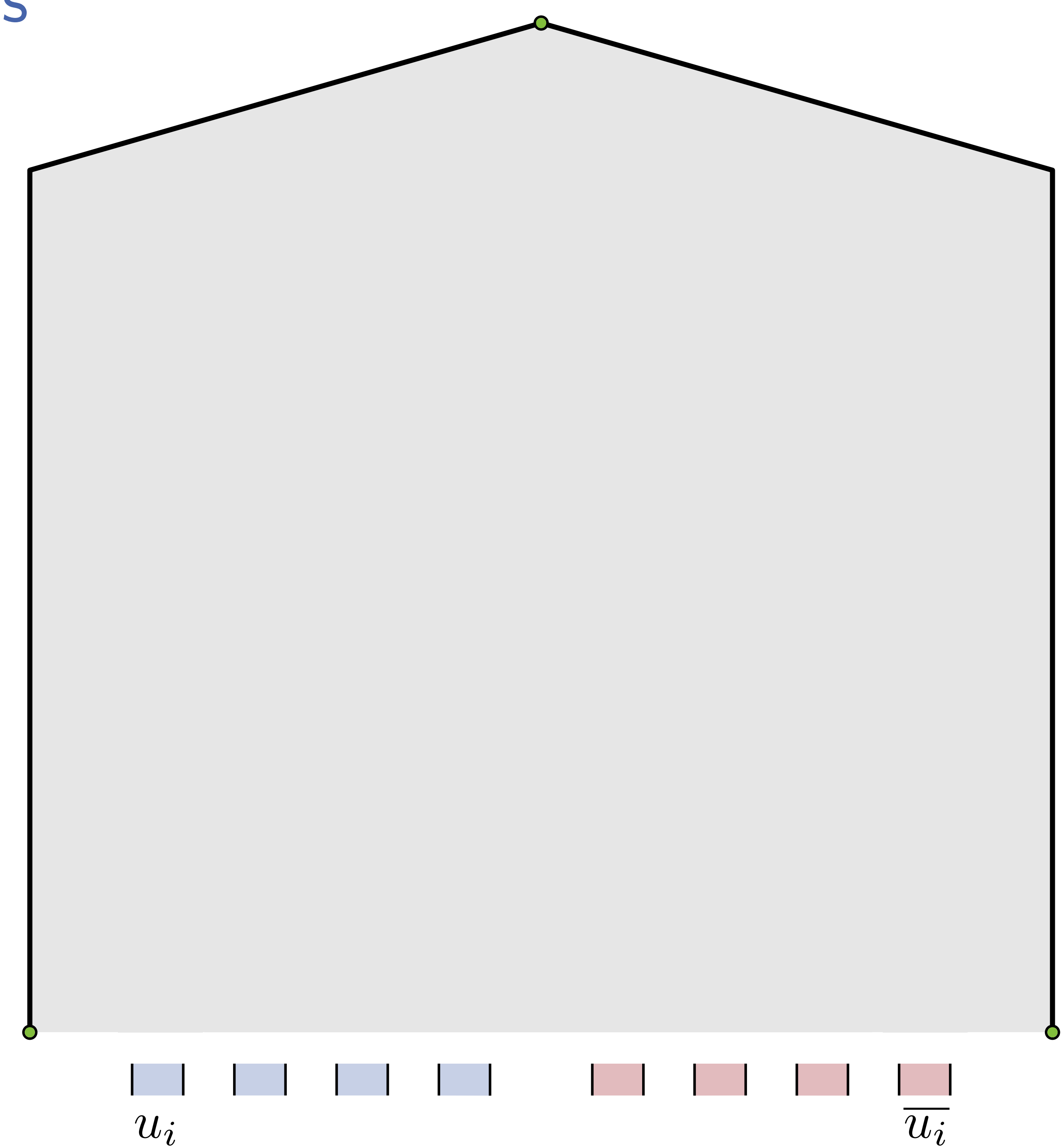
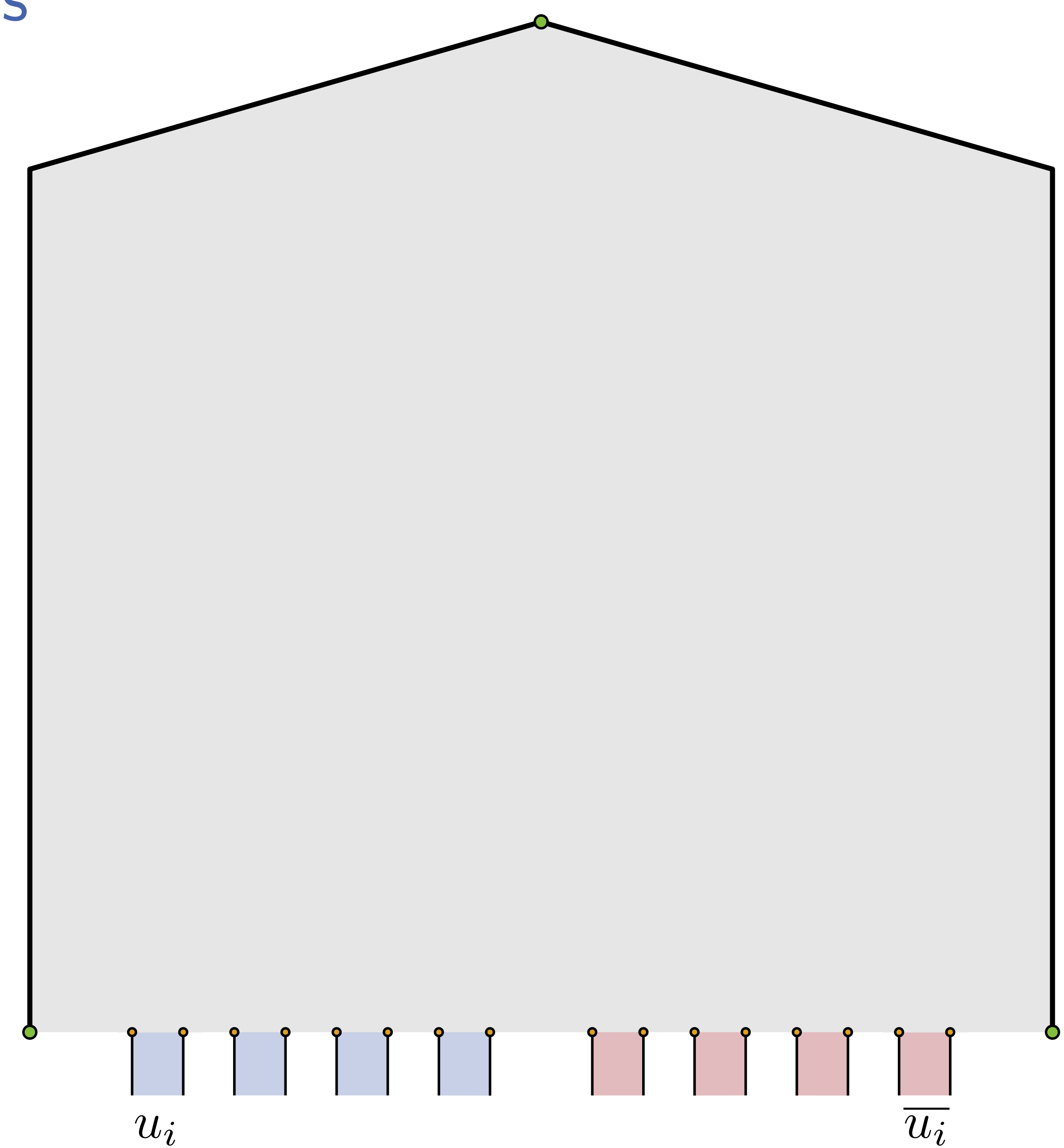


Variable Gadgets

Ensure that for every i either the tunnel u_i or $\overline{u_i}$ can be used, but not both!



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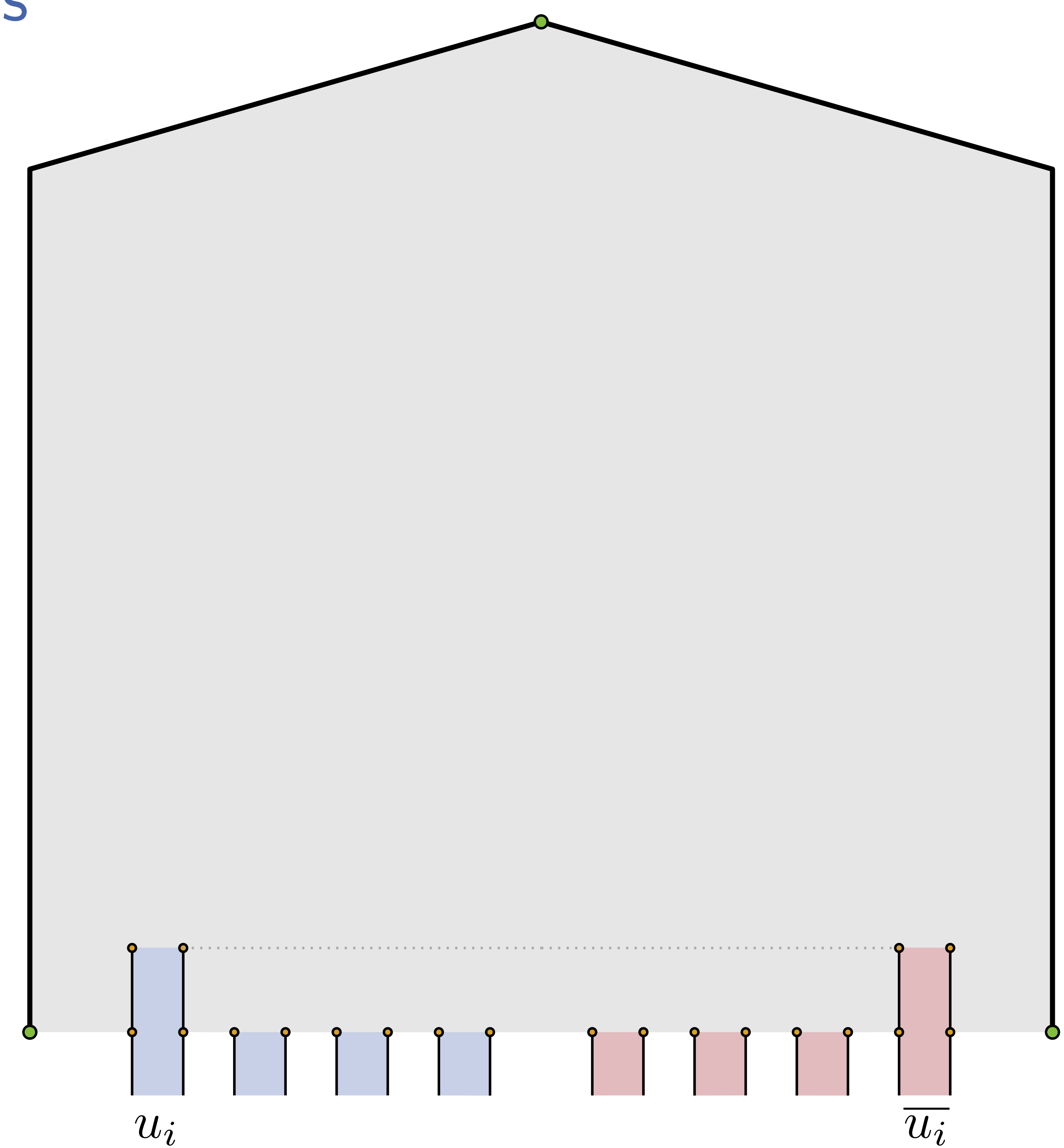

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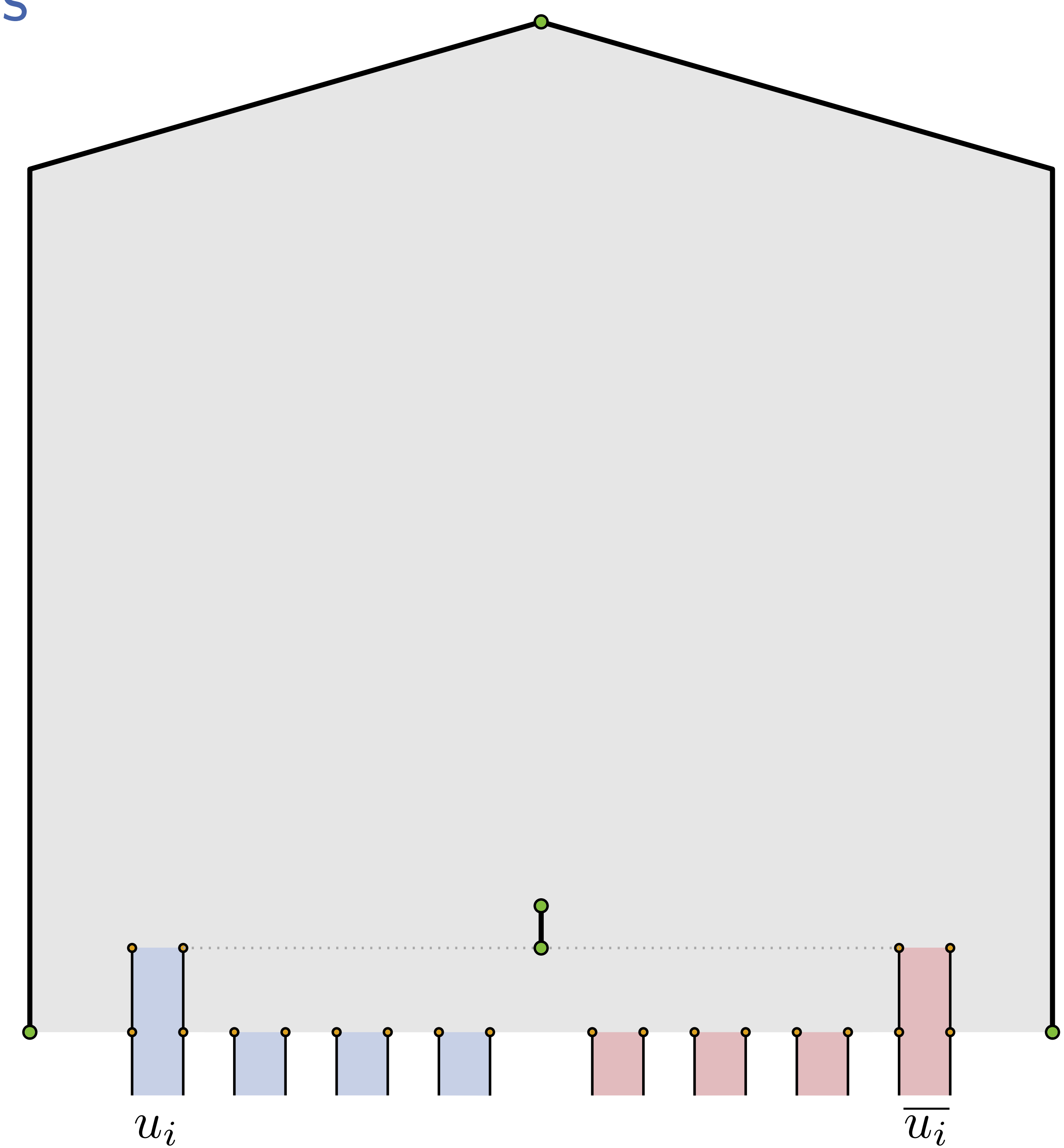

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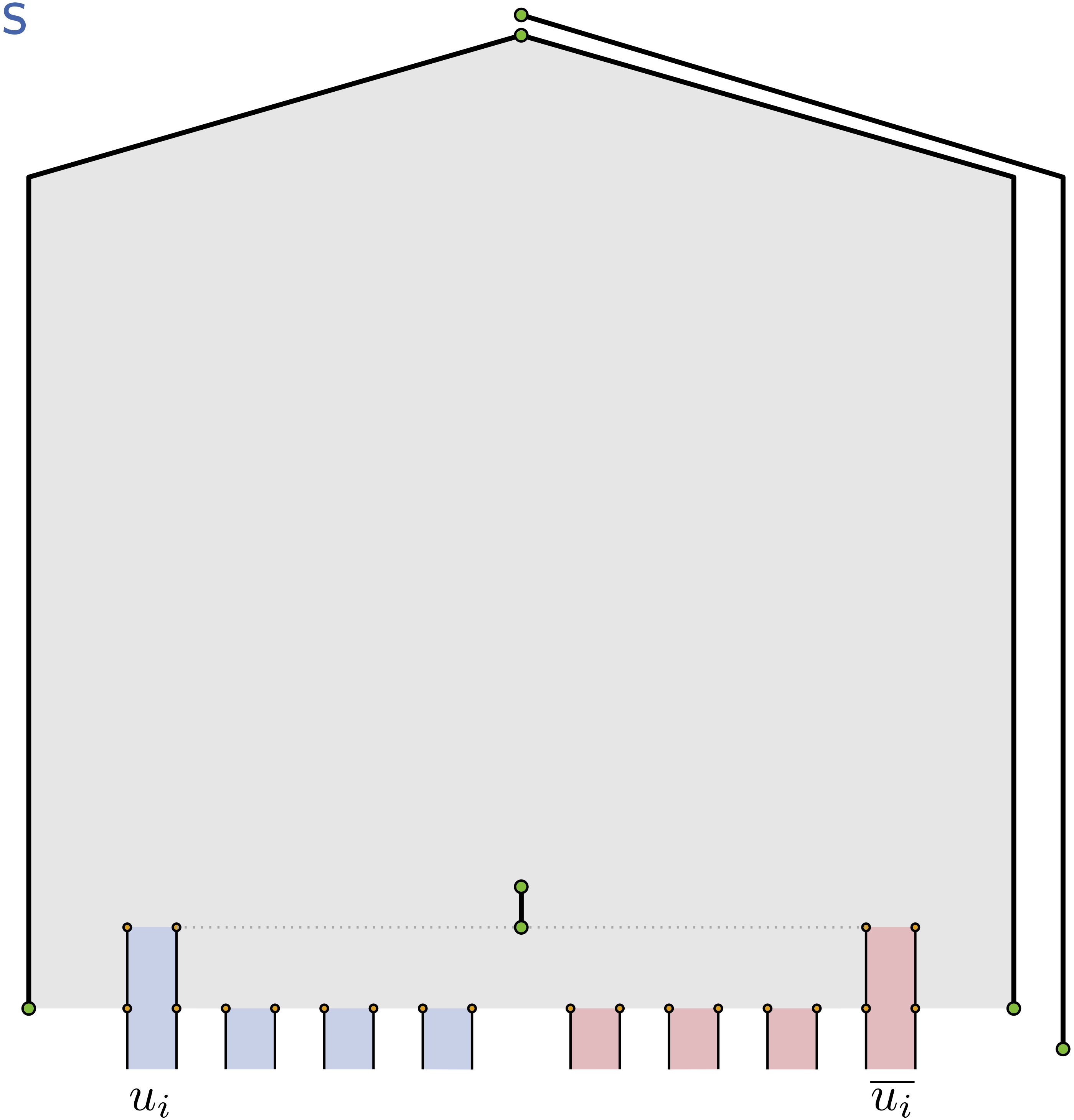

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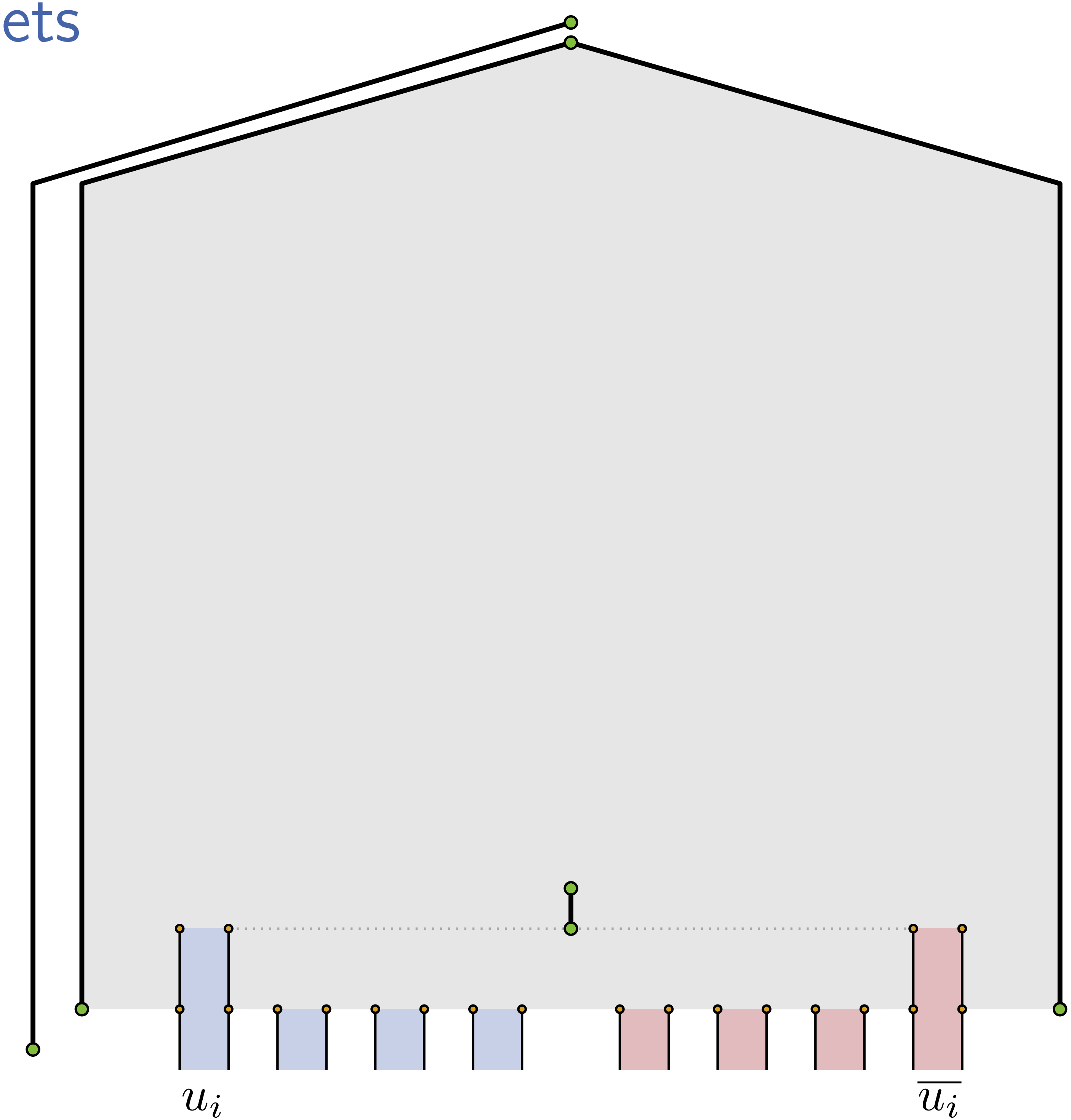

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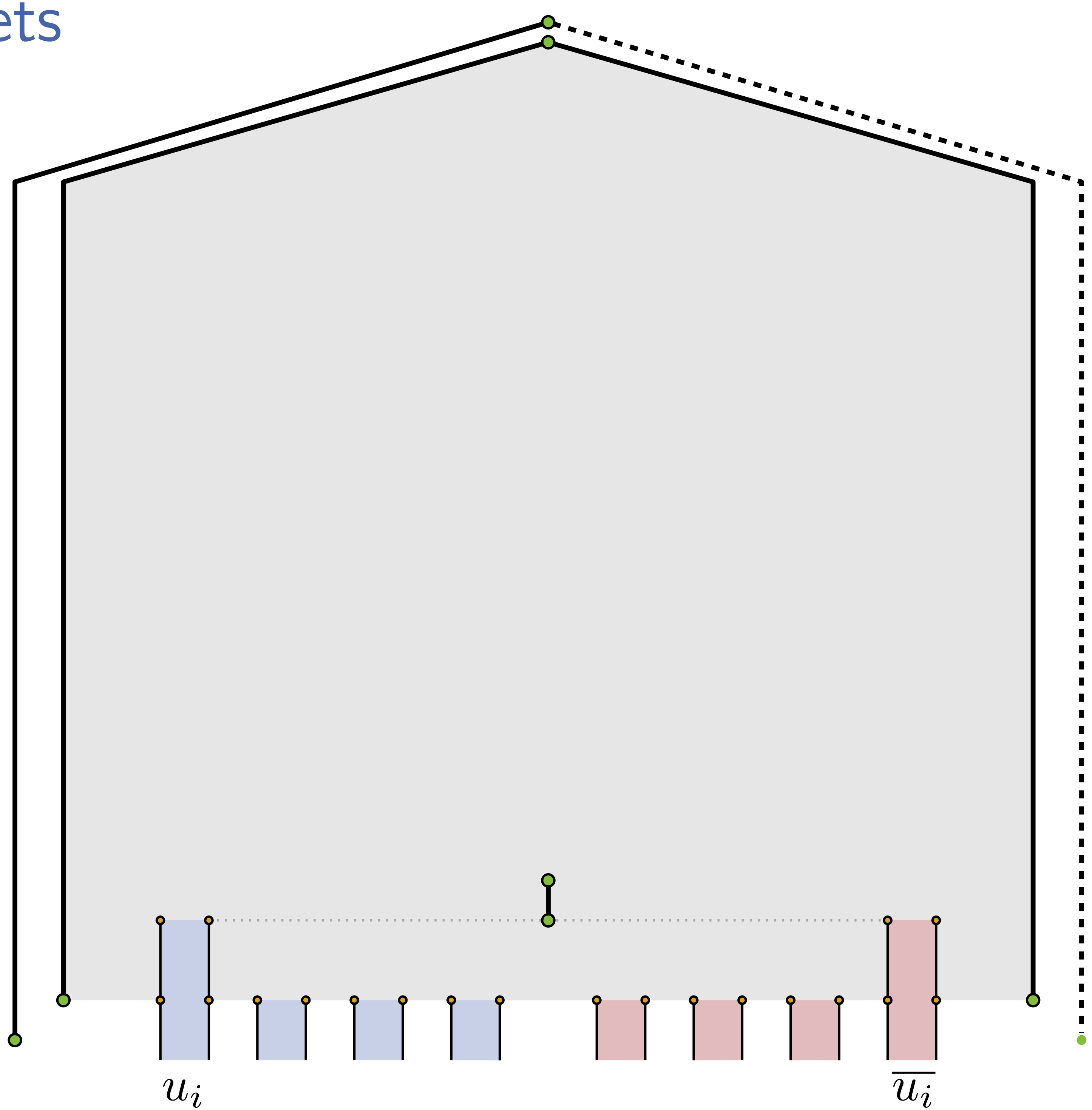

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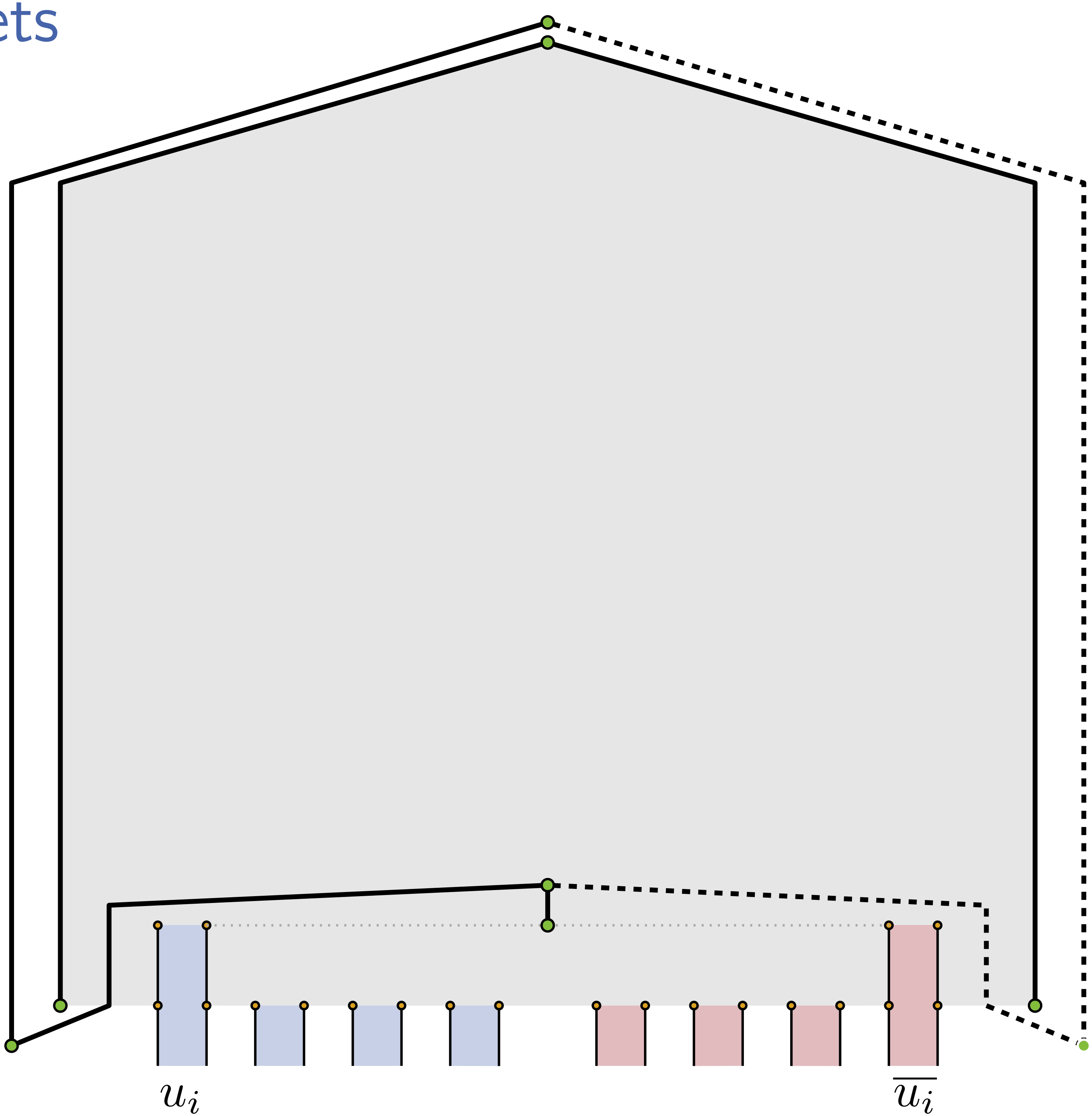

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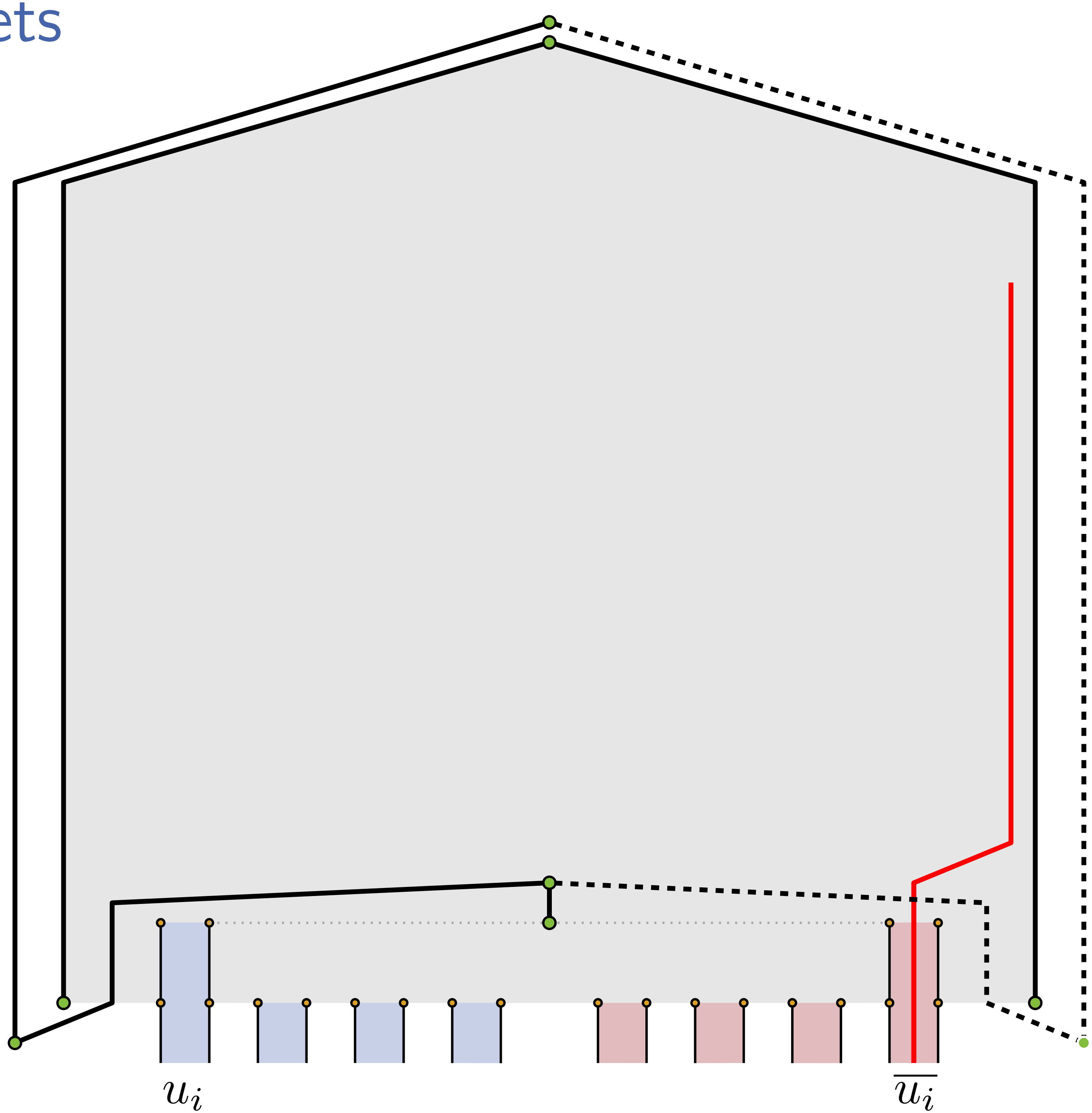

Variable Gadgets

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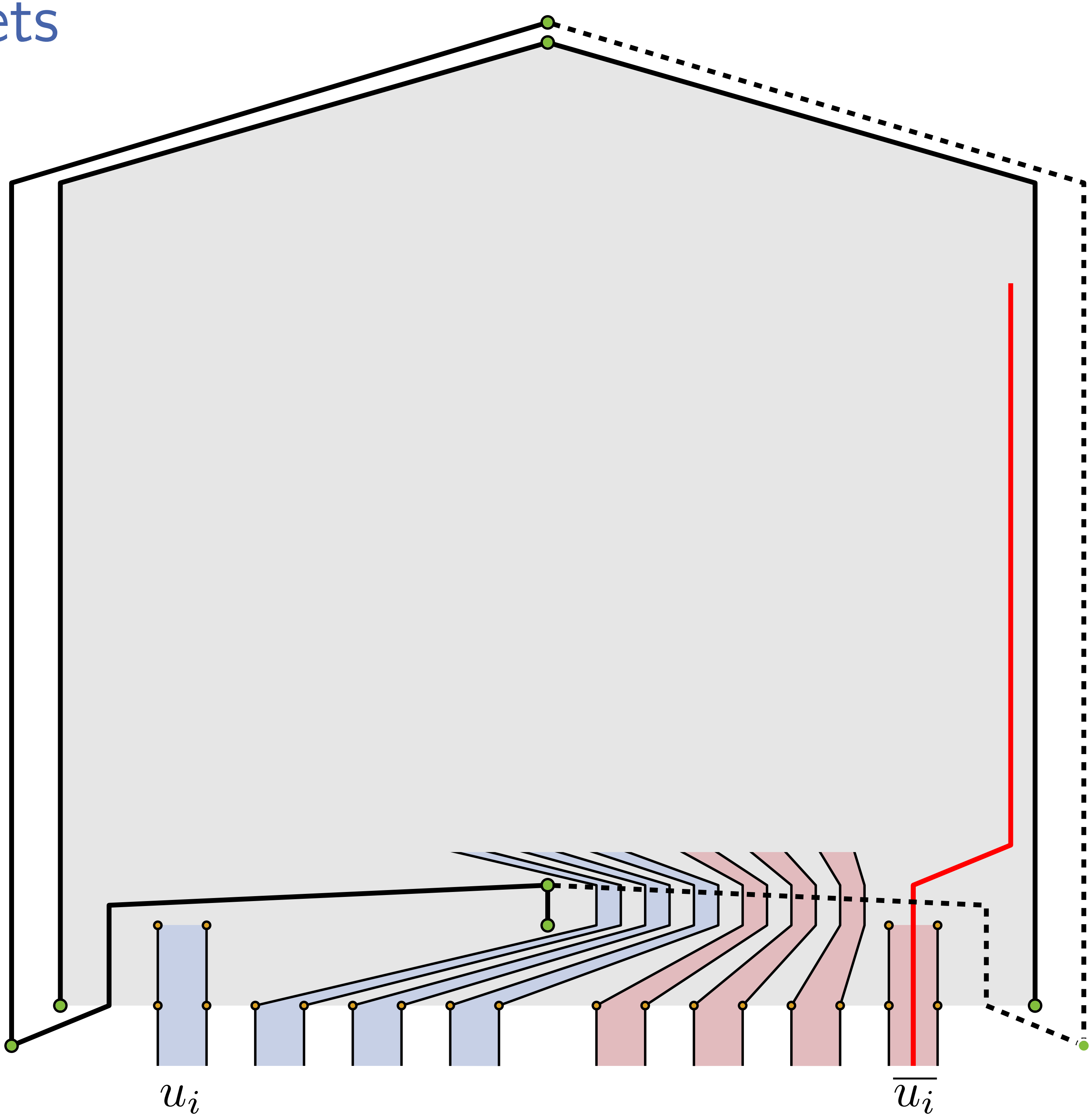

Variable Gadgets

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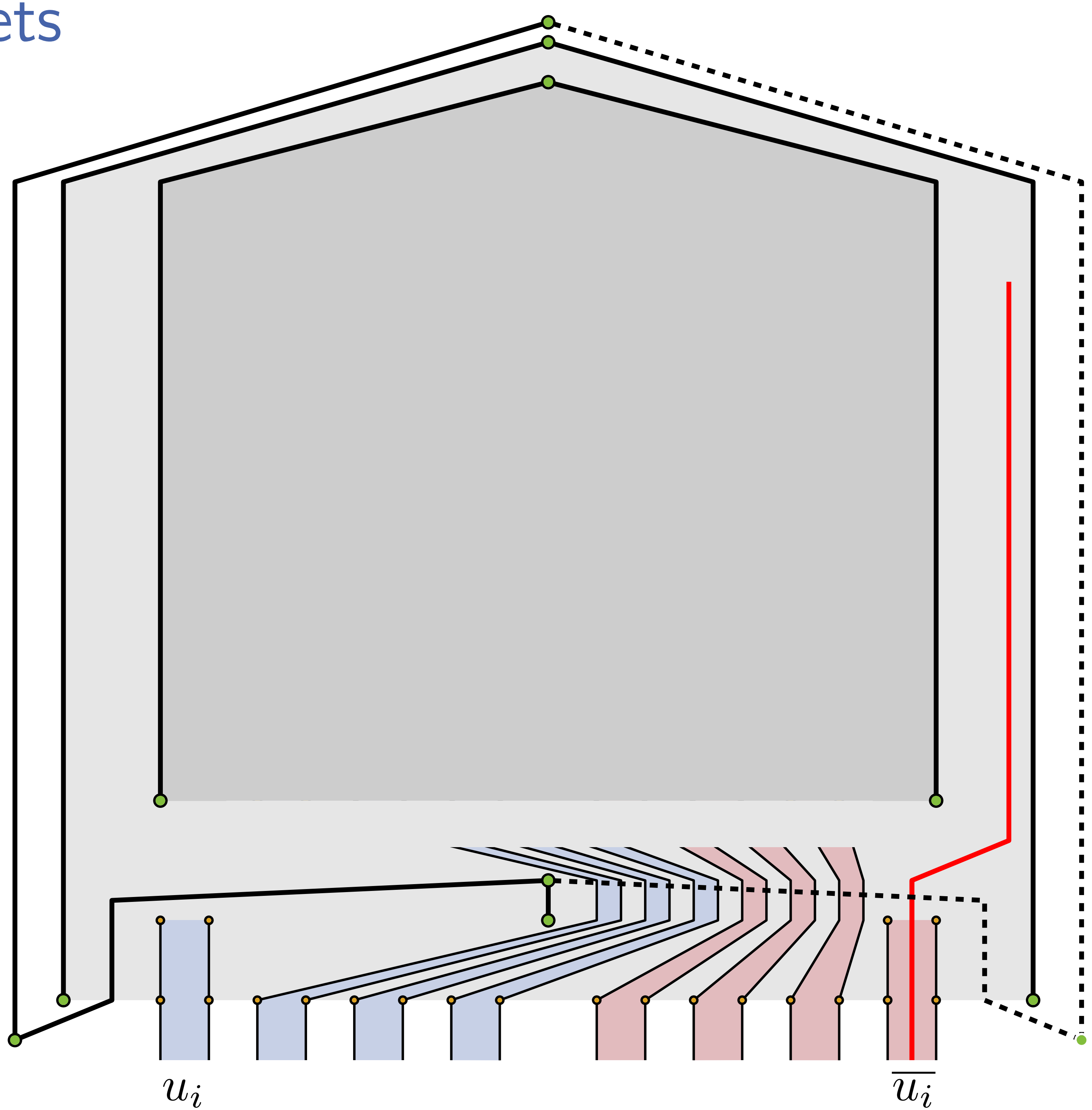

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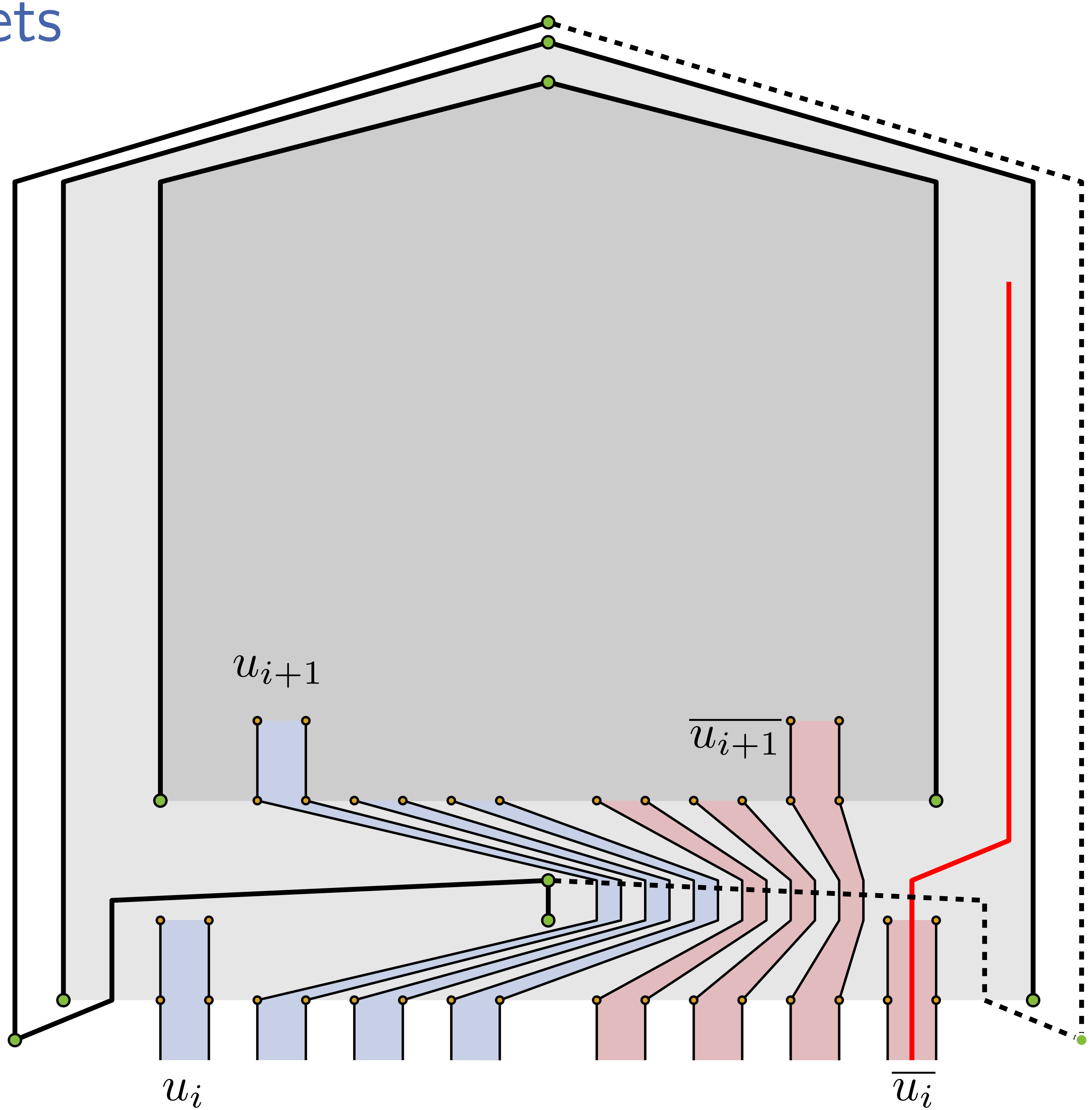

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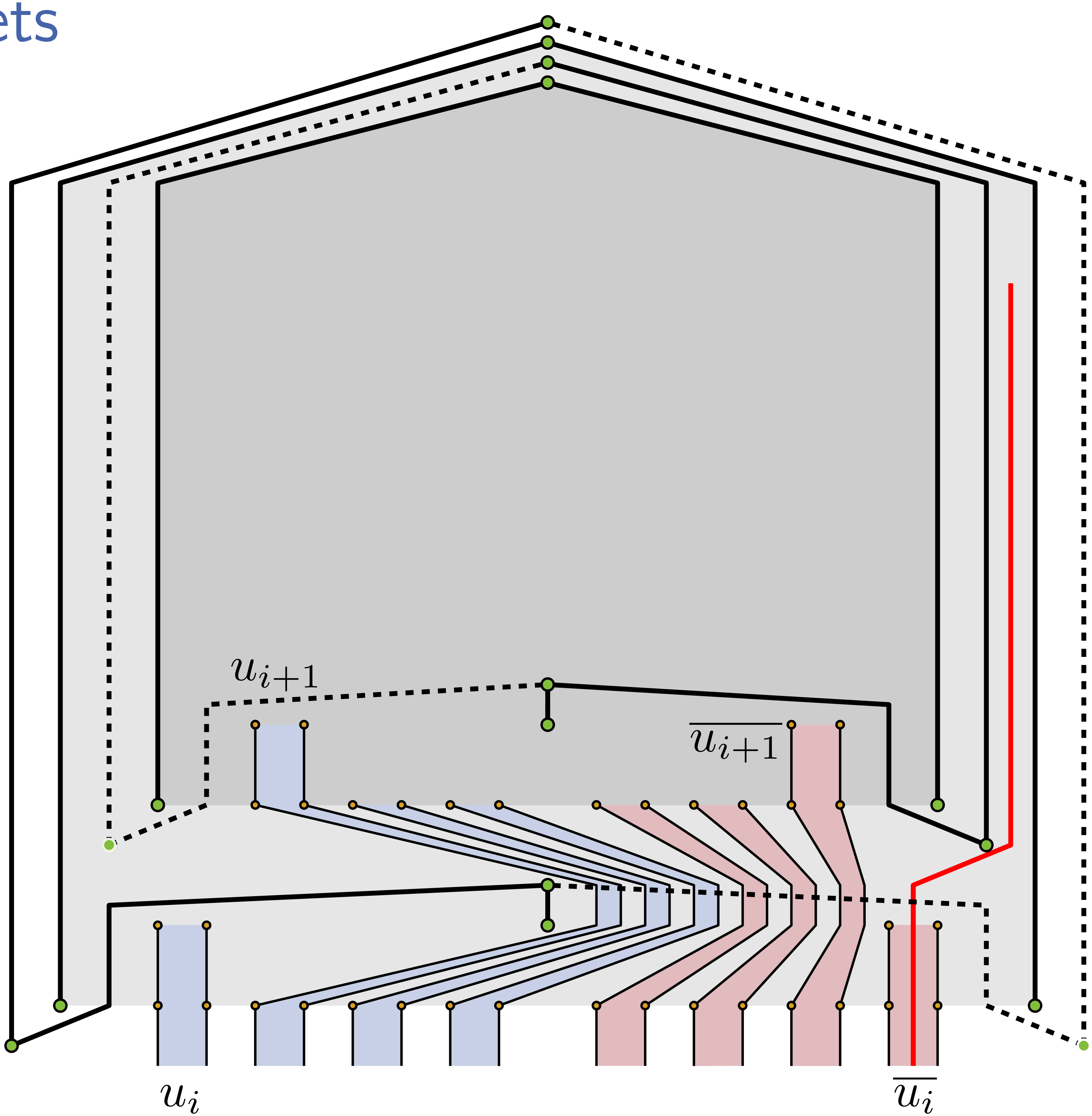

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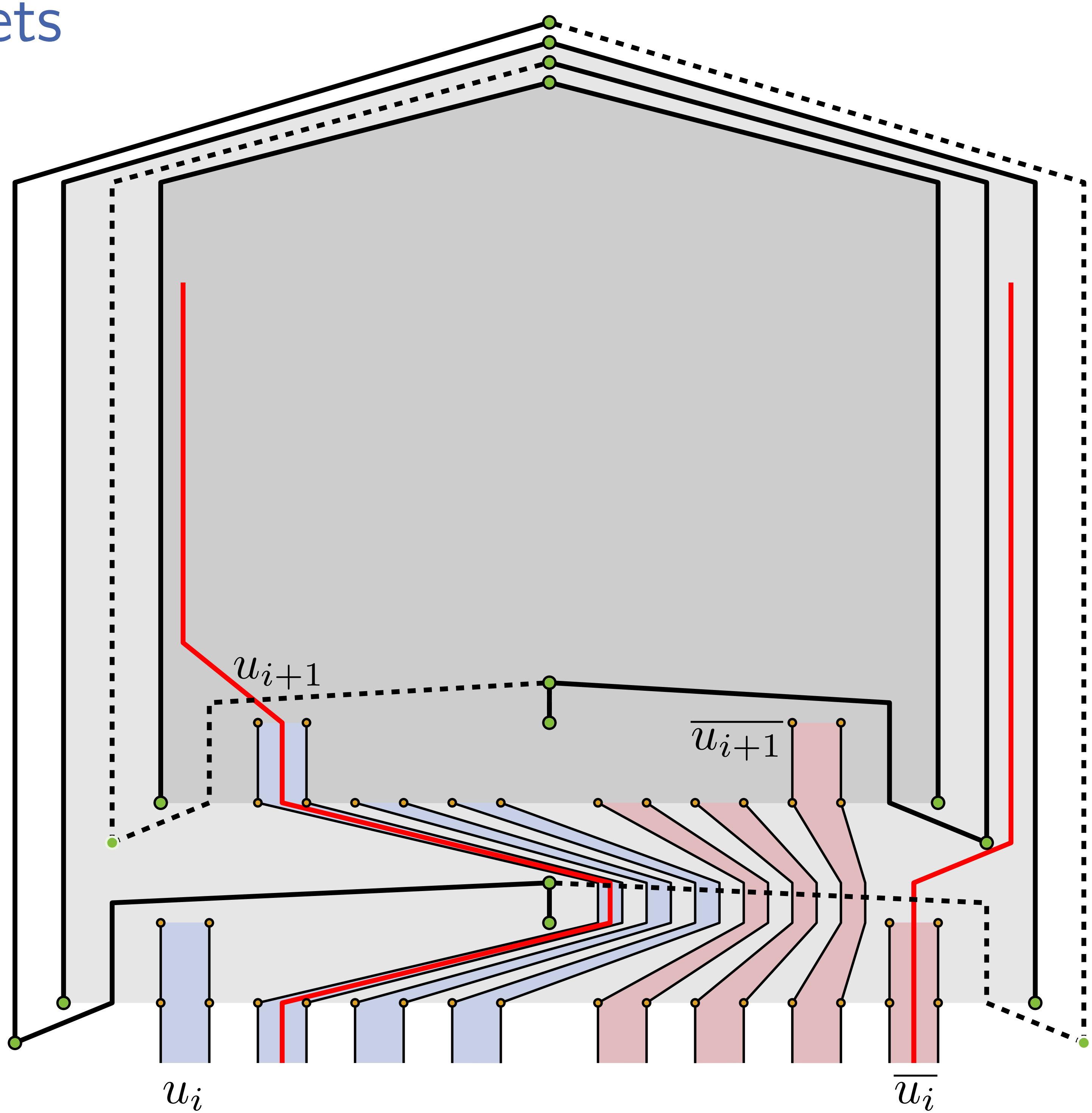

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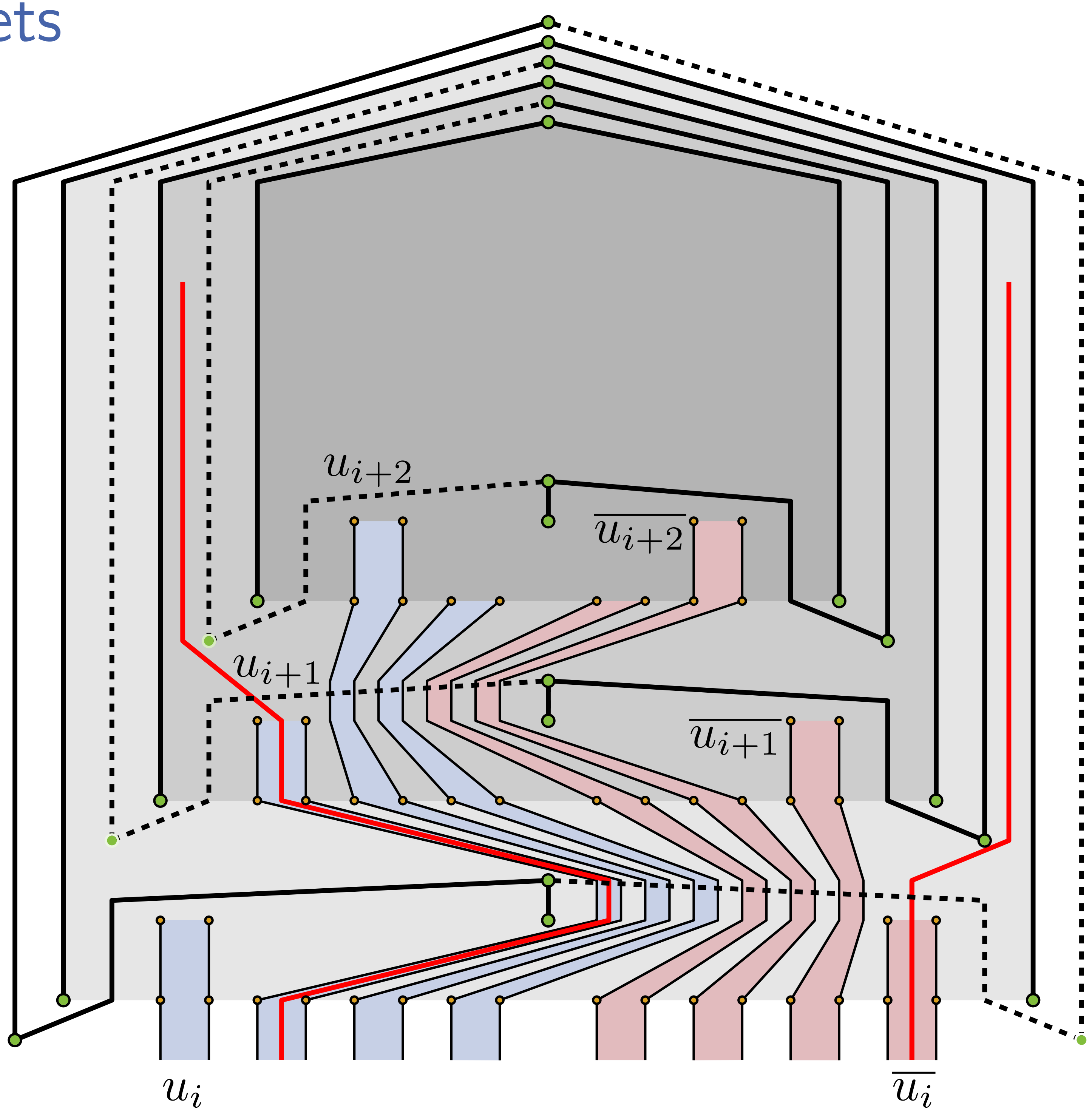

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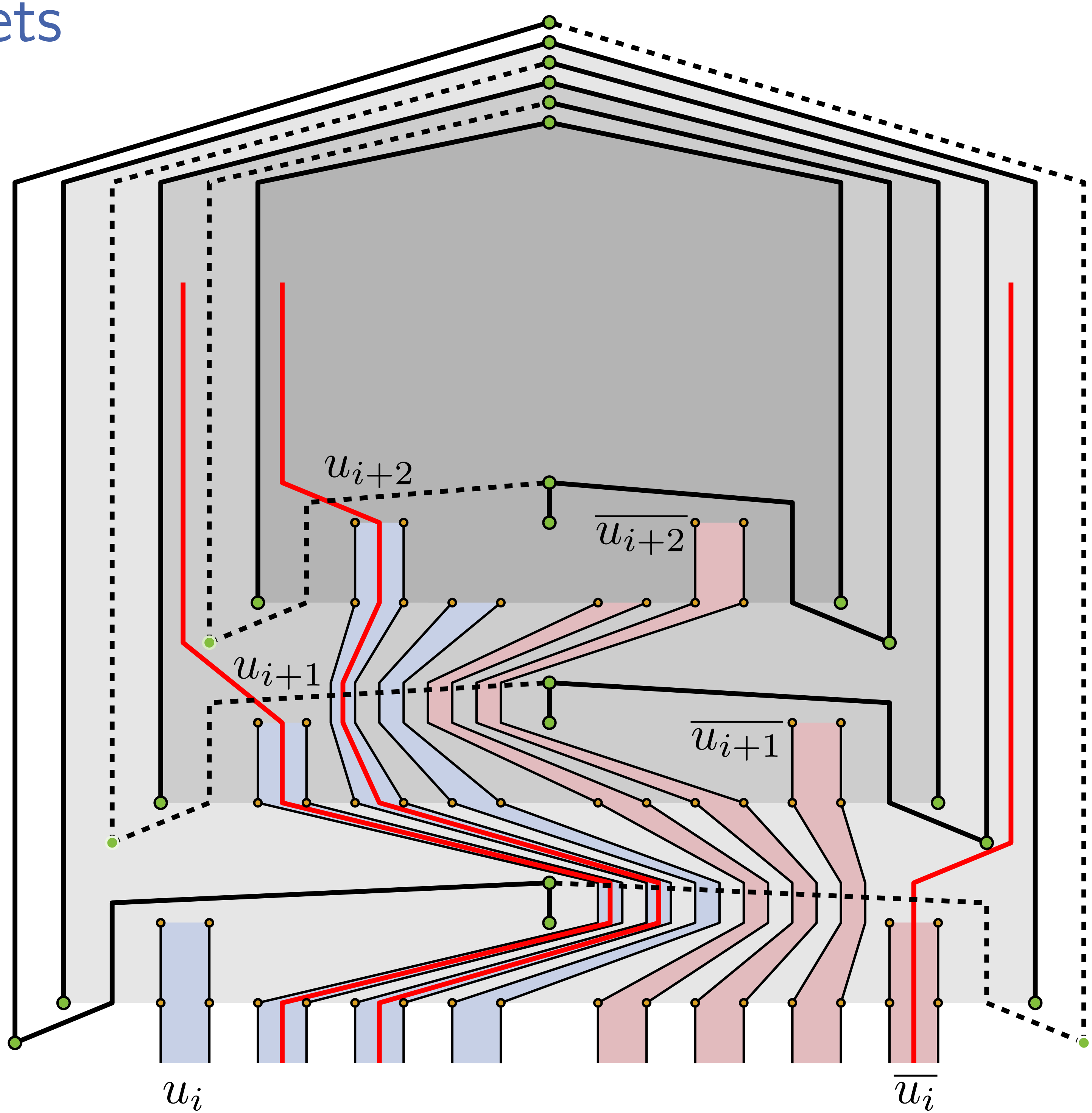

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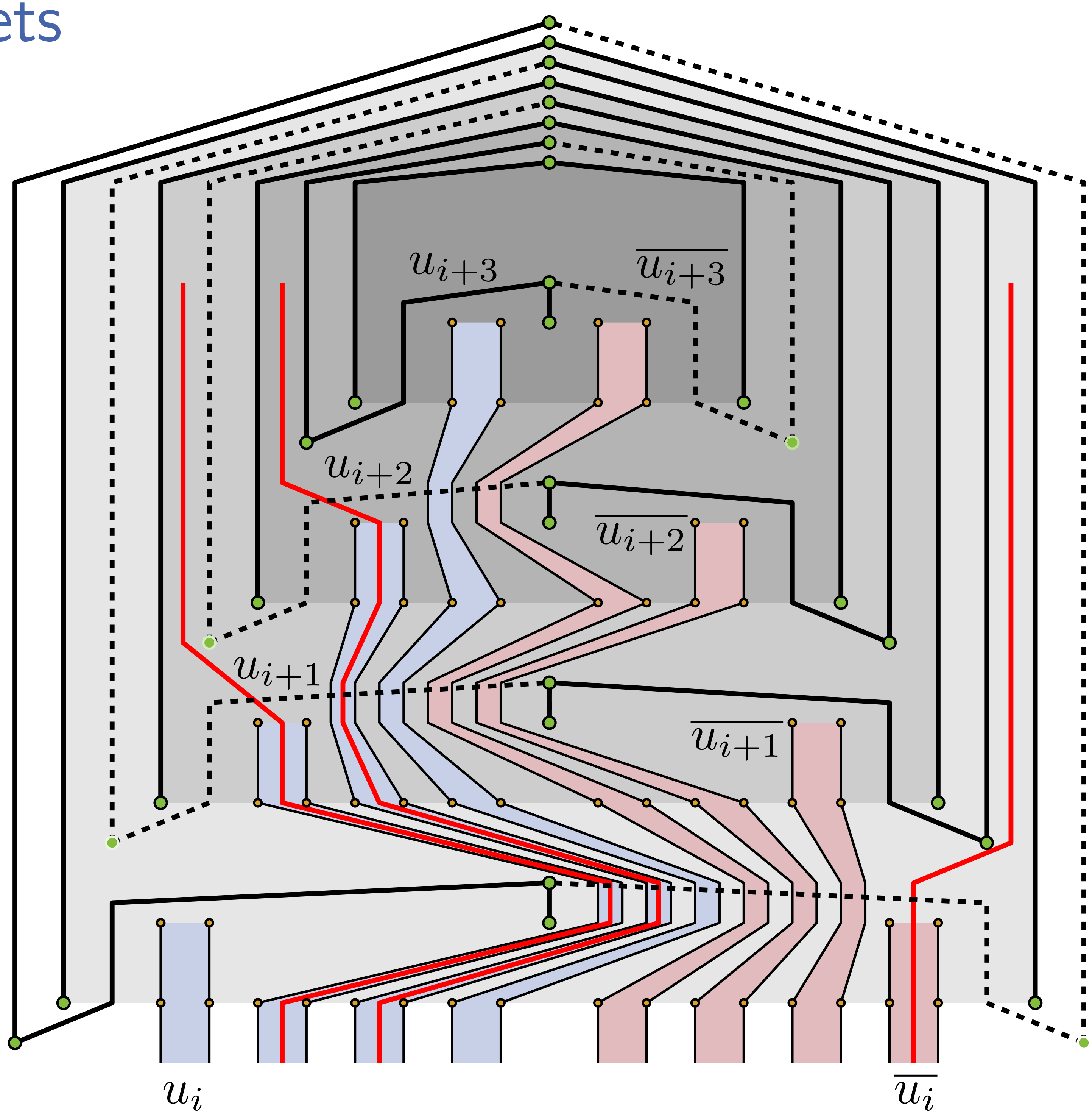

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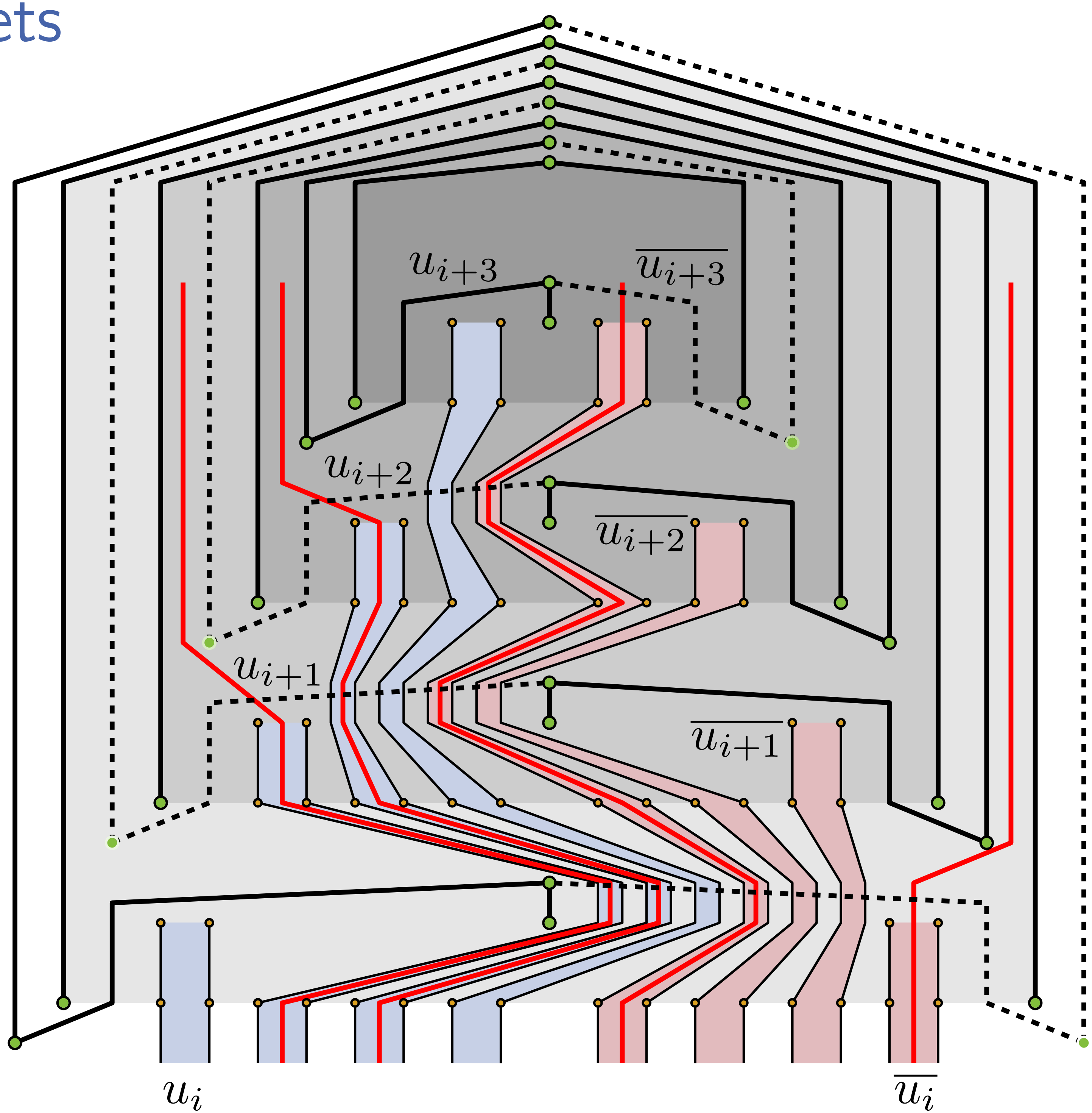

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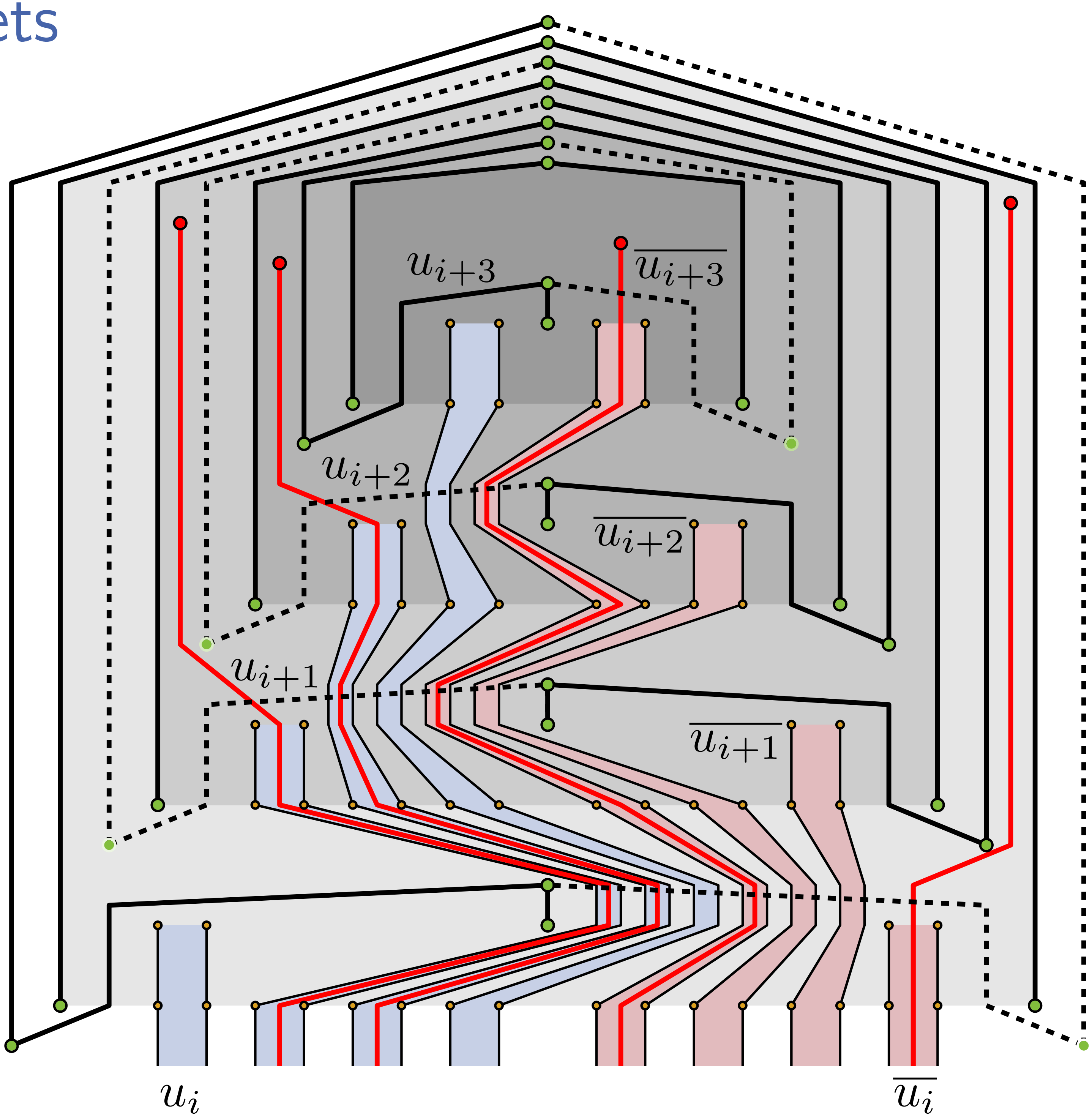
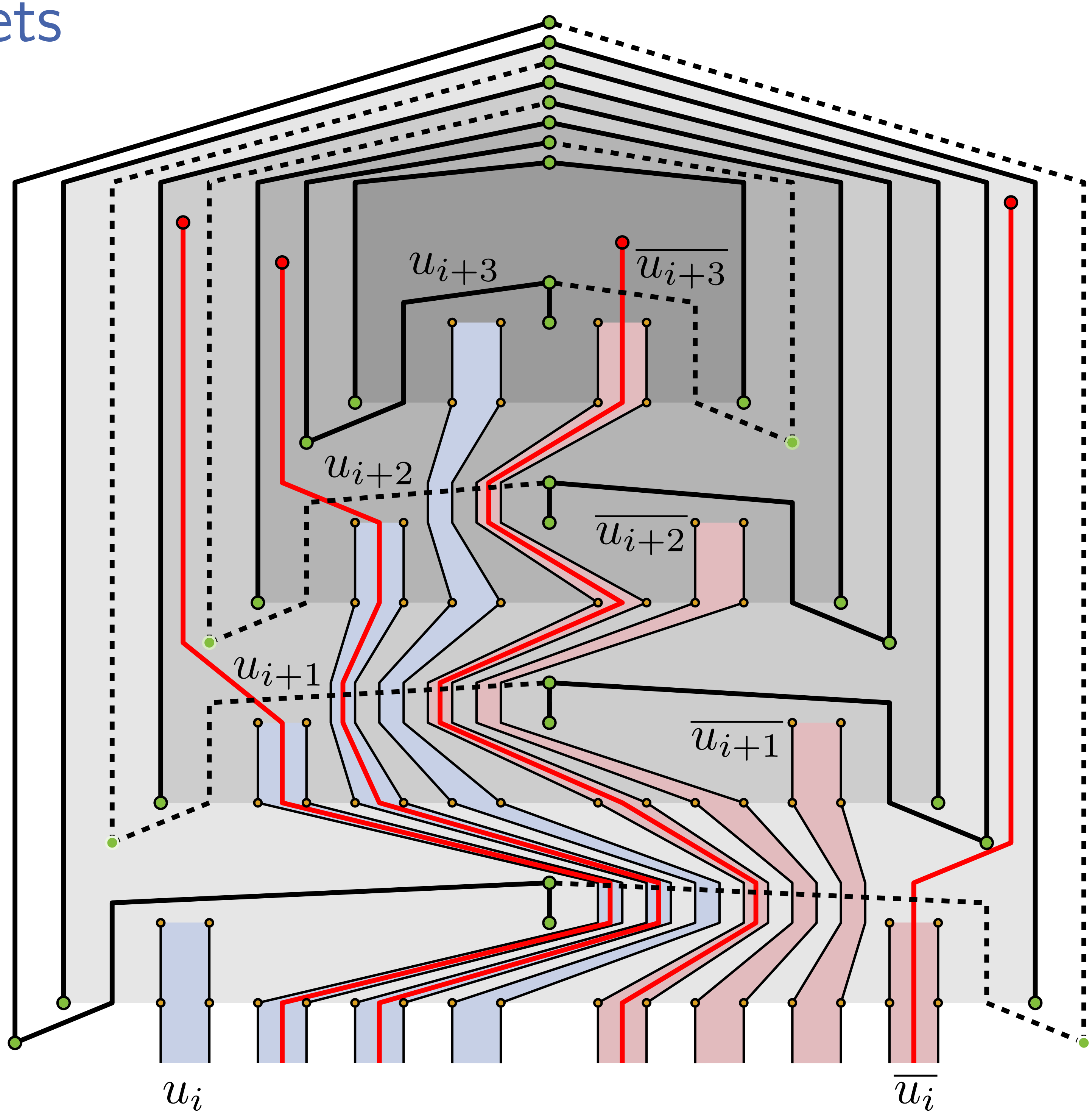


Variable Gadgets

Ensure that for every i either the tunnel u_i or $\overline{u_i}$ can be used, but not both!

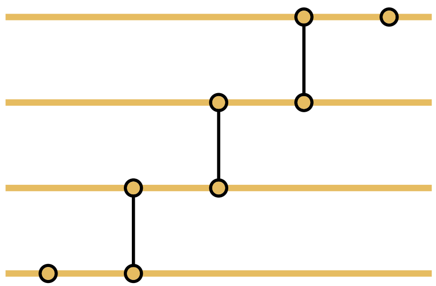
$\Delta = 2$ ✓

$\lambda = 2$?



Reducing to Level Width $\lambda = 2$

$\mapsto$



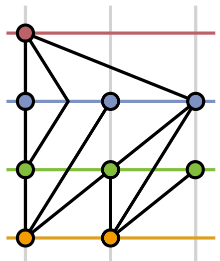
one level, $\lambda = k$

$k - 1$ levels, $\lambda = 2$

Result Overview

level-width λ = max. #vertices per level

Ordered Level Planarity

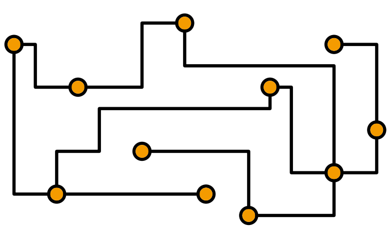


NP-complete even for $\Delta = \lambda = 2$

Polytime for $\Delta_{in} = \Delta_{out} = 1$ or $\lambda = 1$

$\leq$

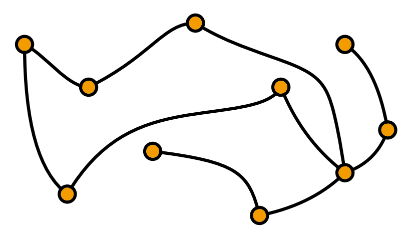
Geodesic Planarity



NP-hard even for matchings in general position

$\leq$

Bi-monotonicity

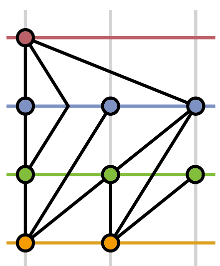


NP-hard even for matchings

Result Overview

level-width λ = max. #vertices per level

Ordered Level Planarity

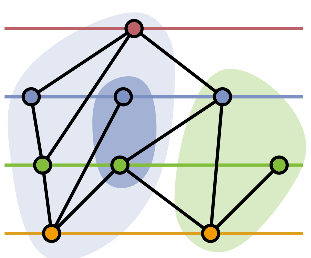


NP-complete even for $\Delta = \lambda = 2$

Polytime for $\Delta_{in} = \Delta_{out} = 1$ or $\lambda = 1$

$\leq$

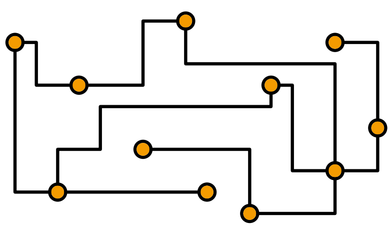
Clustered Level Planarity



NP-complete even for $\Delta = \lambda = 2$ and only 2 clusters

$\leq$

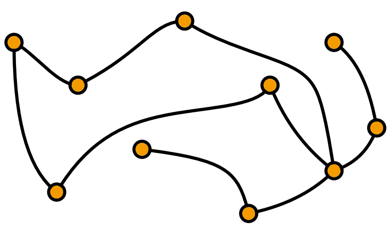
Geodesic Planarity



NP-hard even for matchings in general position

$\leq$

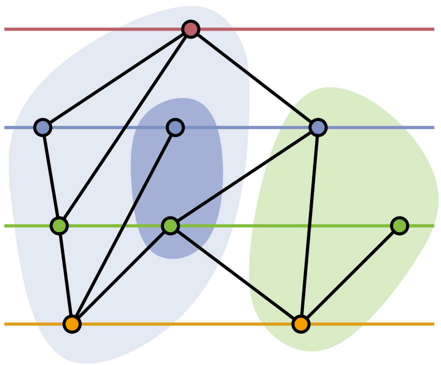
Bi-monotonicity



NP-hard even for matchings

Clustered Level Planarity

Combination of Level Planarity and Cluster Planarity.



[Forster,Bachmaier'04]

NP-complete [Angelini, Da Lozzo, Di Battista, Frati, Roselli'15]

Open: Complexity for flat clustering hierarchies [Angelini et al.'15]

Our result: NP-hardness for $\lambda = \Delta = 2$ and 2 clusters.

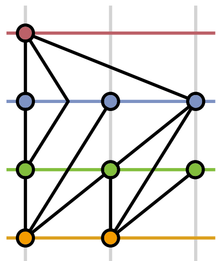
Poly-time algorithms for ...

... some proper instances [Forster,Bachmaier'04]

... all proper instances [Angelini et al.'15]

Result Overview

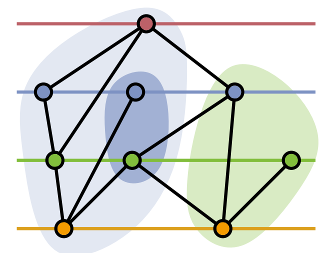
Ordered Level Planarity



NP-complete even for $\Delta = \lambda = 2$

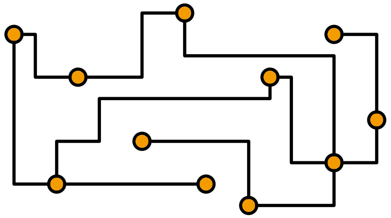
Polytime for $\Delta_{in} = \Delta_{out} = 1$ or $\lambda = 1$

Clustered Level Planarity



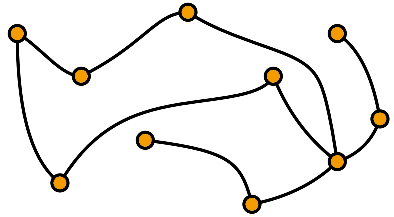
NP-complete even for $\Delta = \lambda = 2$ and only 2 clusters

Geodesic Planarity



NP-hard even for matchings in general position

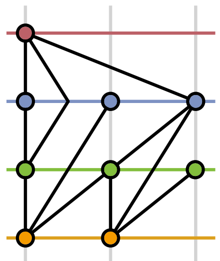
Bi-monotonicity



NP-hard even for matchings

Result Overview

Ordered Level Planarity

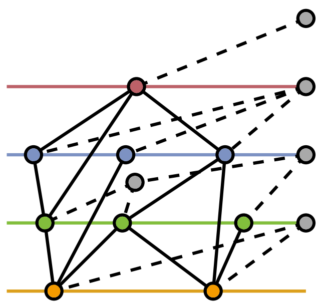


NP-complete even for $\Delta = \lambda = 2$

Polytime for $\Delta_{in} = \Delta_{out} = 1$ or $\lambda = 1$

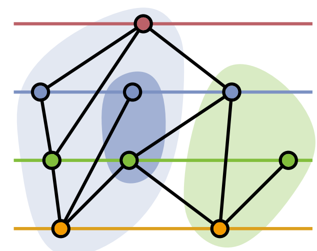
[Angelini et al.'15]

T-Level Planarity



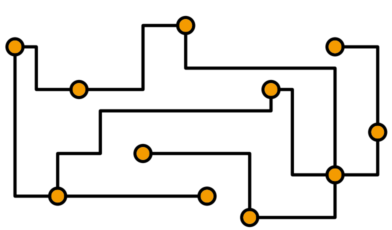
$\leq$

Clustered Level Planarity



NP-complete even for $\Delta = \lambda = 2$ and only 2 clusters

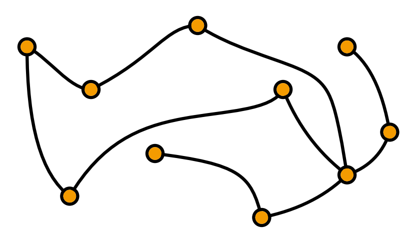
Geodesic Planarity



NP-hard even for matchings in general position

$\leq$

Bi-monotonicity

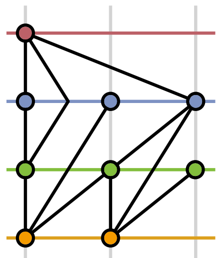


NP-hard even for matchings

Result Overview

[Brückner, Rutter'17]

Ordered Level Planarity



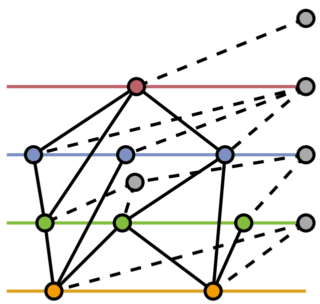
NP-complete even for $\Delta = \lambda = 2$

Polytime for $\Delta_{in} = \Delta_{out} = 1$ or $\lambda = 1$

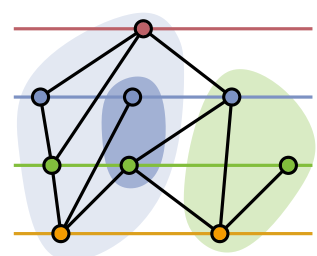
Constrained Level Planarity

NP-complete for $\Delta = \lambda = 2$ and total orders

T-Level Planarity

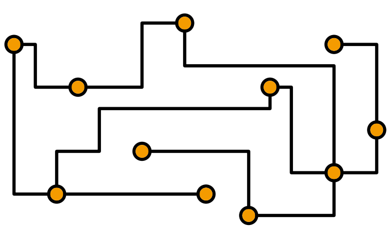


Clustered Level Planarity



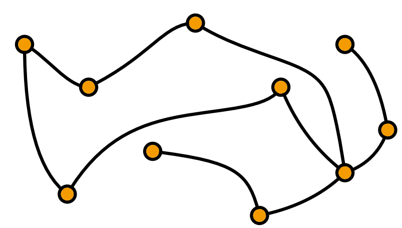
NP-complete even for $\Delta = \lambda = 2$ and only 2 clusters

Geodesic Planarity



NP-hard even for matchings in general position

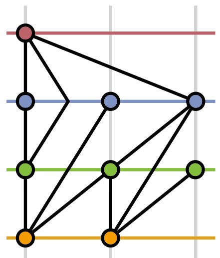
Bi-monotonicity



NP-hard even for matchings

Result Overview

Ordered Level Planarity



NP-complete even for $\Delta = \lambda = 2$

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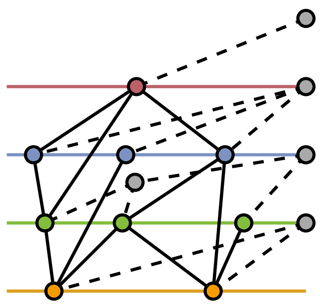
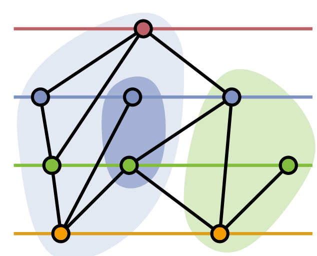
Constrained Level Planarity

NP-complete for $\Delta = \lambda = 2$ and total orders

T-Level Planarity

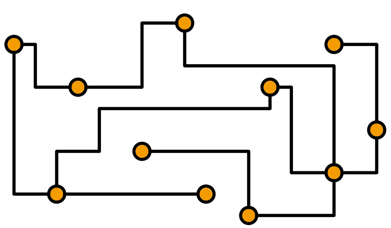


Clustered Level Planarity



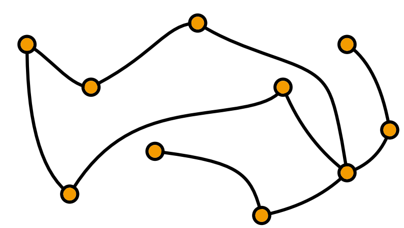
NP-complete even for $\Delta = \lambda = 2$ and only 2 clusters

Geodesic Planarity



NP-hard even for matchings in general position

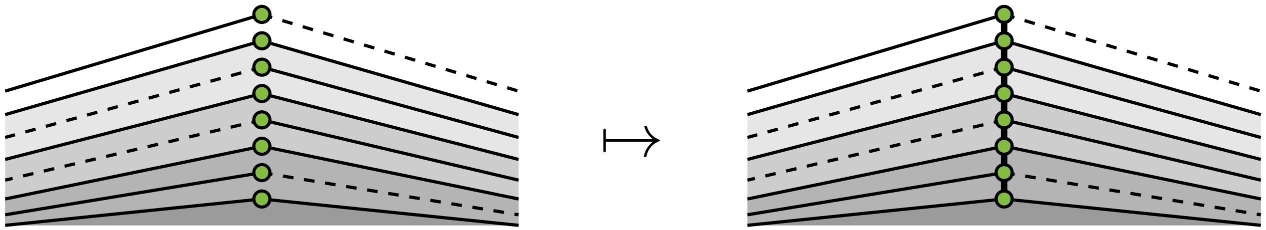
Bi-monotonicity



NP-hard even for matchings

Connectivity

NP-hardness of Ordered Level Planarity also holds for connected instances with $\Delta = 4$ and $\lambda = 2$



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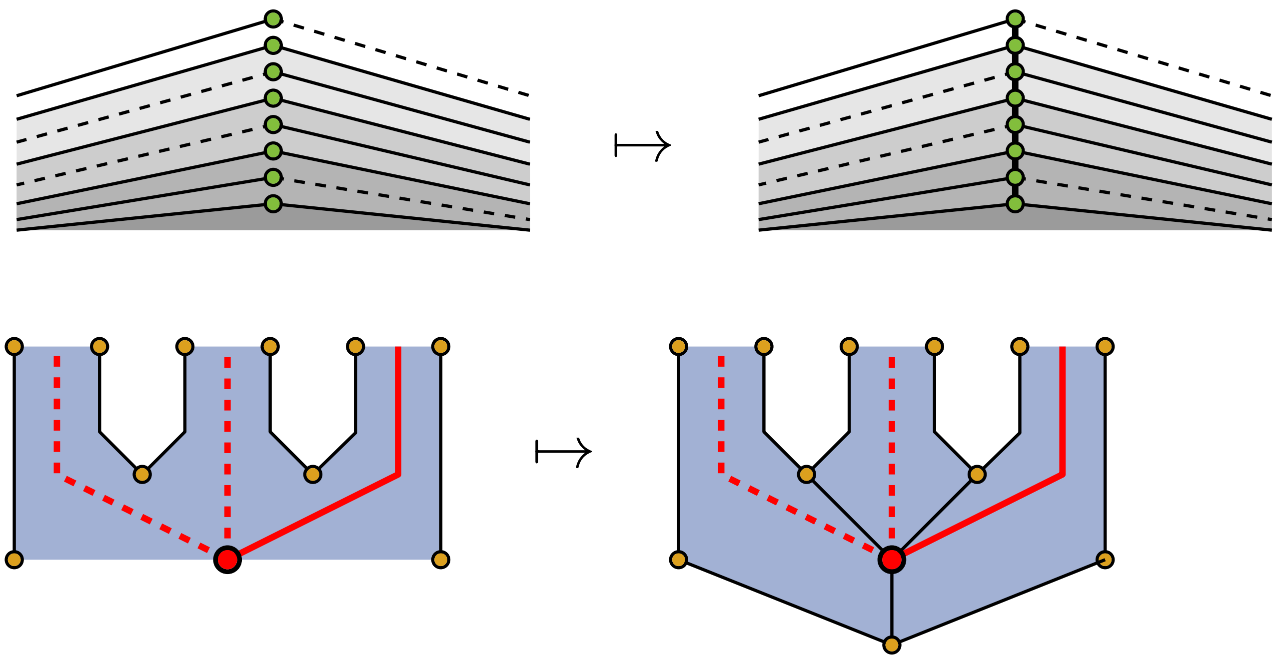
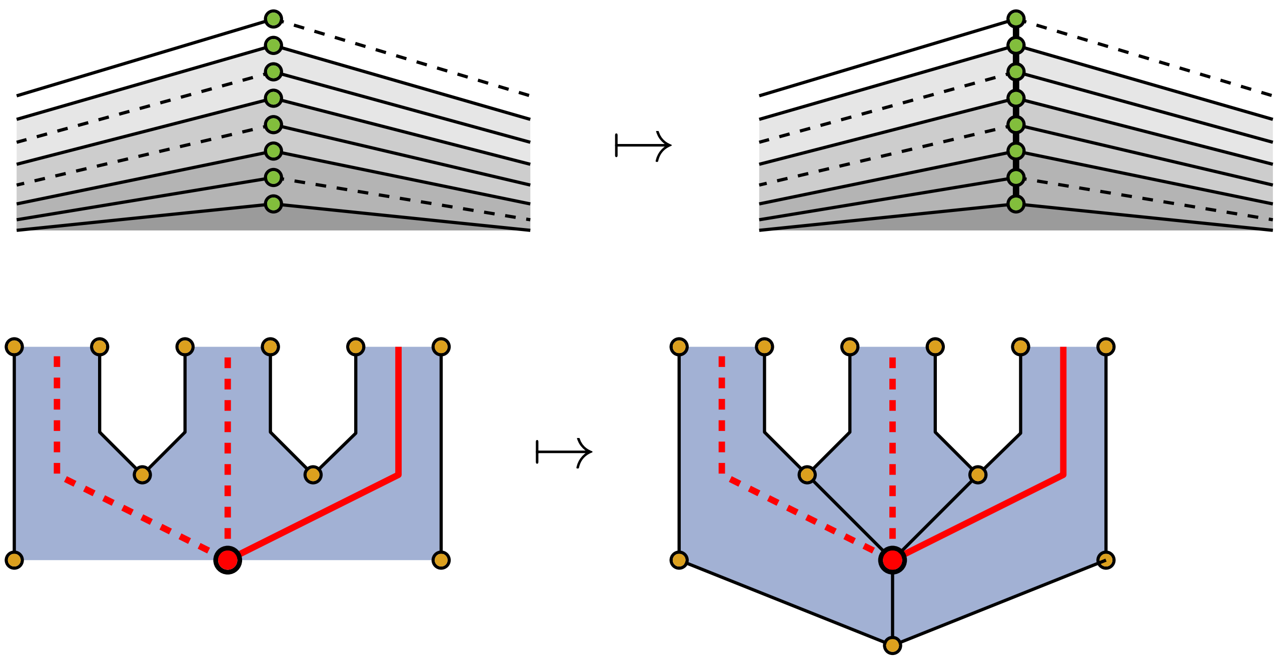


Connectivity

NP-hardness of Ordered Level Planarity also holds for connected instances with $\Delta = 4$ and $\lambda = 2$

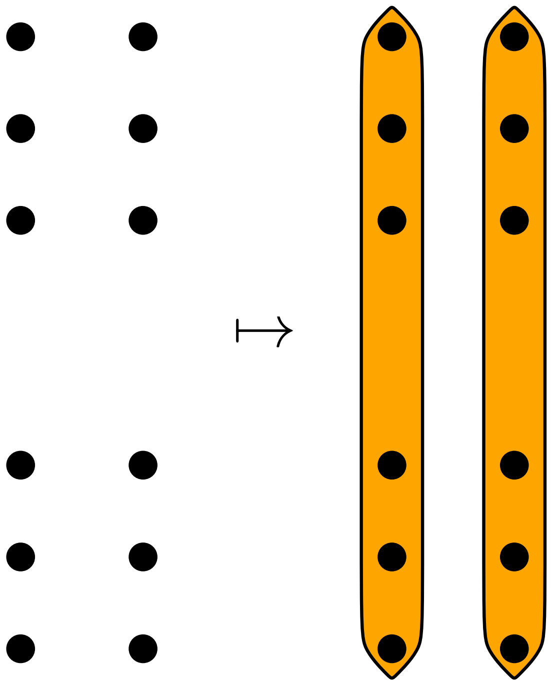


Open: Complexity of Manhattan Geodesic Planarity for connected instances

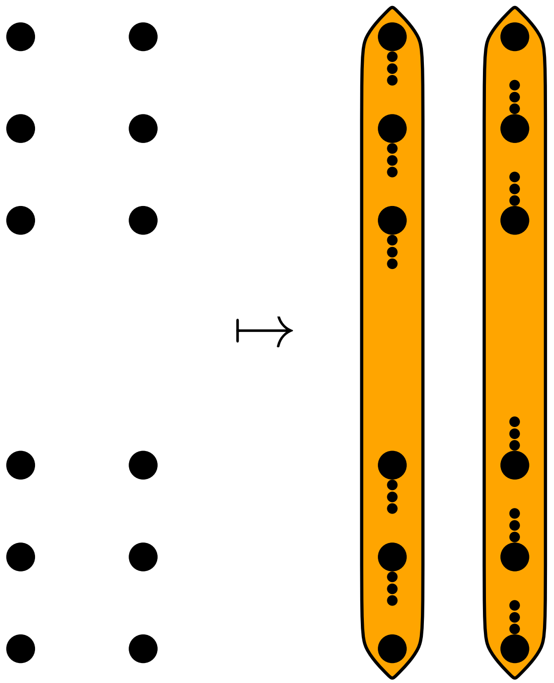
Problem:

$\Delta_{out} \leq 2$ $\qquad\qquad$ $\Delta_{out} \geq 3$

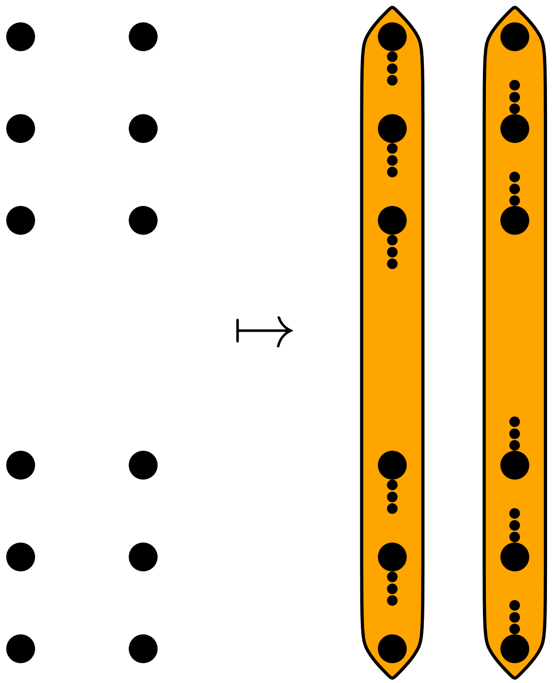
Clustered Level Planarity

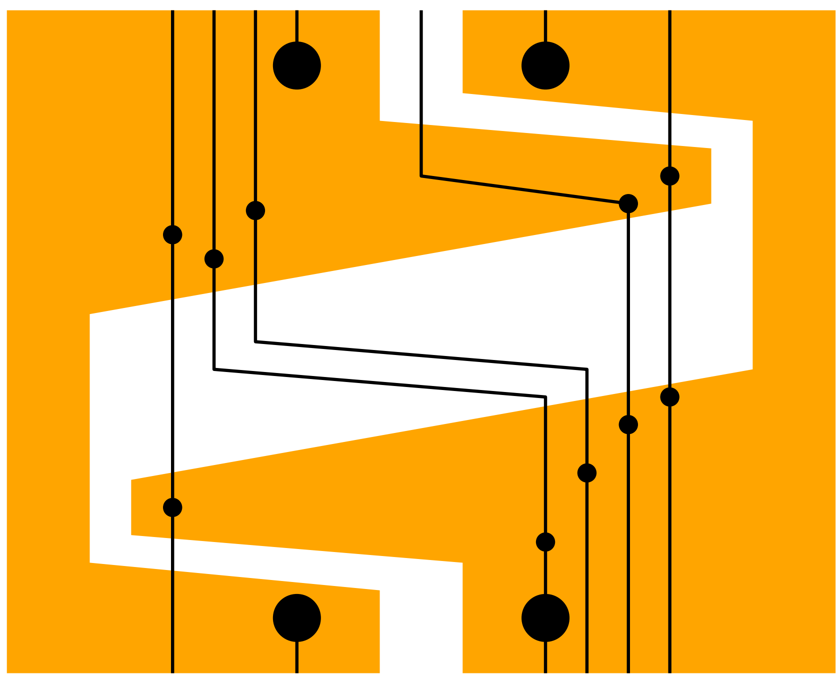


Clustered Level Planarity



Clustered Level Planarity

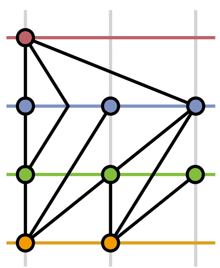




Result Overview

level-width $\lambda = \#$ vertices per level

Ordered Level Planarity



NP-complete even for $\Delta = \lambda = 2$

Polytime for $\Delta_{in} = \Delta_{out} = 1$ or $\lambda = 1$

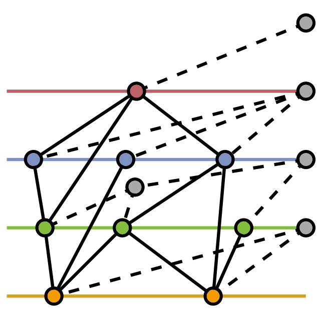
Constrained Level Planarity

NP-complete

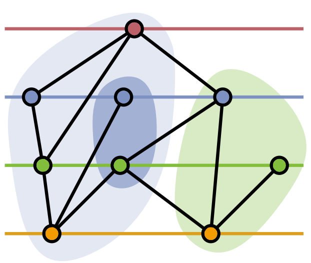
for $\Delta = \lambda = 2$

and total orders

T-Level Planarity



Clustered Level Planarity

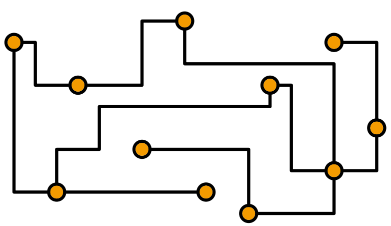


NP-complete even

for $\Delta = \lambda = 2$

and only 2 clusters

Geodesic Planarity



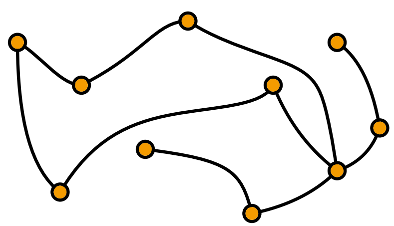
NP-hard even

for matchings

in general position

$\leq$

Bi-monotonicity



NP-hard even

for matchings